Characterization of interfacial waves and pressure drop in horizontal oil-water core-annular flows

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(Received 23 May 2017; accepted 29 July 2017; published online 18 August 2017)

We study the transportation of highly viscous furnace-oil in a horizontal pipe as core-annular flow (CAF) using experiments. Pressure drop and high-speed images of the fully developed CAF are recorded for a wide range of flow rate combinations. The height profiles (with respect to the centerline of the pipe) of the upper and lower interfaces of the core are obtained using a high-speed camera and image analysis. Time series of the interface height are used to calculate the average holdup of the oil phase, speed of the interface, and the power spectra of the interface profile. We find that the ratio of the effective velocity of the annular fluid to the core velocity, \( \alpha \), shows a large scatter. Using the average value of this ratio (\( \alpha = 0.74 \)) yields a good estimate of the measured holdup for the whole range of flow rate ratios, mainly due to the low sensitivity of the holdup ratio to the velocity ratio. Dimensional analysis implies that, if the thickness of the annular fluid is much smaller than the pipe radius, then, for the given range of parameters in our experiments, the non-dimensional interface shape, as well as the non-dimensional wall shear stress, can depend only on the shear Reynolds number and the velocity ratio. Our experimental data show that, for both lower and upper interfaces, the normalized power spectrum of the interface height has a strong dependence on the shear Reynolds number. Specifically, for low shear Reynolds numbers, interfacial modes with large wavelengths dominate, while, for high shear Reynolds numbers, interfacial modes with small wavelengths dominate. Normalized variance of the interface height is higher at lower shear Reynolds numbers and tends to a constant with increasing shear Reynolds number. Surprisingly, our experimental data also show that the effective wall shear stress is, to a large extent, proportional to the square of the core velocity. Using the implied scalings for the holdup ratio and wall shear stress, we can derive an expression for the pressure drop across the pipe in terms of the flow rates, which agrees well with our experimental measurements. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4998428]

I. INTRODUCTION

The high viscosity of heavy oil leads to large pressure drops and energy requirements during its transport in pipelines. Conventional methods of reducing oil viscosity, e.g., mixing with light oils or heating, are not cost-effective for the commercial pipeline transport of heavy-oil.\(^1,2\) It has long been suggested that highly viscous fluids can be transported in a water-lubricated state, known as the core-annular flow (CAF), in which a lubricating layer of water is injected near the pipe wall so that the oil does not come into direct contact with the wall. Lower wall shear stress, due to the much smaller viscosity of water flowing in the annular layer adjacent to the wall, drastically reduces (typically by 80%–95%) the effective pressure drop (and therefore, energy requirements) required for pumping the oil.

Experimental studies of oil–water CAF in pipes have been extensively performed over the past few decades, by several research groups.\(^2-8\) A major goal of these experiments has been to obtain an empirical relationship for the pressure drop across the pipe, as well as the holdup ratio (or volume ratio), as a function of water and oil flow rates. Arney et al.\(^3\) first observed that the oil-to-water holdup ratio appears to depend only on the ratio of oil and water flow rates and proposed a purely empirical correlation based on their data. They also proposed correlations for the effective friction factor in a CAF in terms of an effective Reynolds number by modifying relationships based on a perfect core-annular flow (PCAF), where it is assumed that the interface is undisturbed. A large scatter is observed in the experimental data on the effective friction factor, when compared to the proposed correlations,\(^3\) which is attributed to the higher eccentricity of the core at a lower Reynolds number. It is also not clear how theoretical relationships for a perfect core-annular flow, where viscous stresses dominate the stress balance, can be extended to wavy core-annular flows at high Reynolds numbers, where pressure stresses at the oil–water interface may dominate over the viscous stresses. Rodriguez et al. have similarly proposed a modified pressure drop model, which accounts for slip ratio and buoyancy effects.\(^9\) Here, they have compared their model with data from literature and field experiments, and they propose a correlation for the pressure drop that uses expressions derived assuming PCAF.

The shape of the oil–water interface in CAF plays a key role in determining the overall pressure drop. Several theoretical studies in the past have focused on understanding the evolution and steady-state solution of the interface profile.
The linear stability analysis of CAFs\textsuperscript{10–13} have been extremely valuable for understanding the flow regimes where the interface becomes wavy. Miesen \textit{et al.} have used analytical methods to solve the growth rate problem of interfacial waves in CAF, and presented an asymptotic solution method.\textsuperscript{14} The shape of the interface predicted by stability analysis may agree well with experiments near the PCAF state. Direct numerical simulation (DNS) of periodic wavy CAF,\textsuperscript{15} and numerical simulations of CAF in the lubrication limit\textsuperscript{16} have been performed previously. Both of these studies focused on connecting the shape of the interface with levitation of the core in horizontal CAF. They are however not able to provide a description of the interface at high flow rates, where the interface shape constantly evolves, and non-linear processes (e.g., merging and breaking of waves) become important. Bannwart analyzed the interfacial waves in CAF with kinetic wave theory, and compared the wave velocity ($u_w$) with the core velocity ($u_c$) in horizontal and vertical CAFs.\textsuperscript{17} For a low density oil core, and with the consideration of slip ratio, he reports that: $u_w > u_c$ for the vertical downward flow, $u_w < u_c$ for the vertical upward flow, and $u_w = u_c$ for the horizontal flow.\textsuperscript{17} This work provides an interesting way to measure holdup in CAF using interfacial wave speed; however, it does not provide information on the shape of the interfacial waves.

In Ref. 18, the interface in an oil–water CAF was assumed to be periodic with a certain wavelength, and a constraint was derived for the profile of the interface using the lubrication assumption. The constrained interface profile agreed well with interface shapes recorded in experiments. Statistics on the interface profile were also reported in Ref. 18, although these statistics were not linked to the pressure drop. Volume-of-fluid simulations of CAF in Ref. 19 confirmed some of the statistics were not linked to the pressure drop. We do not use expressions for the pressure drop

II. MATERIALS AND METHODS

A. Experimental setup

A schematic of the experiment setup is shown in Fig. 1. The flow-loop had three loft tanks (from Sintex): (i) a water tank of capacity 200 l, (ii) an oil tank of capacity 300 l, and (iii) a collector tank of capacity 500 l. Acrylic pipes of internal diameter 15.5 mm and total length of about 5 m were used for the development and visualization of CAF. The length of each acrylic pipe was 2 m, and therefore flexible acrylic couplings were fabricated, to ensure smooth connection between two pipes. All other connections were made with chlorinated polyvinyl chloride (CPVC) pipes of 3/4 in. internal diameter. All the pipe connections in the CPVC pipe loop were made...
with standard 3/4 in. CPVC fittings (elbow, tee, couplings, unions, reducers, etc.). CPVC ball valves (size 3/4 in.) were used at different places to redirect and/or control the flow of both the fluids. The flow rates of both the fluids (water and oil) were controlled by globe-valves (GVs) installed in respective loops, as shown in Fig. 1. The water was pumped with a centrifugal pump while the oil was pumped with a gear pump. A turbine type flow-meter was used to measure the flow rate of water, while an oval gear type flow meter was used for the oil. Both the flow-meters were based on Hall effect sensors and gave output in terms of voltage pulses. Electronic pulse counter devices, one each for oil and water, were calibrated to display the flow rates in liters per minute (lpm). The pressure drop, across a length of 2 m in the fully developed section, was measured with a differential pressure transmitter (DPT). The pressure drop and flow rate data were logged with a data acquisition system for different combinations of flow rates of oil and water. An acrylic visualization box was used in the fully developed section (about 2.5 m from the injector), through which the acrylic pipe could pass, and then sealed to prevent any leakage of the fluid filled in it. This box was filled with glycerol (a fluid whose refractive index is matched to that of acrylic) to reduce the lens effect associated with the curvature of the acrylic pipe. A high speed CMOS camera (model: PCO 1200hs) was used to record images of the oil-water core-annular flow through the visualization box, at 1000 frames/s. A 500 W halogen lamp was used to provide an adequate amount of light for image recording. Suitable supports were designed and fabricated to support the acrylic pipes, tanks, and CPVC pipes.

**B. Design of water injector**

The water was introduced into the acrylic pipe through an injector which induces the CAF state by injecting the water along the inside wall of the acrylic pipe. A schematic of the injector used in our experiment is shown in Fig. 2. The injector was made of two parts, which when assembled provides a circumferential groove that acts as a reservoir for water. Two inlets were made in the outer part of the injector through which the water could fill the groove and flow uniformly along the inner wall of the pipe as the annular fluid. The injector components were fabricated from stainless steel (SS-316).

**C. Pressure drop measurements**

The pressure drop per unit length, $\frac{\partial p}{\partial z}$, was measured for two different cases: (i) keeping $Q_c$ constant and varying $Q_a$, and (ii) keeping $Q_c$ constant and varying $Q_a$. Here $Q_c$ is the volumetric flow rate of the core fluid (oil) and $Q_a$ is the volumetric flow rate of the annular fluid (water). The experimental values of flow rates of oil and water for these two cases are shown in Tables I and II, respectively. The pressure drop measurements were performed with a differential pressure transmitter (DPT) across a length of two meters in the fully developed flow section of the acrylic pipe. The percentage reduction in the pressure drop was estimated from the following equation along the inside wall of the acrylic pipe. A schematic of the injector used in our experiment is shown in Fig. 2. The injector was made of two parts, which when assembled provides a circumferential groove that acts as a reservoir for water. Two inlets were made in the outer part of the injector through which the water could fill the groove and flow uniformly along the inner wall of the pipe as the annular fluid. The injector components were fabricated from stainless steel (SS-316).
for the pressure reduction factor (PRF):\(^2\)

\[
PRF(\%) = \left( \frac{\Delta P}{L} \right)_{\text{Oil}} - \left( \frac{\Delta P}{L} \right)_{\text{CAF}} \times 100,
\]

where \((\Delta P/L)_{\text{Oil}}\) is the pressure drop of a pure oil flow (without water lubrication) and \((\Delta P/L)_{\text{CAF}}\) is the pressure drop of the water lubricated core-annular flow. For the pure oil flow, the Reynolds number is always below 150, and therefore \((\Delta P/L)_{\text{Oil}}\) can be calculated assuming the Hagen-Poiseuille flow.

### D. Time series of the interface position

We extract the spatial interfacial height profile from the frames in the high-speed video of the CAF via the following steps. All the steps are automated using standard image processing software and MATLAB programs.

#### 1. Interface height profile at different times

The different flow rate combinations of oil and water over which the flow profiles of the fully developed CAF were recorded are shown in Table III. Note that we record videos for a smaller set of flow rate combinations compared to processing steps. All the steps are automated using standard image processing software and MATLAB programs.

#### 2. Wave speed of the interface

In the frame of reference of the core, the interface shape evolves slowly with time; we use this fact to calculate the wave speed of the interface. For each oil–water flow rate pair, we calculate the average wave-speed of the upper and lower interfaces, \(c_u\) and \(c_l\), respectively, via the following second order correlation of interface height:

\[
r(\Delta z) = \frac{1}{r} \int_{-r}^{r} \tilde{\eta}(z, t) \tilde{\eta}(z + \Delta z, t + \Delta t) dz,
\]

which requires two images spaced apart by time \(\Delta t\). Here, the field of view is defined as \(0 < z < L_z\), and the second order correlation is calculated for \(0 < \Delta z < \zeta\). Wave speed is then given by \(c = \Delta z_{\text{max}} / \Delta t\) (we have omitted the subscript \(u/l\) for brevity), where \(\Delta z_{\text{max}}\) denotes the value of \(\Delta z\) for which \(r(\Delta z)\) attains a maximum. Here, the main restrictions on \(\Delta t\) are that it should be large enough so that two points separated by the distance \(c \Delta t\) are distinctly resolved by the camera, and it should be small enough so that the wave does not change its shape over this time period. For each oil–water flow rate, we calculate \(c\) using several different values of \(\Delta t\) to ensure that it lies within the correct range. Due to the large viscosity of oil, we expect the average wave speed to be almost equal to the average speed of the core. We compare these two quantities later in this section. We found almost negligible difference between \(c_u\) and \(c_l\) for the range of flow rates examined in our experiments, which may also be attributed to the high viscosity of the oil.

#### 3. Time series of the interface profile

For a fixed point \(z_0\) within the field of view, we obtain temporal data for the interface profile \(\tilde{\eta}_u(t) = \tilde{\eta}_u(z, t)\) and \(\tilde{\eta}_l(t) = \tilde{\eta}_l(z, t)\), for the upper and lower interfacial waves, over the total observation time. Further, the glycerol filled visualization box alleviates the lensing effects associated with the curvature of the pipe. However, the upper and lower interfacial wave profiles need to be corrected for the refraction effects associated with the water present in the annular region. The interface profiles were corrected using ray tracing by considering the refractive indices of acrylic \((n_a = 1.49)\) and water \((n_w = 1.33)\). Temporal profiles of upper and lower interfaces, from the centerline of the pipe, for a set of constant water flow rates of \(Q_w = 2\ lpm\) and \(Q_w = 4\ lpm\) are shown in Figs. 3(a) and 3(b), respectively. Clearly, for low oil flow rates, the core has significant eccentricity with respect to the center of the pipe. This eccentricity decreases for higher oil flow rates. Also, it should be noted that, in general, the time series does not look periodic and appears to contain modes with a range of frequencies. We elaborate further on the power spectrum of interface height profile in Sec. IV C.
FIG. 3. Temporal profiles of upper and lower interfaces from centerline of the pipe for different oil flow rates, (a) \( Q_a = 2 \) lpm (constant) and (b) \( Q_a = 4 \) lpm (constant).

4. Spatial profile of the interface height

The spatial profile of the upper interface, \( \eta_u(z) \), and lower interface, \( \eta_l(z) \), can be readily obtained from the corresponding time series data using the expressions \( \eta_u(z) = \eta_u^*(z/c_u) \) and \( \eta_l(z) = \eta_l^*(z/c_l) \); we are essentially using a form of Taylor’s hypothesis here,\(^{25,26}\) which is often used by researchers in the turbulence community to construct two-point correlations in turbulent flows using single-point measurements, by assuming that the turbulent structures in the flow are “frozen” as they get advected across the probed region. The spatial profiles of the interface are defined over a much large range in \( z \) compared to \( \bar{\eta}_u \) and \( \bar{\eta}_c \), and their Fourier transforms can therefore yield high quality power spectra of the interface.

E. Measurement of holdup and bulk velocities

The holdup volume fraction of oil (\( e_o \)) is defined as\(^3\)

\[
e_o(z) = \frac{A(z)}{\pi R^2} = \frac{(\eta_u + \eta_l)^2}{4R^2},
\]

where, for a given length of pipe \( L \) in the fully developed flow region, \( V_o \) represents the volume of oil in the pipe and \( V_w \) is the volume of water in the pipe.

The holdup and average velocities of core and annular fluids were estimated using spatial profiles of the interface. Using cylindrical coordinates, the radial location of the interface at some axial (\( z \)) and azimuthal (\( \theta \)) positions is given by \( \eta(z, \theta) \). If, for example, \( \theta = 0 \) corresponds to the lowest position of the interface, then \( \eta(z, \pi) = \eta_u(z) \) and \( \eta(z, 0) = \eta_l(z) \), respectively. Due to the lack of data on azimuthal variation of the height of the oil–water interface, and also due to the low Bond number of the interface (Sec. III), we assume that the cross section of the core is circular. The cross-sectional area of the core at a given \( z \) location is therefore given as

\[
A(z) = \frac{\pi(\eta_u + \eta_l)^2}{4}. \tag{3}
\]

At a given location, the oil holdup will be given by the ratio of area occupied by oil to the cross sectional area of the pipe,

\[
e_o = \frac{V_o}{V_o + V_w}, \tag{2}
\]

where, for a given length of pipe \( L \) in the fully developed flow region, \( V_o \) represents the volume of oil in the pipe and \( V_w \) is the volume of water in the pipe.

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Next, we define the average velocities of the core and annular fluids. The core flow rate (\( Q_c \)) can be related to the bulk velocity of the core (\( U_c \)) via \( Q_c = \langle A \rangle U_c \), such that

\[
U_c = \frac{Q_c}{\pi R^2 e_o}, \tag{6}
\]

Similarly, the bulk velocity of the annulus is given by

\[
U_a = \frac{Q_a}{\pi R^2 (1 - e_o)}. \tag{7}
\]

In Fig. 4, we compare \( U_c \) to \( c = (c_u + c_l)/2 \). Not surprisingly, \( U_c \) is always slightly higher than \( c \), since parts of the wave are
located closer to the pipe wall, where the speed should necessarily be smaller. On average, the two values differ by 7%, and the maximum deviation is around 20% for small values of $Q_a$ and high values of $Q_a/Q_c$. These data confirm that the interfacial waves are almost stationary with respect to the frame of reference of the core, most likely due to the high viscosity of the oil.

III. DIMENSIONAL ANALYSIS OF ANNULAR REGION

We primarily characterize the interface using the dimensional analysis of the annular region of the CAF. We are interested in obtaining empirical relationships for the fluctuation in the interface profile,

$$\eta'(z) = \eta(z) - \langle \eta \rangle \quad (8)$$

(along with its associated statistics), as well as the effective and high values of the interface profile, respectively. Analysis of the annular region of the CAF. We are interested in obtaining empirical relationships for the fluctuation in the interface profile,

$$\tau = \frac{R \partial p}{2 \partial z}, \quad (9)$$

in terms of the other independent parameters that control the interfacial flow in the annulus. We favor modeling $\tau$ over $\partial p/\partial z$, since it is a localized quantity within the annulus. However, we must also note that the above definition for $\tau$ represents the circumferential average of wall shear stress and that in our experiments, we are not measuring $\tau$ as a function of the azimuthal angle in the pipe.

The average interface thickness is an independent parameter here and is given by

$$a = R - \langle \eta \rangle. \quad (10)$$

For the dimensional analysis, we will assume $R \gg a$, i.e., the annular thickness is small compared to the radius of the core; this is in line with the analysis of viscosity-stratified flows in Ref. 23. The schematic in Fig. 5 shows the important dependent and independent parameters in the problem.

Before we proceed, we state the dimensional values of the relevant quantities in the experiment,

$$(a \sim 10^{-3} \text{ m, } \bar{U}_a \sim \bar{U}_c \sim 0.7 \rightarrow 1.5 \text{ m/s}, \quad g \sim 10 \text{ m/s}^2, \quad \Delta \rho \sim 10 \text{ kg/m}^3, \quad v_a \sim 10^{-6} \text{ m}^2/\text{s}, \quad v_c \sim 10^{-4} \text{ m}^2/\text{s},)$$

where $v_c$ and $v_a$ are the kinematic viscosities of the core and annular fluids, $g$ is the acceleration due to gravity, $\Delta \rho$ is the magnitude of density difference between oil and water, $\bar{U}_a$ and $\bar{U}_c$ are the average velocities of the core and annular fluids, and $\gamma$ is the interfacial tension between the core and annular fluids. $\rho_a$ is the density of the annular fluid (water, in this case). Note that, strictly speaking, $\bar{U}_a$ and $\bar{U}_c$ depend on $\theta$ (azimuthal location on pipe) and are therefore different from $U_a$ and $U_c$ [Eqs. (6) and (7)], which are bulk parameters. In numerical simulations of oil-water CAF in Ref. 19, it has been reported that significant variation of $\bar{U}_a$ may be present along $\theta$. On the other hand, $\bar{U}_c \approx U_c$ is a good approximation, due to the high viscosity of the oil. In the discussion below, we will therefore replace $\bar{U}_c$ with $U_c$.

Next, we note that the most general relationships for the interface fluctuation and wall shear stress are as follows:

$$\eta'(z) = F^*(\rho_a, z, a, U_c, v_c, \bar{U}_a, v_a, \gamma, g, \Delta \rho), \quad (11)$$

$$\bar{\tau} = G^*(\rho_a, a, U_c, v_c, \bar{U}_a, v_a, \gamma, g, \Delta \rho), \quad (12)$$

where, again, $\bar{\tau}$ is the net wall shear stress at a particular azimuthal location on the pipe wall. Since the Reynolds number of the annular fluid is high, we choose the inertial stress scale $\rho_a U_c^2$ to non-dimensionalize $\tau$ as well as interfacial tension $\gamma$. The non-dimensional equations are then

$$\eta'(z) = aF\left(\frac{z}{a}, \frac{U_a}{U_c}, \frac{v_c}{v_a}, \text{We}, \text{Re}_c, \text{Fr}, \text{Bo}\right), \quad (13)$$

$$\bar{\tau} = \rho_a U_c^2 G\left(\frac{\bar{U}_a}{U_c}, \frac{v_c}{v_a}, \text{We}, \text{Re}_c, \text{Fr}, \text{Bo}\right), \quad (14)$$

where

$$\text{Re}_c = \frac{U_c a}{v_a} \quad (15)$$

is the shear Reynolds number, $\text{We} = \rho_a U_c^2 a / \gamma$ is the Weber number, $\text{Fr} = U_c / \sqrt{\rho g a}$ is the Froude number, and $\text{Bo} = \Delta \rho g a^2 / \sigma$ is the Bond number. Based on the dimensional values of parameters, we note that $v_c/v_a \sim 100$ is a fixed quantity for all the experiments, as is the capillary number $\text{Ca} = \mu_c^2 / \rho_a \gamma R = 0.06$.

Despite the large Weber number ($\text{We} \sim 10 \rightarrow 110$), we cannot necessarily assume that interfacial tension is unimportant, since the growth of capillary instabilities will depend on the radius of the core, $R_1 = R - a$. A separate Reynolds number, $\text{Re}_1 = U_c R_1 / \nu_c$, along with the capillary number ($\text{Ca}$), determines whether or not long-wave capillary instabilities are present. In previous studies, the linear
stability analysis of CAF was used to show that surface tension can lead to unstable long-wave modes for \( Re_l < Re_l^* \), where \( Re_l^* = \beta(Ca)^{-1/2} \), and \( \beta \) is a constant that depends on the parameters \( a/R_1, \nu_a/\nu_c \). High shear rates stabilize the flow for \( Re_l > Re_l^* \). For the parameter range relevant to our experiments (\( \nu_a/\nu_c \rightarrow 0, 0.05 < a/R_1 < 0.3 \)), it has been shown\(^{10} \) that \( \beta \in [0.8, 1.4] \), and therefore \( Re_l^* < 15 \). For our experiments, \( Re_l \in [30, 60] \), and therefore we do not expect to see interfacial instability due to interfacial tension.

The Froude number is large (\( Fr \sim 7 \rightarrow 15 \)), implying that gravity does not directly affect interfacial dynamics in the annular region. The Bond number is small (\( Bo \sim 5 \times 10^{-3} \)), implying that surface tension dominates gravitational forces within the annular region. In horizontal CAF, the Froude number and Bond number based on the core radius \( R_1 \) (instead of \( a \)) together determine the eccentricity of the core and the average cross-sectional shape of the core, respectively. The Bond number based on \( R_1 \) is around 0.2, implying that the average cross section of the core should be approximately a circle.

In our experiments, where \( \nu_a/\nu_c \) and \( Ca \) are constant, we can simplify Eqs. (11) and (12) to

\[
\eta'(z) = aF \left( \frac{z}{a}, \frac{U_a}{U_c}, Re_c \right), \tag{16}
\]

\[
\tau = \rho_a U_c^2 G \left( \frac{U_a}{U_c}, Re_c \right). \tag{17}
\]

In the rest of the paper, we will further characterize \( \eta'(z) \) and \( \tau \) using the above non-dimensional forms. In our experiments, we do not explicitly measure \( U_a/U_c \); however, as we will see below, the relevant non-dimensional flow and interfacial statistics appear to depend more strongly on \( Re_c \), for which we have been able to take accurate measurements. Similarly, even though we do not measure \( \tau \) in our experiments, we will find that the net wall shear stress depends strongly only on \( U_c \), which does not vary azimuthally.

IV. RESULTS AND DISCUSSION

A. Flow development and pressure drop

Representative images showing the development of the core-annular flow for a fixed value of \( Q_a \) are shown in Fig. 6. At low oil flow rates, we observe the formation of small slugs in the core [Fig. 6(a)]. As the oil flow rate is increased, these slugs combine to experience lower drag and coalesce with each other, forming longer slugs [Fig. 6(b)]. A further increase in the oil flow rate typically results in the core-annular flow with a continuous core [Figs. 6(c)–6(f)]. The eccentricity of the core with respect to the center of the pipe reduces at higher flow rate ratios of oil to water, although we do not present a systematic study of eccentricity in this paper. The images in Fig. 6 were obtained using a high speed camera operating at 1000 frames/s.

The pressure drop, in terms of water head per meter length of pipe \( (\eta w^{-1}) \), for different flow rate combinations, is shown in Figs. 7(a) and 7(b). Figure 7(a) represents six different cases where \( Q_c \) is kept constant and \( Q_a \) is varied while Fig. 7(b) shows six complementary cases where \( Q_a \) is held constant and \( Q_c \) is varied. The plots here compare well with similar data presented in Figs. 5–7 in the work of Arney et al.,\(^3 \) where crude oil and fuel oil were used as core fluids, and the pipe had inner diameter very similar to our setup. The percentage reduction in the pressure drop in terms of the pressure reduction factor (PRF), calculated using Eq. (1), is shown in Fig. 8. It can be seen that the pressure drop with water-lubrication can be typically reduced by up to 97%, when compared to the pressure drop required to pump only oil. Therefore, as reported by numerous researchers in the past,\(^2–6 \) the pressure drop (and therefore power) required to transport highly viscous oil can indeed be significantly reduced by injecting water in the annulus.

B. Velocity and holdup ratios

The ratios of average velocities of annular (water) and core (oil) fluids, estimated using the image analysis [Eqs. (6) and (7)], for different flow rate ratios, are shown in Fig. 9(a). We observe that the velocity ratio (over the entire data set) varies between \( 0.6 < U_a/U_c < 0.86 \) and takes a mean value of 0.74. As pointed out previously,\(^{15} \) the value of \( U_a/U_c \) should lie between 0.5 (for the PCAF state, or \( \eta' = 0 \)) and 1 [when the wave amplitude is equal to the annular thickness, or \( max(\eta') = a \)]. The average value of \( U_a/U_c = 0.74 \) is close to the best fit value of 0.72 observed in Ref. 12. However, the scatter in the value of \( U_a/U_c \) is in fact quite large, which should not be surprising; the dimensional analysis of full CAF (instead of just the interface region) shows that this ratio will itself depend on the flow rate ratio \( Q_a/Q_c \), along with the Reynolds number \( U_c R / \nu_c \), Froude number \( U_c / \sqrt{gR} \), etc. We note that the scatter is high for lower values of \( Q_a/Q_c \), where \( \langle \epsilon_o \rangle \) is higher.
FIG. 7. Pressure gradient in terms of water head per meter length of the pipe (h_wl^-1) for (a) Q_c = constant and varying Q_a and (b) Q_a = constant and varying Q_c.

We now repeat the analysis used previously,\textsuperscript{12} where it is shown that the oil holdup ratio can be directly related to the velocity ratio and flow rate ratio. If we assume that \( \frac{U_a}{U_c} \) stays close to its average value \( \alpha = 0.74 \), then we can readily relate the flow rate ratio and holdup using Eqs. (6) and (7) as

\[
\frac{Q_a}{Q_c} = \alpha \left[ 1 - \langle e_o \rangle \right].
\]

(18)

Thus,

\[
\langle e_o \rangle = \frac{1}{1 + \frac{1}{\alpha} \frac{Q_a}{Q_c}}.
\]

(19)

The above expression agrees well with our experimental data for holdup [calculated via the image analysis, from Eq. (5)], as shown in Fig. 9(b). To explain the good agreement for the holdup ratio between model and experimental measurements in spite of the large scatter in \( \alpha \), we can carry out the following sensitivity analysis. Using Eq. (19), we can see that, for a small change \( \delta \alpha \) in the velocity ratio, the fractional change in the holdup ratio is \( \delta \langle e_o \rangle / \langle e_o \rangle \approx (1 - \langle e_o \rangle) \delta \alpha / \alpha \). Therefore, for \( \langle e_o \rangle \approx 1 \), the value of the holdup ratio is fairly insensitive to the value of \( \alpha \). In our data, the maximum value of \( \delta \alpha / \alpha \approx 0.23 \) occurs at the holdup ratio \( \langle e_o \rangle \approx 0.7 \), for which the percentage error in predicting holdup is only around 7%. Thus, due to the low sensitivity of the holdup ratio to \( \alpha \), the model prediction and experimental data for \( \langle e_o \rangle \) agree reasonably well with each other, when we use the average value \( \alpha = 0.74 \) in the model.

C. Characteristics of interfacial waves

To analyze the properties (i.e., shape and magnitude) of fluctuations in the interface height \( \eta'(z) \), we use the non-dimensional form in Eq. (16), to observe that \( \eta'(z)/a \) will primarily depend on \( z/a \), the shear Reynolds number, \( \text{Re}_c \), and the velocity ratio \( \alpha \). In Fig. 10, we have plotted the standard deviation \( \sigma = \sqrt{\langle (\eta')^2 \rangle} \), normalized with average annular thickness \( a \), as a function \( \text{Re}_c \). First, note that \( \text{Re}_c \) varies over a rather large range, from 250 to 2500. There is a significant scatter in the data at intermediate values of \( \text{Re}_c \), most likely due to the scatter in \( U_a/U_c \). Despite the scatter present in the data, we observe a clear trend, which shows that \( \sigma/a \) decreases with increasing \( \text{Re}_c \) and converges to a value of around 0.5 at large \( \text{Re}_c \).
FIG. 10. Standard deviation of the interface $\sigma/a$, plotted as a function of $Re_c$, for all flow rate ratios and for both lower and upper interfaces.

In previous work,$^{18}$ the interfacial wave amplitude was linearly correlated to the holdup ratio with a negative slope. The wave amplitude was calculated by taking the difference between the maximum radial locations of the interface and subtracting it from the average radius of the interface; such a measure of interface variation can be justified if the interface profile is periodic over a certain wavelength. We, however, avoid using such a characterization, since the height profile of the interface (Fig. 3) does not appear to show such obvious spatial periodicity.

Next, we focus on how the wave content of the interfacial height profile changes with $Re_c$ by examining the normalized power spectra of $\eta'(z)$. The power spectra of the interface fluctuation, $E(k)$ (here $k$ is the wavenumber of the Fourier mode), can be easily calculated from $\eta'(z)$ using a standard discrete Fourier transform (DFT). The variance in $\eta'$ is related to $E(k)$ by

$$\sigma^2 = \langle \eta'^2 \rangle = \int_0^{k_{\text{max}}} E(k) dk,$$

where $k_{\text{max}} = 2\pi/\Delta z$ and $\Delta z$ is the sampling width on the $z$ axis. This equation can be written in the non-dimensionalized form as

$$\int_0^{k_{\text{max}}} E^*(k) dk^* = 1,$$

where $E^*(k) = \frac{E(k)}{\sigma^2 a}$ is the non-dimensional energy spectrum and $k^* = ka$ is the non-dimensional wavenumber. The above non-dimensionalization ensures that the area under the curve $E^*$ versus $k^*$ is equal to 1; we are therefore purely examining the wave content (and not magnitude) of the interfacial fluctuations here. In Fig. 11, we have first ordered the spectra according to their Reynolds number $Re_c$, and we then plot the spectra within different ranges in $Re_c$. It can be seen that the spectra at low $Re_c$ are predominantly from the upper interface, while the spectra at high $Re_c$ are all from the lower interface. Spectra from the intermediate range of $Re_c$ are from both upper and lower interfaces, as expected.

Several interesting features emerge from the power spectrum. Negligible energy is present in waves with $k^* > 1$, which corresponds to wavelengths $\lambda < 2\pi a$. Clearly, almost all of the energy in the interface is stored in modes with wavelengths much larger than the annular thickness. For low values of $Re_c$, the energy in the spectrum peaks close to $k^* \approx 0$ and then decreases almost monotonically and smoothly with increasing wavenumber. At intermediate values of $Re_c$, the spectrum is bimodal, with a second peak emerging at $k^* \approx 0.6$. At high values of $Re_c$, the first peak at $k^* \approx 0$ almost disappears, and the second peak at $k^* \approx 0.6$ dominates. The spectra corresponding to larger values of $Q_a/Q_c$ do not always follow this trend. The reason for this discrepancy may be that these cases also correspond to large values of $a$ (or smaller values of oil-to-water holdup), for which $a/R \ll 1$ may no longer be a good approximation. To further quantify the shifting of energy in the interface from low to high wavenumbers, we also calculate the weighted average of $k$, given by

FIG. 11. Normalized energy spectra of the interface $E^*$ versus $k^*$, plotted for ranges in $Re_c$, for both lower and upper interfaces. Within each graph, the curves are ordered according to $Re_c$ (lower to higher).
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Fig. 13(a), it appears that in fact
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the data, it is clear that the average wavelength of the interface
decreases with increasing Re
The results presented here are consistent with previous observations on two-layer viscosity-stratified flows, where an instability, with the highest growth rate for modes with
ka ~ 1, was predicted at large values of Re
. The large energy at
ka ≈ 0 for low Re
may correspond to a long wavelength instability first derived by Yih for the same category of flows. A more detailed comparison of our results on energy spectra of the interface with linear stability analysis will be performed in the future.

D. Scaling of wall shear stress
Using the non-dimensional relation for τ [Eq. (17)], we can expect τ/(ρaUa2) to depend mainly on Re
 and Ua/Uc. In Fig. 13(a), it appears that in fact τ is approximately linearly proportional to Ua2, suggesting that G(Re
) [Eq. (17)] depends weakly on Re
and Ua/Uc. This is a bit surprising, given the large range of Re
in our experiments, along with the fact that the characteristics of interfacial waves do vary significantly with Re
(Sec. IV C).

We can fit a straight line to the linear relationship between τ and ρaUa2 and obtain the following best fit expression:

\[
\tau = C_\tau \frac{1}{2} \rho_a U_a^2,
\]

where C\_τ = 7.04 \times 10^{-3} is the friction factor, with an L2 error norm of less than 10% for the fit, over all data points; the maximum error of 20% occurs for low values of Re
(discussed below). This result implies that τ in fact depends on inertial fluid forces, rather than viscous forces, and that these inertial forces are almost completely driven by the core velocity. In Fig. 13(b), we have plotted C\_τ/2 = τ/(ρaUa2) as a function of Re
, in which we can see that C\_τ in fact varies from 9 \times 10^{-3} (at low Re
) to 6.5 \times 10^{-3} (at high Re
).

Physically, wall shear stress may arise due to a combination of viscous stresses (enhanced due to turbulent mixing of momentum) and form drag acting on the oil-water interface. In Ref. 27, experimental data for shear stress in the turbulent Couette flow between two plane walls separated by distance 2h have been reported, in which the lower wall is stationary, and the upper wall is moving at a speed Uc. The Reynolds number of the Couette flow is defined as Re\_h = hUc/νh. Due to the high viscosity of the oil, for a/R ≪ 1, the flow in the CAF annulus can be approximated as a Couette flow, with the upper wall moving at speed Uc. Therefore, we can directly compare the data on the friction factor C\_f = 2τ/(ρaUa2) reported in Ref. 27 with C\_τ from our experiments, keeping in mind that since 2h = a, therefore Re\_h = Re\_c/2. Additionally, we have to ignore the waves on the interface to make this comparison; the possible contribution of the wavy interface to the net shear stress is discussed below. For 125 < Re\_h < 1250 (corresponding to the range of Re
in our CAF experiments), the friction factor C\_f varies between 0.016 (at Re\_h = 125) and 7 \times 10^{-3} (at Re\_h = 1250). The best fit value of C\_f = 7.04 \times 10^{-3} from our experiments lies on the lower end of this range, and Fig. 13(b) shows that C\_τ also varies with Re\_c in a similar manner as C\_f. Thus, turbulent mixing of momentum is a possible reason for the generation of shear stress within the annulus. Eccentricity of the core may explain why C\_τ is always on the lower side, since, even if the localized value of Re\_c for the upper interface is small, the value of Re\_c on the lower interface may be high due to the eccentricity of the core, which in turn can reduce the average value of C\_τ.

To understand how the shape of the interfacial profile affects the net shear stress, we first assume that the interface height is a periodic wave with wavelength λ\_avg = 2π/k\_avg

![FIG. 12. k\_avg = k\_avg plotted with respect to Re\_c.](image1)

![FIG. 13. Scaling of wall shear stress, (a) τ versus U_a^2 and (b) C_τ/2 = τ/(ρ_a U_a^2) versus Re_c. Here Re_h = U_a/ν_h is based on the lower or upper interface value of a.](image2)
and with amplitude proportional to $\sigma$. In the context of our experiments, $k_{\text{avg}}$ and $\sigma$ have already been defined and characterized earlier in Sec. IV C. We again assume that the core is moving like a solid body, with velocity $U_c$, and that the flow in the annulus is completely driven by the core velocity; in effect, we are assuming that the flow in the annulus is similar to the Couette flow between a wavy upper wall that is moving and a flat lower wall which is stationary. This is a fair assumption, since the effect of the mean axial pressure gradient will be small in the thin annulus, compared to the effect of the local pressure distribution induced by the interface shape. Dimensional analysis shows that the percentage of stress due to form drag is then a function of $\sigma k_{\text{avg}}$, as well as $k_{\text{avg}} a$. In our experiments (Sec. IV C), we observe that $\sigma k_{\text{avg}} \in [0.1 0.3]$ and $0 < k_{\text{avg}} a < 1$. There is little direct experimental or computational data on how the percentage of stress due to form drag depends on these parameters, even in the prior literature on DNS of CAFs.\cite{29,15} In Ref. 29, DNS of the Couette flow over wavy walls at a high Reynolds number was carried out, where it was observed that the percentage of shear stress due to form drag increases with increasing $k_{\text{avg}} \sigma$ and reaches around 20% of the net shear stress for $k_{\text{avg}} \sigma \sim 0.2$. However, for these simulations, $k_{\text{avg}} a = 2 \pi$ was used, which, for a fixed value of $k_{\text{avg}} \sigma$, corresponds to a much smaller blockage ratio of the annular gap compared to our experiments, where $k_{\text{avg}} a < 1$. We expect the percentage of shear stress due to form drag to be higher in our experiments because of the larger blockage ratios, but we are not able to give a clear quantitative estimate of this percentage due to the lack of prior data. At any rate, it appears that the friction factor $C_f$ depends weakly on $Re_c$, even though the percentage of form drag and viscous drag acting on the interface may vary with flow parameters.

E. Pressure drop vs. flow rate relationship

In this section, we use the inertial scaling for $\tau$ [Eq. (23)], along with the model for holdup [Eq. (19)] to obtain an expression for pressure drop $\frac{\partial p}{\partial z}$ in terms of flow rates $Q_a$ and $Q_c$. Equation (23) along with the definition for $U_c$ [Eq. (6)] and holdup $\langle e_o \rangle$ [Eq. (5)] yields

$$\tau = C_f \frac{1}{2} \rho_a \frac{Q_a^2}{\langle \lambda \rangle^2} = C_f \frac{1}{2} \rho_a \frac{Q_a^2}{\pi^2 R^4 \langle e_o \rangle^2}. \quad (24)$$

We next use the relationship between $\tau$ and $p$ [Eq. (9)], along with the model for holdup [Eq. (19)] to obtain a model for pressure gradient,

$$\frac{\partial p}{\partial z} = C_f \rho_a \frac{Q_a^2}{\pi^2 R^8} \left[ 1 + \frac{1}{\alpha} \frac{Q_a}{Q_c} \right]^2. \quad (25)$$

In Fig. 14, we compare our experimental data with the above model, for the different sets of oil and water flow rates. Overall, the model and experimental data appear to match reasonably well, with an average L2 norm error of 12% between model and experiment. There is one outlier data point at $Q_a = 1.5$ lpm and $Q_c = 4$ lpm (which we have not included in the overall error estimation), which shows the maximum deviation of 60% between the model and experimental predictions. The deviation between model and experiment can be traced back to two sources of error. First, we are using a fixed value of $U_a / U_c = 0.74$ in our model, while the experimental data show a scatter for $U_a / U_c$ between 0.6 and 0.86. Using the correct, measured value for $U_a / U_c$ for each data point significantly reduces the discrepancy between the model and data (not shown here). A second source of error between the model and prediction lies in the model for shear stress [Eq. (23)], which can show up to 20% error between model and experiment.

The implications of Eq. (25) are quite straightforward. For a fixed value of the core flow rate $Q_c$, the pressure drop increases with increasing annular flow rate. For a fixed value of the flow rate ratio $Q_a / Q_c$, the pressure drop is proportional to $Q_a^2$. For $Q_a / Q_c \ll 1$, the pressure drop will be insensitive...
to the flow rate ratio and will again be proportional to $Q_c^2$. The pressure drop does not explicitly depend on the viscosity of the annular fluid, as long as $Re_c$ is large enough. Equation (25) will probably not hold at very low values of $Q_c/Q_a$, where the core starts to foul the pipe wall.

The viscosity ratio $\nu_c/\nu_a$ should affect the dominant modes in the interface height profile. However, based on our experimental data, it appears that the friction factor $C_f$ depends weakly on the wave content of the interface height profile. Therefore, we expect the pressure drop itself to depend weakly on the viscosity ratio of the core and annular fluid, over a wide range of viscosity ratios. However, we will need to perform additional experiments with different core fluids to completely characterize this dependence.

V. CONCLUSION

In this work, we have presented a novel localized analysis of experimental data of the annular region in the core-annular flow (CAF). It is clear that the oil-water interface shape is strongly dependent on the shear Reynolds number $Re_c$. Specifically, the wavelength of the most dominant mode of the interface, normalized by the annular thickness, reduces with increasing $Re_c$ and tends to a constant value at large $Re_c$. Another major insight from our data is that the effective wall shear stress $\tau$ appears to depend linearly on $\rho_a U_c^2$. More specifically, the friction factor, $C_f = 2\tau/\rho_a U_c^2$, depends weakly on the shear Reynolds number $Re_c$. Due to the lack of measurements and prior data on the how the pressure and viscous stresses are related to the shape of the interface, it is not possible to exactly link the interface shape to the net wall shear stress. However, it is plausible that viscous stresses, enhanced by turbulent mixing of momentum in the annulus, as well as form drag acting on the interface, are responsible for the net shear stress. We also found that a reasonably good agreement between experimental data and model for the oil holdup fraction can be obtained by using an averaged value for the ratio of annular to core velocity, $\alpha$. Using the model for oil holdup and the scaling $\tau \sim \rho_a U_c^2$ emerging from our experiments, we are able to build a semi-empirical model for the pressure drop that matches well with the experimental measurements for a large range of parameters. Given that the interface height profile is in fact composed of modes spanning a range of wavelengths, a more thorough understanding of the relationship between drag acting on the interfacial waves and the power spectra of the interfacial waves is required here. The scatter in values of annular-to-core velocity, $\alpha$, is a major source of discrepancy between model and experimental measurements of both the holdup-ratio and wall shear stress. In the future, we will attempt to understand how this ratio depends on the various independent parameters in CAF.

ACKNOWLEDGMENTS

The authors are grateful to Orica Limited (Australia) for financial support and useful discussion on the injector design. The authors are also thankful to the PIV Lab, Mechanical Engineering Department, IIT Bombay for providing space for the experimental setup.


