

Counting Labelled 3-Connected Graphs

Nicholas Wormald
UNIVERSITY OF NEWCASTLE

Labelled connected graphs were first counted by Riddell [3], as were unlabelled connected graphs and labelled 2-connected graphs. Robinson [4] developed the only known method of counting unlabelled 2-connected graphs. We have found a recursive method of counting labelled 3-connected graphs. The corresponding problem for unlabelled graphs remains unsolved.

Our counting method is based on a characterization of the graphs which can be formed when a line is deleted from a 3-connected graph. Consequently, it is necessary to include the number of lines as a parameter of the enumeration.

The characterization result also allows us to count labelled 3-connected cubic graphs. Read [1] and [2] has already counted labelled cubic graphs and labelled connected cubic graphs. Since no cubic graph is 4-connected, it only remained to enumerate labelled 2-connected cubic graphs. We have accomplished this by taking into consideration graphs which are cubic except for one or two points of degree 2. The method can also be used to derive the recursive formulae of Read [2] for the numbers of labelled cubic graphs and labelled connected cubic graphs. In addition, we have used related techniques to show that almost all labelled cubic graphs are 3-connected, the analog for cubic graphs of the fact that almost all labelled graphs are 3-connected.

References

- [1] R. C. Read, The enumeration of locally restricted graphs II. *J. London Math. Soc.* 35 (1960) 344–351.

- [2] R. C. Read, Some unusual enumeration problems. *Annals N.Y. Acad. Sci.* 175 (1970) 314–326.
- [3] R. J. Riddell, Contributions to the theory of condensation. Dissertation, University of Michigan, Ann Arbor (1951).
- [4] R. W. Robinson, Enumeration of nonseparable graphs. *J. Combinatorial Theory* 9 (1970) 327–356.