Using challenging and consolidating tasks to improve mathematical fluency

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Following an overview of teaching with challenging tasks, we explore the nexus between using both challenging and consolidating tasks to simultaneously develop conceptual understanding and procedural fluency. In particular, we argue that it is critical that students are provided with parallel opportunities to work on consolidating tasks, in order to connect conceptual understanding to improved strategy efficiency. This discussion makes reference to the Big Ideas in (primary) mathematics (Charles & Carmel, 2005) and provides three examples of challenging and consolidating tasks, each of which support students in grappling with a different 'Big Idea'.

CHALLENGING TASKS, PROMPTS AND DIFFERENTIATION[1]

Challenging tasks are complex and absorbing mathematical problems with multiple solution pathways, where the whole class works on the same problem (Sullivan & Mornane, 2013). Challenging tasks can be viewed as an interpretation of cognitively demanding tasks that meet specific criteria that are outlined below (adapted from Sullivan et al., 2011).

The challenging task must:

- be solvable through multiple means (i.e., have multiple solution pathways) and may have multiple solutions;
- involve multiple mathematical steps (i.e., as opposed to a single insight facilitating completion of the problem);
- have at least one enabling prompt and one extending prompt developed prior to delivery of the lesson;

The challenging task should:

- be initially perceived as challenging by the majority of students;
- engage students (i.e., students are motivated to solve the problem);
- involve students spending considerable time working on the task (although the exact length of time will vary substantially, depending on the nature of the task, the age group and the student in question, it is generally expected that students will spend at least 10 to 15 minutes engaged with the problem);

As indicated above, challenging tasks are differentiated through the use of enabling and extending prompts. Enabling prompts are designed to reduce the level of challenge through simplifying the problem, changing how the problem is represented, helping the student connect the problem to prior learning and/ or removing a step in the problem (Sullivan, Mousley, & Zevenbergen, 2006). When developing enabling prompts, it is critical that they do not undermine the primary learning objective of the lesson by 'giving too much away'. By contrast, extending prompts are designed for students who finish the main challenge and expose students to an additional task that is more challenging but requires them to use similar mathematical reasoning, conceptualisations and representations as the main task (Sullivan et al., 2006).

When engaged in a challenging task, students should be encouraged to access enabling prompts at their own initiative. Enabling prompts should be a student's first point of call if they feel they need some assistance to make progress with the problem (i.e., rather than immediately asking for support from the teacher/ a fellow student). Consequently, students need explicit support around how to use enabling prompts most effectively and when to use them, particularly if they have not previously been exposed to challenging tasks. It is important, therefore, that the teacher ensures that all students know where the enabling prompts are in the room, and that there is no stigma associated with accessing an enabling prompt (e.g., an overly competitive classroom climate, where it is assumed that 'good mathematicians don't need help').

In my classroom (first author), I call the enabling prompts the 'hint sheet', and print one prompt on each side of this sheet. During each challenging task, I include a pile of hint sheets up the front of the classroom on a chair, so students know exactly where they are. By contrast, I call the extending prompt the 'super challenge' and generally place the extending prompt on the flip-side of the challenging task.

When working on a challenging task, the expectation is that all students engage with the same primary learning objective; however, accessing enabling or extending prompts may modify, add or remove secondary learning objectives (Russo & Hopkins, 2017a). It is important that all students work on a similar core task with a focus on the same primary learning objective to lay the foundation for a highly-participatory, meaningful and inclusive discussion around the relevant mathematics.

STRUCTURING LESSONS INVOLVING CHALLENGING TASKS

It is often assumed that work on the challenging task should precede any teacher-facilitation discussion of the key mathematical ideas (Sullivan et al., 2014; although see Russo and Hopkins, 2017b, for an exception). Consequently, teaching with challenging tasks typically involves a three-stage process: the *launching* of the task by the teacher, the *exploration* of the task by the students, and the whole-class, student-centred *discussion* (and teacher-led *summary*) of student solutions and the underpinning mathematical concepts (Stein, Engle, Smith, & Hughes, 2008).

The teacher begins by *launching* the challenge, which involves presenting the problem, engaging students in the relevant mathematical mindset, and highlighting resources students have at their disposal (e.g., enabling prompts, concrete mathematical materials). After the challenge is launched, students *explore* the task, either individually or collaboratively, and the teacher encourages students to develop at least one potentially appropriate solution. The next stage of the lesson involves the teacher facilitating a whole-group *discussion*, which provides students with an opportunity to present their particular approach to solving the task.

As highlighted by Stein et al. (2008), this discussion component generally involves the teacher organising student responses in increasing order of mathematical sophistication. This sequential structure supports meaningful student participation and helps to build on the discussion of key mathematical concepts. However, the Stein et al. note that effectively coordinating this discussion can require considerable practice, skill and planning. Consequently, teachers beginning to experiment with challenging tasks in their classrooms should view it as a learning opportunity and not be overly self-critical or immediately discouraged if the discussion does not flow in the manner they expect. The teacher will usually close the lesson by offering a brief *summary*, reiterating the learning objective(s) and presenting a sample of student work that supports this objective.

More recently, an additional phase to lessons involving challenging tasks has been put forward. Specifically, there has been an emphasis on students engaging with supplementary tasks to consolidate their understanding following the discussion phase of the challenging task, and prior to the teacher summary (Russo & Hopkins, 2017c; Sullivan et al., 2014). Therefore, the three-stage lesson structure suggested by Stein et al., (2008) could be re-construed as a five-stage structure, as it now makes sense to separate out the teacher-led discussion and the teacher-directed summary stages. This revised five-stage structured for teaching with challenging tasks is presented below, with approximate periods for each phase indicated in parentheses (assuming a 60-minute block):

1. Launch (5 minutes)
2. Explore (20 minutes)
3. Discuss (15 minutes)
4. Consolidate (15 minutes)
5. Summarise (5 minutes)

Developing meaningful consolidating tasks that complement a given challenging task is an integral focus of the remainder of this paper.

CHALLENGING AND CONSOLIDATING TASKS, AND THE 'BIG IDEAS' IN MATHEMATICS

Challenging and consolidating tasks operate in unison to allow students to effectively grapple with the Big Ideas in mathematics. A Big Idea refers to 'an idea that is central to the learning of mathematics… that links numerous mathematical understandings into a coherent whole' (Charles & Carmel, 2005, p. 10). For example, a Big Idea in primary school mathematics is the notion that 'the base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value' (Charles & Carmel, 2005, p. 13).

In the first instance, the challenging task builds conceptual understanding through providing a context in which students can engage with the relevant Big Idea. It is a reflection of the deliberate design of such tasks (including, but not limited to, the high level of cognitive demand) that it is expected that several students will consider these Big Ideas simply through their engagement with the task itself, and independent of any teacher instruction or explanation. For other students, engagement with these Big Ideas is instead more likely to occur during the discussion phase of the lesson, when students have an opportunity to reflect on how they approached the task and become both more explicitly aware of the mathematics they employed and the relative efficiency of their particular strategy. The key is that, in either instance, students are not having the Big Idea explicitly unpacked and explained to them. Rather, the Big Idea appears to emerge organically out of the process of students undertaking the task, justifying their solution method, listening to other student approaches, and, importantly, listening to the teacher make links between student approaches and the associated mathematical concepts. The major role of the teacher during the discussion phase is therefore to contextualize, organise and shape student work, mapping it onto the underlying mathematical structure (i.e., Big Idea). This is what is meant by the phrase a 'student-centred discussion'.

Having primed student engagement with the relevant Big Idea, the consolidating task(s) offers students the opportunity for problem-based practice working on tasks which are similar in structure to the main task, however involve a lower level of cognitive demand. The goal of the consolidating task(s) is to build mathematical fluency in the context of student engagement in a given Big Idea. This in turn implies that there are multiple mathematical fluencies, each associated with a particular Big Idea.

We will now present three examples of challenging tasks, with the associated consolidating tasks included. Each task will relate to a different Big Idea in mathematics, as presented by Charles and Carmel (2005). The tasks are targeted at Year 1 and Year 2 students.

TASK 1: SKIP-COUNTING PATTERNS

Big Idea: *Patterns*. 'Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways' (Charles & Carmel, p. 17).

Primary Learning Objective: Overlaying multiple skip counting sequences will result in some numbers being covered more than once (i.e., numbers with many factors) and some numbers not being covered at all (i.e., potential prime numbers).

Challenging Task: *Twos, threes, fours and fives; which numbers will survive?* Starting at 0, I skipped counted by 2's to 40, crossing off the numbers as I went. Then I did the same thing, but instead skip counted by 3's. Next, I did it by 4's. Finally, I skip counted again, but counted by 5's. Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 10 numbers survived? Now check if you are right.

Enabling Prompt: Hint about skip-counting patterns (see Figure 1).

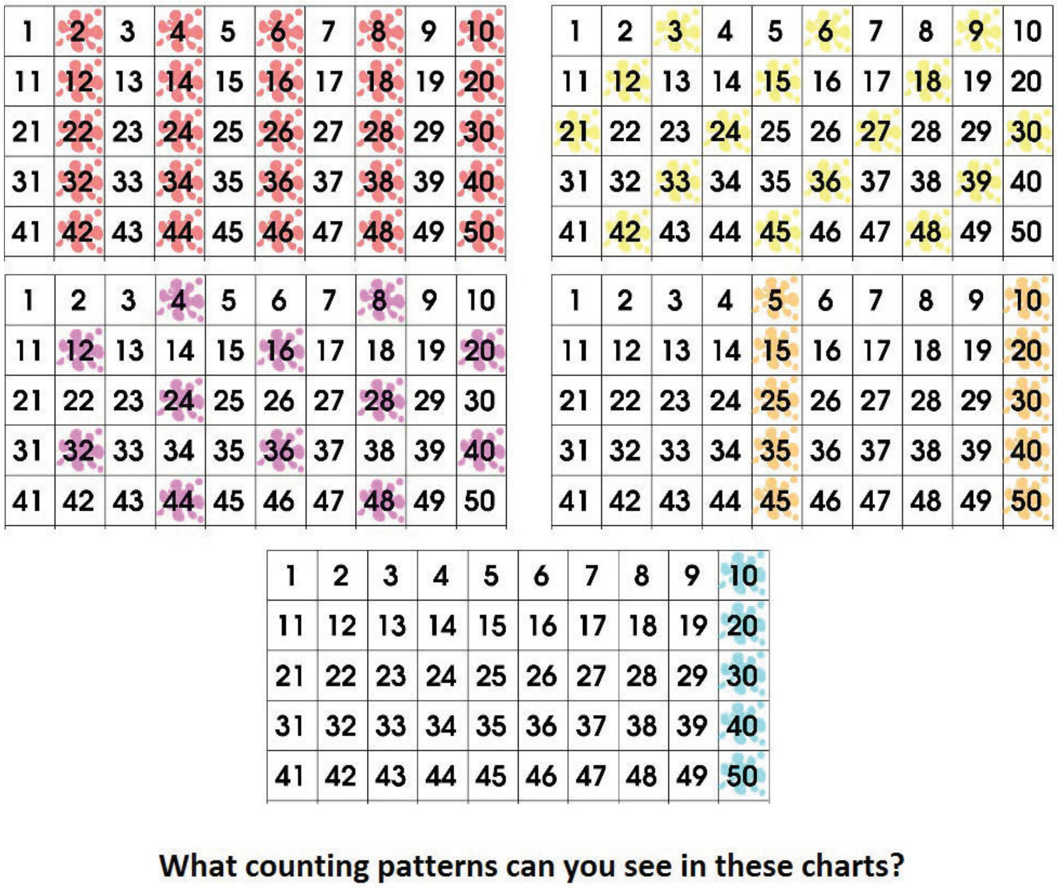

Figure 1. A hint to students about the relevant skip-counting patterns.

Extending Prompt: What if I also skip counted by 6's, 7's, 8's, 9's and 10's? Would all 10 numbers still survive? How many more numbers would get crossed off?

Consolidating Tasks:

- Starting at 0, I skip counted by 2's to 20, crossing off the numbers as I went. Next, starting at 0, I skip counted by 3's to 20, crossing off the numbers as I went. Some numbers were crossed off more than once, but some numbers survived – they weren't crossed off at all. Can you guess which 7 numbers survived? Now check if you are right.
- Starting at 0, I skip counted by 2's to 20, placing a counter on all the numbers I landed on. Next, I skip counted by 5's to 20, again placing a counter on all the numbers I landed on. Finally, I skip counted by 10's to 20, again placing a counter on all the numbers I landed on? What are the numbers with three counters on them – the numbers I landed on three times?
- Starting at 0, I skip counted by 3's to 20, placing a counter on all the numbers I landed on. Next, I skip counted by 5's to 20, again placing a counter on all the numbers I landed on. What is the only number with two counters on it?

TASK 2: EQUIVALENCE

Big Idea: *Equivalence*. 'Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.' (Charles & Carmel, p. 14).

Primary Learning Objective: Towers of a given height can be represented in a number of different ways.

Challenging Task: *Towers* (adapted from Sullivan, 2017). The class is presented with an image of Unifix towers (or actual Unifix towers), as per Figure 2. Students are given the instructions:

1. Create the towers like the ones in the picture.
2. Can you create two towers of equal height using *all* of these smaller towers? Record your solution.

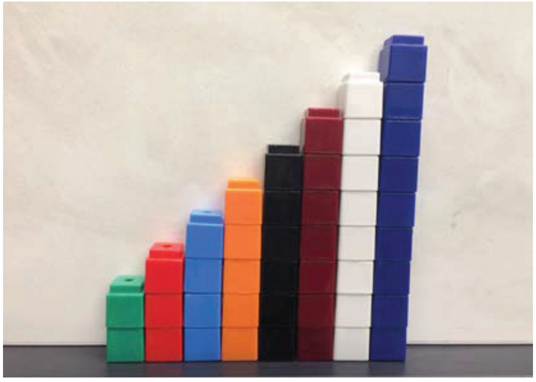

Figure 2. Image of the smaller towers for Tower challenge.

Enabling prompt: The student is presented with an adapted image of Unifix towers (see Figure 3). Students are given the instructions:

1. Create the towers like the ones in the picture.
2. Can you create two towers of equal height using *all* of these smaller towers? Record your solution.

Figure 3. Enabling prompt for Tower challenge.

Extending prompt: Can you do it another way? How many solutions can you find?

Consolidating Task:

1. Create the towers like the ones in the picture (see Figure 4).
2. Can you create a tower that is exactly 10 blocks tall from these smaller towers? Record your solution.
3. There are exactly nine solutions. Can you find them all?

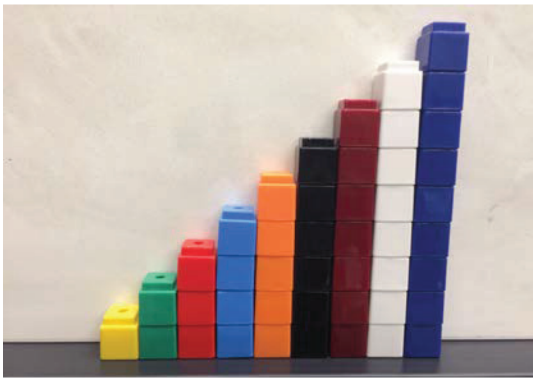

Figure 4. Consolidating task for Tower challenge.

TASK 3: PARTITIONING

Big Idea: *Basic Facts and Algorithms*. 'Basic facts and algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones.' (Charles & Carmel, p. 16).

Primary Learning Objective: When adding a sequence of numbers, partitioning and regrouping into lots of 10 can make addition easier.

Challenging Task: *Christmas Shopping*. Jeffrey did some Christmas shopping for his two sisters. He decided to get them both tickets to see Katie Perry in concert. The tickets cost $99 each. He also got his dog, Sook, a plastic bone for $2. How much money did he spend on his Christmas shopping?

Enabling prompt: Students are presented with a simpler problem (19 + 19 + 2 =), and a hint about the relevant partitioning (see Figure 5).

- 19 + 19 + 2 = ?
- If we break the 2 into 1 and 1, we can rewrite the number sentence as 19 + 19 + 1 + 1= ?

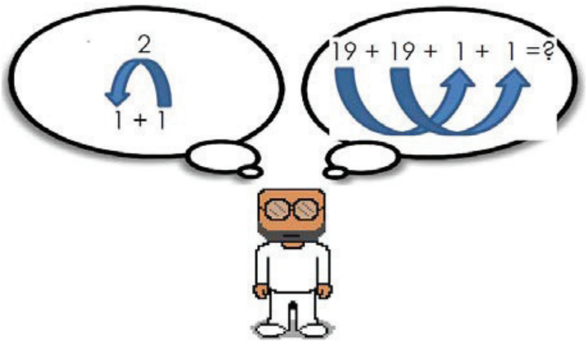

Figure 5. Enabling prompt for Christmas Shopping challenge.

Extending prompt: Jeffrey forgot that his 4 cousins and their dog Fletcher were also going to be at his house on Christmas day. He decided to buy his cousins entrance to Luna Park, which cost $49 each. For Fletcher, he bought a new collar for $5. How much more money did poor Johnny have to spend?

Consolidating Tasks:

- 9 + 4 = ?
- 9 + 7 = ?
- 3 + 9 = ?
- 19 + 6 = ?
- 29 + 29 + 2 = ?
- 19 + 9 + 19 + 3 = ?
- 49 + 39 + 2 = ?
- 4 + 18 + 19 + 9 = ?
- 5 + 19 + 18 + 19 = ?

CONCLUDING REMARKS

Consistent with Sullivan et al. (2014), we have argued that consolidating tasks should be considered a critical aspect of teaching with challenging tasks and importantly, should be presented in the same lesson. Specifically, we have argued that such tasks are vital for building mathematical fluency, after exploration and discussion of the challenging task has primed student engagement with the relevant Big Idea. The three examples of consolidating tasks that we have provided suggest that such tasks can come in a variety of forms. Task 1 provides a series of three consolidating tasks, each that has a similar structure to the original challenging task, however is less complex and cognitively demanding. By contrast, the consolidating task for Task 2 has the structure of an additional challenging task in itself. The key difference is that students are more likely to have success with this consolidating task than with the main challenge (i.e., to find at least one solution). However, the invitation for students to find all solutions associated with the consolidating task (and the implication that they need to be systematic in order to do so) ensures that the task still has an appropriately high ceiling. Finally, Task 3 provides nine consolidating tasks designed to encourage students to continue to experiment with partitioning strategies to support addition. The tasks are presented in order of increasing complexity, again to ensure that students are confronted with an appropriate level of challenge.

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[1] In response to reviewer feedback requesting more information about how to teach with challenging tasks, the first part of this paper draws heavily on an article that appeared in *Australian Primary Mathematics Classroom* (Russo, 2016).