A New Model for Rail-Based Park-And-Ride with Feeder Bus Services

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Abstract

Many studies have been done to model and design park and ride scheme, since park and ride has long been considered as a strategy to alleviate traffic congestion and reduce emission. However, due to the difficulty in modelling an urban transport network system, research on park and ride still needs to overcome some limitations. This paper proposes a robust model by combining the driver’s mode choice and route choice, which are based on a combined cross nest logit (CNL) and user equilibrium (UE) model. Mathematical programming and variational inequality are used to solve a network optimization. The main contributions of this paper are: (1) considering travel time uncertainty on roads which can affect both modal split and route choice. (2) Taking multi-class demands into consideration because different people have different requirements for the travel time and level of service. In other words, mean-excess travel time (METT) model are proposed in this paper to depict uncertainty environment. Heterogeneity is considered by commuters’ variations of METT parameter which depends on travelers’ trip characteristic and income level.

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1. Introduction

PNR is a multi-mode travel mode that combines auto and public transport together (Parkhurst, 1995). Park and Ride (PNR) refers to a travelling mode that travelers first drive their vehicles to a parking site, access to a public transport facility, and take public transport to finish the remaining part of their trip (Noel, 1988).

The traffic congestion and air emission in CBD is a big challenge for its sustainable development (Dijk, de Haes, & Montalvo, 2013). In this sense, peripheral PNR is capable of solving these problems at some extent by attracting travelers who previously drive directly to the city center to park their vehicles at a parking site located in peripheral area of the city center and then take public transport to finish the remaining part of their trip (Cairns, 1998). It is obvious that if the private vehicle demand can be transferred to public transport, the traffic congestion should be reduced which ultimately leads to a reduction in emission (Duncan, 2010). In suburb areas with low residential density, PNR provides a better accessibility to public transport services. In other words, vehicles can be expanded to the catchment of public transport services (Cairns, 1998). In summary, PNR can positively improve the operational efficiency by reducing demand on the auto side (W. Liu, Yang, & Yin, 2014).

However, PNR did not always show its advantages in practice. The main problem lies on the demand split and route choice (X. Chen, Liu, Islam, & Deng, 2014). Many researches based on the statistical analysis of observations and surveys indicated that PNR in some places are unattractive to auto drivers, but public transport users (W. Liu et al., 2014). As the demand for auto mode is unreduced, traffic congestion and air emission are not mitigated (Truong & Marshall, 2014). Approaches to study PNR have been quite different such as survey, GIS, expert system, but this paper will focus on the network equilibrium approach to model the PNR (Meek, Ison, & Enoch, 2010). Since equilibrium theory has more advantages for it consider demand split and route choice together and evaluate the role of PNR on the urban transport network instead of just focus on single or several sites (X. Y. Chen, Chen, Liu, & Deng, 2014).

The route choice for people who choose PNR is also different from a general route choice which leads to a redistribution of traffic flow on the transport network (Parkhurst, 2000). As a result of flow redistribution, some problems such as longer travel distance trip generation and increased air pollution were identified (Duncan & Cook, 2014).

After discussion the general concept of PNR, it is also necessary to discuss the role of public transport which is an important part of PNR. Public transport services for PNR can be generally categorized into two parts: rail-based and bus-based. Rail-based PNR utilizes train or tram, while bus-based PNR utilizes bus. Compared to bus services, travel time on rail is more reliable. In other words, rail based public transport services are more reliable than bus based public transport services. Thus, this paper will focus on the rail based peripheral PNR.

Li (2007) utilized a combined modal split and traffic assignment with elastic demand to model PNR in a bi-modal transport network. To be specific, Li utilized a multi-nominal logit (MNL) model for the modal split and MNL based stochastic user equilibrium (SUE) for the route choice. Then, he built a variational inequality (VI) model to analysis PNR in an urban transport network and utilized a decomposition method to solve it. This paper believes that MNL cannot appropriately reflect the complex of mode choice of PNR because MNL cannot reflect the similarity between different modes (Papola 2004). Generally, studies for PNR consider three modes: auto, rail, and PNR. PNR is a combination of auto and rail, which has overlap with auto in the first part of their trip and with rail in the last part of their trip. It is unreasonable to treat them as independent. Therefore, cross nest logit (CNL) model for modal split is used (Marzano & Papola, 2008). CNL is capable of modelling a similarity as a generalization of the two levels hierarchical logit model (Marzano, Papola, Simonelli, & Vitillo, 2013). For route choice, this paper utilizes user equilibrium (UE) theory to reflect users’ route choice. Furthermore, this paper considers travel time uncertainty and multiclass demand.

Uncertainty is unavoidable for travelers making decisions, such as modal choice, route choice, and transfer location choice. In this paper, we propose the - reliable mean-excess travel time that explicitly considers both reliability and unreliability aspects of travel time variability in the route choice decision process. Unlike travel time budget model (TTB), it answers “how much time do I need to allow?” and “how bad should I expect from the worse cases?” (Chen & Zhou, 2010). Therefore, travelers’ behavior is evaluated in a more accurate way under an uncertain circumstance.
This paper is structured as follows: section 2 develops a mathematical programming model, discusses its mathematical property and solution algorithms. Section 3 describes the model by another modelling tool: Variational inequality (VI). Section 4 takes travel time uncertainty and multi-class into consideration. Section 5 makes discussion and conclusion of this paper.

2. Mathematical programming model

2.1. Assumption and notation.

This section shows formulate a MP model for combined model, which combine CNL and UE. Given a strongly connected and directed transportation network \( G ( N, A) \), network attributes are denoted by notations as follows:

\( N \): set of nodes
\( A \): set of links
\( A^a \): set of auto links
\( A^r \): set of rail links
\( W \): set of OD pairs
\( R_{od} \): Set of all the paths between OD pair \( od \)
\( q_{od} \): Travel demand between OD pair \( od \)
\( q_{om} \): Travel demand for mode \( m \) understand nest \( e \) between OD pair \( od \)

\( \mathbf{q} \): Column vector of all the OD travel demands, \( \mathbf{q} = (q_{od})^T \)

\( f_{k}^{od} \): Traffic flow on path \( k \in R_w \) between OD pair \( w \in W \)

\( f_{k,em}^{od} \): Traffic flow on path \( k \in R_w \) for mode \( m \) understand nest \( e \) between OD pair \( od \)

\( v_a \): Traffic flow on link \( a \in A \)

\( \mathbf{v} \): Column vector of all the link traffic flows, \( \mathbf{v} = (v, a \in A)_a^T \)

\( t_a (\mathbf{v}) \): Travel time on link \( a \in A \), and it is a function of link traffic flow vector \( \mathbf{v} \)

\( \mathbf{t}(\mathbf{v}) \): Column vector of all the link travel time functions, \( \mathbf{t}(\mathbf{v}) = (t_a (\mathbf{v}), a \in A)_a^T \)

2.2. Mathematical programming model and solution

The mathematical programming model can be formulated as follows:
Where $Z_1$ is the well known Beckman transformation for the user equilibrium problem (i.e., congestion effect); $Z_2$ and $Z_3$ are respectively the conditional and marginal entropy terms corresponding to the CNL two-level probability structure (Bekhor and Prashker 1999); Eq.(2) is the demand conservation constrains. Eq.(5) is the flow conservation constrains. Eq.(3) and eq.(4) are non-negative constraints.

**Proposition 1** The MP formulation in Eqs. (1) through (6) gives the modal split and link choice solution of the combined modal split (CNL) and traffic assignment (UE) model.

**Proposition 2** The demand and link flow solution of CNL-UE model is unique.

This model can be easily solved by a two stage algorithm proposed by Evans (Evans, 1976; Sheffi, 1985). The convenience property of the algorithm are also proved in this paper. The general steps of this algorithm are as follows:

**Step1:** Calculate the shortest path for each mode.

**Step2:** Calculate the auxiliary demand $q_{od}^a$

**Step3:** Assign auxiliary demand $q_{od}^a$ to the path based on all or nothing assignment and obtain auxiliary link flow $P_a$.
Step4: Obtain the descent direction \((\dot{q}^d_m - q^d_m), (\ddot{v}_a - v_a)\).

Step5: Line search to obtain the optimal step size. (golden section)

Step6: Based on the optimal step size, we got new solution \(v_a\) and \(q^d_m\)

Step7: Convergence test (difference between link flow).

2.3. VI model

We are going to develop a VI models which is equivalent to the above MP model. We rewrite this model, because VI is more general than MP. Based on VI, we are able to take more factors into consideration. For example, Crowdedness of public transport also be considered into this formula. This VI model is developed based on fundamental build up by the MP model.

According to network economics (Nagurney, 2013), MP can be rewrite into VI into two forms which comes as follows:

\[
\begin{align*}
&\langle C^d_{k,em}^T, f^d_{k,em} - f^d_{k,em}^* \rangle \\
&\quad + \left( \frac{\mu}{\theta} q^d_{em} \ln \left( \frac{q^d_{em}}{\left( \alpha^d_{em} \right)^\frac{1}{\theta}} \right) + \frac{1 - \mu}{\theta} \left( \sum_m q^d_{em} \right) \ln \left( \sum_m q^d_{em} \right) \right)^T \left( q^d_{k,em} - q^d_{k,em}^* \right) \geq 0
\end{align*}
\]

Where \(f^d_{k,em}\) and \(q^d_{em}\) subject to the eqs.(2) through (5).

Proposition 3 The VI formulation in Eq (7). gives the modal split and route choice solution of the combined modal split (CNL) and traffic assignment (UE) model.

Proposition 4 The demand solution of CNL-UE model is unique, the route choice solution of this model exists.

\[
\begin{align*}
&\langle C^d_a^T, v_a - v_a^* \rangle \\
&\quad + \left( \frac{\mu}{\theta} q^d_{em} \ln \left( \frac{q^d_{em}}{\left( \alpha^d_{em} \right)^\frac{1}{\theta}} \right) + \frac{1 - \mu}{\theta} \left( \sum_m q^d_{em} \right) \ln \left( \sum_m q^d_{em} \right) \right)^T \left( q^d_{k,em} - q^d_{k,em}^* \right) \geq 0
\end{align*}
\]

Where \(v_a\) and \(q^d_{em}\) subject to the eq.(2) through (6).

The difference between this two forms is that first VI model solves the route flow while the second model solves the link flow.

Proposition 5 The VI formulation in Eq. (8) gives the modal split and link choice solution of the combined modal split (CNL) and traffic assignment (UE) model.
**Proposition 6** The demand and link flow solution of CNL-UE model is unique.

We choose PC algorithm (He & Liao, 2002) to solve this VI models for they are one of effective algorithm which convergence at linear rate. PC algorithm is a one of Projection method (Meng & Liu, 2012), and already show its advantage in solving transportation assignment problems (Meng, Liu, & Wang, 2014).

### 2.4. Travel time uncertainty and multi-class.

Section headings should be left justified, bold, with the first letter capitalized and numbered consecutively, starting with the Introduction. Sub-section headings should be in capital and lower-case italic letters, numbered 1.1, 1.2, etc, and left justified, with second and subsequent lines indented. All headings should have a minimum of three text lines after them before a page or column break. Ensure the text area is not blank except for the last page.

In this section, we incorporate travel time uncertainty into consideration. Travel time uncertainty exert a huge influence for travelers’ travel decision (Mahmoud, Habib, & Shalaby, 2014). Travelers would rather to take a higher price (e.g. leave much earlier, take a detour, choose another mode) to avoid uncertainty to ensure their arrival punctual. In this paper, we choose mean-excess travel time (MET) to computer travel time uncertainty (A. Chen & Zhou, 2010).

\[ i_k^d (\alpha) = \mathbb{E} \left[ T_k^{rs} \bigg| T_k^{rs} \geq \hat{i}_k^d (\alpha) \right] \quad \forall k \in R^d \tag{9} \]

Where \( T_k^{rs} \) is a random travel time on route \( k \) from origin \( o \) to destination \( d \). \( \mathbb{E}[\cdot] \) is the expectation operator. \( \hat{i}_k^d (\alpha) \) is the TTB on route \( k \) from origin \( o \) to destination \( s \) defined by the travel time reliability chance constraint at a confidence level \( \alpha \) in eq.(10). (Du & Wang, 2014; Lo, Luo, & Siu, 2006). eq.(9) refers to the mean-excess travel time \( \hat{i}_k^d (\alpha) \) for a route \( k \in R^d \) between origin \( o \) to destination \( d \) with a predefined confidence level \( \alpha \) is equal to the conditional expectation of the travel time exceeding the corresponding route \( \hat{i}_k^d (\alpha) \).

\[ \hat{i}_k^d (\alpha) = \min \left\{ \zeta \left| \mathbb{P}(T_k^{rs} \leq \zeta) \geq \alpha \right. \right\} = \mathbb{E}(T_k^{rs}) + \gamma_k^{rs}(\alpha) \quad \forall k \in R^d \tag{10} \]

where \( \zeta \) is the travel time budget (a threshold value in the probabilistic constraint) required to ensure on-time arrival at a confidence level \( \alpha \), and \( \gamma_k^{rs}(\alpha) \) is the extra time added to the mean travel time as a ‘buffer time’ to ensure more frequent on-time arrivals at the destination under the travel time reliability requirement at a confidence level \( \alpha \). Generally we assume \( T^{rs} \) follow normal distribution \( N(\mu, \theta^2) \) where \( \mu \) and \( \theta^2 \) are parameter waiting to be calibrate. Due to the string monotony of \( i_k^d (\alpha) \), we can simple replace \( c_k^{rs} \) in eq.(7) with \( \hat{i}_k^d \). Then, we get the combined CNL and UE model with \( \alpha \)-reliable mean-excess stochastic travel times.

Heterogeneity across commuters is considered in this study. A variety of characteristics can be used to define the heterogeneity among commuters. We mainly focus on the commuters’ variations of confidence level \( \alpha \). Therefore, the commuters can be classified into \( G \) groups with different heterogeneity parameters, indexed by \( g = 1, 2, 3...G \). By simply dividing users into several classes with different confidence level \( \alpha \), we get a multi-class combined CNL and UE model with \( \alpha \)-reliable mean-excess stochastic travel times.
3. Conclusion

In this paper, a multi-class combined CNL and UE model with $\alpha$-reliable mean-excess stochastic travel times was developed to model PNR in a bi-modal urban transport network. The combined modal split and traffic assignment model are the fundamental of this model. With this model, we are able to get the pattern of modal split and route choice for the whole urban transport network which is an extremely powerful tool for evaluating the potential performance of PNR and optimizing the design of PNR. In this paper, we adopt CNL for modal split and considered stochastic travel time. With this improvement, it can be expected that we can obtain more reasonable mode choice and route choice pattern of the urban transport system, which is fundamental for planning and designing a PNR scheme in a city.

In conclusion, the contribution of this paper is 1) Utilize CNL to model mode similarity. 2) Proposes a VI formulated mode to combine mode split and route choice. 3) Consider travel time uncertainty which influence route choice greatly. 4) Heterogeneity are also modelled in this paper.

Compare to other paper which study PNR, this paper are able to get a more reasonable result because it studied three important factors: mode similarity, travel time uncertainty, and heterogeneity. This model are useful in the phrase of planning PNR schemes. During this phrase, we are very interested in the pattern of travel choice and distribution of users. This model are capable of providing planners sufficient information on the influence of PNR schemes in a urban transport network.

References


