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Reliable Sensor Location for Object Positioning and Surveillance via Trilateration

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Abstract

Object positioning and surveillance has been playing an important role in various indoor location-aware applications. Signal attenuation or blockage often requires multiple local sensors to be used jointly to provide coverage and determine object locations via mobile devices. The deployment of sensors has a significant impact on the accuracy of positioning and effectiveness of surveillance. In this paper, we develop a reliable sensor location model that aims at optimizing the location of sensors so as to maximize the accuracy of object positioning/surveillance under the risk of possible sensor disruptions. We formulate the problem as a mixed-integer linear program and develop solution approaches based on a customized Lagrangian relaxation algorithm with an embedded approximation subroutine. A series of hypothetical examples and a real-world Wi-Fi access point design problem for Chicago O’Hare Airport Terminal 5 are used to demonstrate the applicability of the model and solution algorithms. Managerial insights are also presented.

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1. Introduction

High-accuracy object positioning has been playing a critical role in various application contexts such as: (i) civilian uses, including vehicle navigation, driver guidance, activity tracking; (ii) industrial uses, such as aircraft tracking, regional surveillance, extrasolar planets detection; and (iii) military uses, such as search/rescue missions, missile and projectile guidance. In recent several decades, many object positioning systems with different infrastructures and architectures have been developed. Examples include the Global Positioning System (GPS), cellular phone based systems, computer vision systems (Krumm et al., 2000) – each has its own property, configuration, and reliability. Among them, GPS is the most popular and widely-used system. However, location detection is generally less accurate by GPS inside a blocked space (e.g., inside a building or an underground area) due to attenuation or blockage of satellite or phone signals. In recent years, massive availability of mobile devices has stimulated demand for indoor
High-accuracy object positioning has been playing a critical role in various application contexts such as: (i) civilian uses, including vehicle navigation, driver guidance, activity tracking; (ii) industrial uses, such as aircraft tracking, rescue missions, missile and projectile guidance. In recent several decades, many object positioning systems with different applications have been developed. Examples include the Global Positioning System (GPS), cellular phone based architectures have been developed. Among them, GPS is the most popular and widely-used system. However, location detection is generally less accurate due to signal scattering and blockage, the collected or calculated distance information may be inaccurate, and the error (illustrated by the size of the shaded area) normally increases with the distance between the sensor and the object. In such cases, a location cannot be precisely identified from three distance measures. Navidi et al. (1998) used statistical methods to quantify the uncertainty in trilateration results from the potential error of distance measurements. It is worth noting that the positioning error can be reduced if accurate information from additional sensors is used, as indicated by the smaller green area in Fig. 1(c). Therefore, the effectiveness of an indoor positioning system highly depends on the quality (working range and precision level) and quantity of sensor coverage in the local area. Nevertheless, the number and location of sensors should be carefully determined, because high-precision sensors could be expensive to deploy, the system architecture becomes more complex, and the trilateration algorithm requires more computation time when more sensors are used. We would like to achieve the best positioning accuracy with a reasonable investment in sensor installations.

Extensive studies have been conducted to determine the optimal deployment of sensors. Gentili and Mirchandani (2012) provided a comprehensive literature review on existing sensor location models in traffic networks. Many of those studies aim at maximizing sensor coverage or minimizing the error/cost of estimation. Mirchandani et al. (2010) addressed the problem of locating surveillance infrastructure to cover a target surface; possible barriers that may block sensing signals were considered. Erdemir et al. (2008) developed models to study a location covering problem with consideration of both nodal and path-specific demand. Geetla et al. (2014) studied the deployment of omni-directional audio sensors that can detect vehicle crashes on a roadway. Eisenman et al. (2006) proposed a sensor location problem based on a simulation-based real-time network traffic estimation and prediction system. Fei and Mahmassani (2011) presented a multi-objective model that deploys a minimal number of passive point sensors in a roadway network considering link information gains and origin-destination demand coverage. Dancyzyk et al. (2016) developed a sensor location model to minimize the error of monitoring freeway traffic condition. Various customized solution methods...
for sensor location problems have also been developed. Among them, Wang et al. (2005) partitioned the sensing field into smaller sub-regions and deployed sensors in these sub-regions when the working range of a sensor forms an arbitrarily shaped region (i.e., polygon). Clouqueur et al. (2003) developed a sequential decision-making approach to maximize the exposure of network travel paths to a set of sensors. The overall goal was to minimize the system cost needed to achieve a desired exposure rate. Zou and Chakrabarty (2004) proposed a virtual force strategy for sensor deployment and a probabilistic target localization algorithm to enhance sensor coverage. He (2013) presented a graphical approach to find the smallest set of network links to locate sensors, so as to infer the traffic flow on all other links. Ouyang et al. (2009) and Peng et al. (2011) investigated ways to deploy wayside sensors in a railroad network to monitor railcar traffic. Studies on optimal sensor placement, especially those in the context of trilateration, are quite limited. While deploying directional sensors that collectively form regular convex polygons, Xie and Dai (2014) optimized the number of edges and length of these polygons so as to maximize coverage accuracy. As sensor deployment on a regular lattice is usually not optimal for trilateration, Roa et al. (2007) proposed a diversified local Tabu search method where omni-directional sensors can follow a non-regular configuration. De Stefano et al. (2015) investigated the placement of sensors on an engineering structure to detect the existence, location and extent of internal damage. These studies, however, assumed that the sensing targets are homogeneously distributed in a 2-dimensional plane; this is often unrealistic in the real world. Indeed, sensor locations are critical to the overall performance of the surveillance system. For example, Ahmed et al. (2014) demonstrated the significance of sensor location in influencing real-time traffic state prediction after traffic crashes.

These above-mentioned studies generally assumed that sensors would be always functioning once deployed. However, in reality, installed sensors are subject to operational disruptions from time to time due to technological defects, adverse environmental conditions, deliberate sabotages, etc. If a sensor is disrupted, no useful information can be collected, and the effectiveness of object positioning may be affected. This is illustrated in Fig. 1(d): if three sensors are needed to “cover” an object or an area via trilateration, when a first-choice sensor is disrupted, a more remote sensor will be used and yet this may yield a larger error (as compared to the scenario in Fig. 1(b)). Therefore, the impacts of sensor disruptions should be considered in a reliable sensor deployment framework such that the overall expected error across the normal non-failure scenario and various sensor failure scenarios is minimized. There have been several research efforts on reliable facility location in the context of logistics networks (Snyder, 2006; Cui et al., 2010; Li and Ouyang, 2010; Xie et al., 2016). More recently, Li and Ouyang (2011, 2012) investigated reliable traffic sensor deployment in a discrete transportation network to estimate OD flow volume, congestion state, and path travel time. Adjacent sensors along each flow path pair up to monitor the road segment in between, and yet sensors could be disrupted with site-dependent probabilities. Existing studies, therefore, have considered reliable facility/sensor deployment when sensors work either independently or in pairs. To the best of our knowledge, no studies have considered reliable sensor location problem in the context of trilateration, where at least three sensors are required to work together to position a target (or to cover an area).

This paper aims to fill this gap by incorporating the impacts of sensor disruptions into a reliable sensor deployment framework for positioning or surveillance via trilateration. An object is positioned based on distance measurements received from a combination of at least three sensors. Since various sensor combinations could share some common unreliable sensors, failure of a combination could be directly related to that of another combination. This leads to internal correlation among the functionality of sensor combinations. In this case, where to deploy sensors, which combinations of sensors to use, and in what sequence and probability to use backup combinations in case of disruptions, are nontrivial questions. It remains an open challenge to optimize sensor deployment locations that maximize the overall system-wide surveillance or positioning benefits under the risk of site-dependent sensor failures. In this paper, we address the problem by combining and extending the ideas of assigning back-up sensors (Li and Ouyang, 2010, 2011, 2012) as well as correlation decomposition via supporting stations (Li et al., 2013; Xie et al., 2015, 2016). A compact mixed-integer mathematical model is developed to determine the optimal sensor location, sensor level assignment and combination selection plans. A customized solution algorithm based on Lagrangian relaxation and branch-and-bound is developed, together with an embedded approximation subroutine for sub-problems. A series of hypothetical and empirical case studies are conducted to illustrate the applicability and performance of the proposed methodology.

The remainder of the paper is organized as follows. Section 2 introduces the methodology we develop for the reliable sensor deployment problem, including the effectiveness measurements and the model formulation. Section
3 presents the solution algorithm. Section 4 demonstrates the applicability of the model and solution algorithm on a series of examples. Section 5 summarizes the paper and discusses future research directions.

2. Methodology

We consider an area (e.g., airport, shopping mall) which contains a set of spatial neighborhoods \( I := \{i\} \) that need surveillance coverage. In airports, such neighborhoods can be security check gates, boarding gates and restaurants where accidents are more likely to occur due to crowds’ gathering. Each point \( i \in I \) attracts \( v_i \) customers per day. Let \( J \) be the set of candidate locations for potential sensor installations. At most one sensor can be installed at each location \( j \in J \) at a construction cost \( f_j \). Let \( d_{ij} \) denote the distance from surveillance neighborhood \( i \) to sensor location \( j \). A sensor located at \( j \) could be disrupted with a probability of \( p_j \). For a neighborhood \( i \in I \), the sensors are assigned with different backup levels. We assume the receiver (can be the mobile device/object itself) always uses \( N \), where \( N \geq 3 \), sensors with the lowest backup levels to calculate the position of the object. Without loss of generality, for modeling convenience, \( N \) dummy sensors (located at \( |J| + 1, \ldots , |J| + N \)) are added to the system to ensure there are always at least \( N \) sensors available even under the worst case scenario in which all sensors are disrupted. Let \( \tilde{J} \) be the set of dummy sensors and \( \mathcal{J} = \tilde{J} \cup \tilde{J} \) be the set of all sensors. The dummy sensors incur 0 installation cost and are not subject to failure, but make no contribution to object positioning. Let \( K \) be the set of candidate sensor combinations to locate customers. Each combination \( k \in K \) contains exactly \( N \) sensors (including the dummy ones) and could monitor \( i \) with accuracy \( e_{ik} \). Let \( \alpha \) be the monetary value of sensing accuracy. We introduce incidence matrix \( \{a_{kj}\} \) to represent the mapping relationships between combinations and sensors, where \( a_{kj} = 1 \) if combination \( k \) contains sensor \( j \), or 0 otherwise. The maximum number of combinations is \( \sum_{r=0}^{N} \left( \begin{array}{c} N \\ r \end{array} \right) \), where \( t \) indicates that \( t \) regular sensors and \( N - t \) dummy sensors are used in the combination.

As such, the receiver/object will search from the sensor with the lowest backup level until \( N \) sensors have been found. The key decision variables \( X := \{X_j\} \) determine sensor locations, where \( X_j = 1 \) if a sensor is installed at location \( j \) or \( X_j = 0 \) otherwise. For each surveillance neighborhood, the installed sensors are assigned to it at different levels. Variables \( Z := \{Z_{ijr}\} \) determine the relative sensor levels, where binary variable \( Z_{ijr} = 1 \) if sensor \( j \) is installed and is assigned with level \( r \) to neighborhood \( i \), or 0 otherwise; \( Y := \{Y_{ikr}\} \) denote the sensor combination assignment to the customers, where \( Y_{ikr} = 1 \) if neighborhood \( i \) uses combination \( k \) whose highest level element sensor has level \( r \), or 0 otherwise. Note that a combination \( k \) corresponds to only one level \( r \), while there may exist multiple combinations corresponding to the same level \( r \). The backup levels are initially assigned to the sensors. A level \( r \), if associated with a sensor combination \( k \), indicates the highest level of any sensor contained in \( k \); it can be uniquely determined from the backup levels of sensors that are assigned to an object, i.e., \( \{Z_{ijr}\} \). \( P := \{P_{ikr}\} \) are quasi-probability variables where \( P_{ikr} \) defines the probability to use combination \( k \) to monitor neighborhood \( i \) if \( Y_{ikr} = 1 \), and is a state variable if \( Y_{ikr} = 0 \).

This sensor location problem (SLP) can now be formulated as the following mixed-integer non-linear program:

\[
\text{(SLP)} \quad \min_{X,Y,Z,P} \sum_{j \in J} f_j X_j - \alpha \sum_{i \in I} \sum_{k \in K} \sum_{r=1}^{\left[ \frac{N}{r} \right]} v_i e_{ik} P_{ikr} Y_{ikr} \tag{1a}
\]

s.t. \( \sum_{r=1}^{\left[ \frac{|J|}{r} \right]} Z_{ijr} \leq X_j, \forall j \in J, i \in I, \) \hspace{1cm} \tag{1b}

\( \sum_{r=1}^{\left[ \frac{|J|}{r} \right]} Z_{ijr} \leq 1, \forall j \in J, i \in I, \) \hspace{1cm} \tag{1c}

\( \sum_{r=1}^{\left[ \frac{|J|}{r} \right]} Z_{ijr} = 1, \forall j \in \tilde{J}, i \in I, \) \hspace{1cm} \tag{1d}

\( Z_{ijr} = Z_{i,j+1,r+1}, \forall r = 1, 2, \ldots , |\mathcal{J}| - 1, j \in \tilde{J} \setminus (|J| + N), i \in I, \) \hspace{1cm} \tag{1e}

\( \sum_{j \in J} Z_{ijr} + \sum_{s=1}^{|\mathcal{J}|} Z_{i,jr+1,s} = 1, \forall r = 1, \ldots , |\mathcal{J}|, i \in I, \) \hspace{1cm} \tag{1f}
Proof. Consider another solution $Y_{ikr}$, where $P_{ikr} Y_{ikr}$ is the probability that combination $k$ is used by neighborhood $i$ at the $r$-th level. Constraints (1b) ensure that customers can only use installed sensors. Constraints (1c) indicate that for a certain surveillance neighborhood, each regular sensor can only be assigned to at most one level. Constraints (1d) ensure for a certain surveillance neighborhood, each dummy sensor must be assigned to it at a certain backup level. The same dummy sensor could be assigned to other surveillance neighborhoods at different levels. Constraints (1e) postulate that if a dummy sensor $j \in J$ is assigned to surveillance neighborhood $i$ at level $r$, then dummy sensor $j + 1$ must be assigned to $i$ at level $r + 1$. Constraints (1f) require that at each level $r$, a surveillance neighborhood $i$ either uses a regular sensor, or it has used the first dummy sensor at level $s \leq r$. Constraints (1g) enforce that combination $k$ is available to surveillance neighborhood $i$ only if the $N$ sensors in $k$ are all installed. Constraints (1h) require that combination $k$ is available to surveillance neighborhood $i$ when its highest level element serves at level $r$. Constraints (1i) and (1j) recursively define the assignment probability $P_{ikr}$ for $Y_{ikr}$, where the indicator function $1_{[\cdot]} = 1$ when condition $[\cdot]$ holds, or 0 otherwise. Please note that $P_{ikr}$ does not have physical meaning when $Y_{ikr} = 0$ and its value will not affect the value of the objective function. Given that the lower level sensors are used earlier, a combination $k$ is used if and only if its element sensors are all functioning, and the other constructed sensors which has level lower than the highest level in $k$ are all disrupted. The derivation of $P_{ikr}$ is shown as follows.

**Proposition 1.** The assignment probability $P_{ikr}$ (for $Y_{ikr} = 1$ to happen) can be calculated recursively by (1i), given its initial state value defined by (1j).

**Proof.** We substitute $P_{ikr-1}$ in the right hand side of (1i) by a function of $P_{ikr-2}$ and repeat this procedure for the new right hand side value until $r = 0$. The quasi-probability of neighborhood $i$ using combination $k$ at level $r$ can be written as

$$P_{ikr} = \prod_{j \in J} (1 - p_j)^{a_{kj}} \prod_{j \in J} \frac{\sum_{s \leq r} Z_{ijr} (p_j)^{a_{ks}}}{\sum_{s \leq r} Z_{ijr} (p_j)^{1_{[s \leq r]}}},$$

(2)

When combination $k$ contains a dummy sensor, the indicator function excludes the dummy sensor from the $P_{ikr}$ calculation so as to prevent the case $P_{ikr} = 0$, from happening. If $Y_{ikr} = 1$, the element sensors in combination $k$ must be all functioning and the other constructed sensors which has a backup level lower than $r$ are all disrupted. In this case, the probability that combination $k$ is used by neighborhood $i$ can thus be written as $\prod_{j \in J} (1 - p_j)^{a_{kj}} \prod_{s \leq r} \sum_{j \in J} Z_{ijr} (p_j)^{1_{[s \leq r]}}$, which can be shown to equal (2) after some simple algebraic calculation. Namely, the quasi-probability calculated by (2) is equivalent to the assignment probability that neighborhood $i$ uses combination $k$ (i.e., when $Y_{ikr} = 1$).

Since $Y_{ikr} = 0, \forall r \neq r$ must hold if $Y_{ikr} = 1$, the corresponding $P_{ikr}$ only serves as a state variable (not a real probability), which will facilitate the calculation of the actual assignment probability $P_{ikr}$ given that $Y_{ikr} = 1$. However, the value of the non-probability variable $P_{ikr}$ will not affect SLP as $Y_{ikr} P_{ikr} = 0$ when $Y_{ikr} = 0$ in the objective function (1a).

This recursive formula (1i), together with the initial value $P_{ik0}$ specified by (1j) jointly define the quasi-probability in (2), which is also equivalent to the actual assignment probability $P_{ikr}$ for $Y_{ikr} = 1$ to happen.
In (SLP), the surveillance neighbourhood $i$ can choose any installed sensors and assign them to various levels flexibly to minimize the inaccuracy penalty. However, at optimality, each neighbourhood $i$ will use all installed sensors, and the backup level of each sensor solely depends on its relative distance to the neighbourhood (i.e., irrelevant to its failure probability), as proved in the following proposition.

**Proposition 2.** In any optimal solution $\{X, Z, Y\}$, for each surveillance neighborhood $i$, an installed sensor must be assigned to a backup level, and a nearer sensor must be assigned to an earlier level; i.e. the following two properties must hold (i) if $X_j = 1$, then $\sum_{r=1}^{|J|} Z_{ijr} = 1$; (ii) if $Z_{ijr} = Z_{ij,r+1} = 1$ for some $i, r$, then $d_{ij} \leq d_{ij'}$.

**Proof.** We first prove property (i) Suppose sensors $J'$ are installed and are used to monitor neighbourhood $i$ with an accuracy contribution of $C_1$. Suppose the newly installed sensor $j$ where $j \notin J'$ is assigned to the last level $|J'| + 1$ (i.e., $Z_{ij,|J'|+1} = 1$) and $j$ together with $J'$ provide a total accuracy contribution of $C_2$. By doing so, all combinations generated by $J'$ and their probabilities to happen are unchanged in the new system $j \cup J'$. The sensor $j$ brings new combinations and thus additional accuracy contribution. (i.e., $Y_{ik,j,|J'|+1} \neq 0$ for some $k$). Hence we have $C_2 \geq C_1$. Since assigning sensor $j$ to the last level $|J'| + 1$ is only a feasible solution to the level assignment problem for sensors $j \cup J'$. $C_2$ is a lower bound to the maximum accuracy $C_3$ provided by sensors $j \cup J'$. Hence we have $C_3 \geq C_2 \geq C_1$. As such, a sensor must be assigned at a certain level to monitor the neighbourhoods once it is installed.

We now prove the second property (ii) by contradiction. Suppose there exist $i, j_1, j_2$ and $r$ such that $Z_{ij_1,r} = Z_{ij_1,r+1} = 1$ and $d_{ij_1} > d_{ij_2}$. We consider a combination $k_{A,j}$ where $j_1$ is its highest level element sensor ($j_1$ has level $r$) and $A$ represent the other element sensors in combination $k$. Assume that the probability for sensors $A$ to be functioning and all the remaining sensors assigned at levels 1 to $r-1$ to be disrupted is $P_{r-1}$. The expected accuracy contribution by $k_{A,j}$ can be written as $\left(1 - P_{r-1}\right)$. We consider another combination $k_{A,j'}$ constituted by sensors $A \cup j_2$. Given that $j_2$ is assigned to level $r+1$, the expected accuracy contribution by $k_{A,j'}$ is $\left(1 - P_{r+1}\right)$. Then the expected accuracy contribution associated with these two backup levels $r$ and $r+1$ is $C_4 = \sum_k e_{ik} P_{r-1}(1 - P_{r-1}) + e_{ik} P_{r-1}(1 - P_{r+1})$. Consider another solution $\{X, Z, Y\}$ where $P_{r-1} = P_{r+1} = 1$. The associated expected accuracy contribution is $C_5 = \sum_k e_{ik} P_{r-1}(1 - P_{r-1})$. Since $e_{ik} < e_{ik'}$ holds according to $d_{ij_1} > d_{ij_2}$, simple algebra shows that $C_5 > C_4$ and the expected accuracy contribution for assignments $Y$ at other levels are exactly the same in these two solutions. As such, the latter solution $\{X, Z, Y\}'$ is better than $\{X, Z, Y\}$, which poses a contradiction. This completes the proof. 

The current model is nonlinear due to the existence of nonlinear terms $P_{ikr}Y_{ikr}$ in (1a) and $Z_{ijr}P_{ikr-1}$ in (1i). Linearization techniques introduced by Sherali and Almedinne (1992) (similar to those in Li and Ouyang (2012)) can be applied: i.e., we replace each $P_{ikr}Y_{ikr}$ and $Z_{ijr}P_{ikr-1}$ by new continuous variables $W_{ikr}$ and $V_{ikr}$, respectively, and enforce their equivalence by adding the following sets of constraints where $M_k$ is the maximum value of $P_{ikr}$ with $M_k = \prod_{j \in J}(1 - p_j)^{a_{ij}}$.

\[
\begin{align*}
W_{ikr} &\leq P_{ikr} + M_k (1 - Y_{ikr}), \forall k \in K, r = 1, \cdots, |J|, i \in I, \\
W_{ikr} &\geq P_{ikr} + M_k Y_{ikr} - 1, \forall k \in K, r = 1, \cdots, |J|, i \in I, \\
W_{ikr} &\leq M_k Y_{ikr}, \forall k \in K, r = 1, \cdots, |J|, i \in I, \\
W_{ikr} &\geq -M_k Y_{ikr}, \forall k \in K, r = 1, \cdots, |J|, i \in I, \\
V_{ikr} &\leq P_{ikr-1} + M_k (1 - Z_{ijr}), \forall k \in K, j \in J, r = 1, \cdots, |J|, i \in I, \\
V_{ikr} &\leq P_{ikr-1} + M_k (Z_{ijr} - 1), \forall k \in K, j \in J, r = 1, \cdots, |J|, i \in I, \\
V_{ikr} &\leq M_k Z_{ijr}, \forall k \in K, j \in J, r = 1, \cdots, |J|, i \in I, \\
V_{ikr} &\geq -M_k Z_{ijr}, \forall k \in K, j \in J, r = 1, \cdots, |J|, i \in I.
\end{align*}
\]

The original (SLP) is then transformed into the following mixed integer linear program, which we call the linearized sensor location problem (LSLP). It remains an NP hard problem, but small instances can be readily solved by existing
solvers (such as CPLEX).

\[
\begin{align*}
\text{(LSLP)} \quad \min_{X,Y,Z,P,W,V} & \quad \sum_{j \in J} f_j X_j - \alpha \sum_{i \in I} \sum_{k \in K} \sum_{r = 1}^{|J|} v_i e_{ik} W_{ikr} \\
\text{s.t.} & \quad (1b) - (1h), (1j) - (1k), (3a) - (3h), \\
& \quad P_{ikr} = \sum_{j \in J} p_{ij}^{k|\mathcal{N}} V_{ikjr}, \forall k \in K, r = 1, \ldots, |\mathcal{J}|, i \in I, \\
& \quad W_{ikr} \geq 0, V_{ikjr} \geq 0, \forall k \in K, j \in \mathcal{J}, r = 1, \ldots, |\mathcal{J}|, i \in I.
\end{align*}
\]

Owing to the formidable size of variables in the model, solving (LSLP) by commercial solvers is still not an easy job. CPLEX fails to obtain a feasible solution for a small size network even after several hours of computation. In the following section, more sophisticated solution approaches are developed to overcome such computational difficulties.

3. Solution Algorithm

3.1. Lagrangian Relaxation

In (LSLP), the sensor location variables \(X\) are correlated with the sensor level assignment variables \(Z\) by constraints (1b), which is further correlated with the sensor combination assignment variables \(Y\) through constraints (1g) and (1h). Such correlation complicates the model and makes the computation challenging. Moreover, a great amount of continuous variables are introduced for linearization, which adds to the computation burden significantly. In the following, we will work with the original (SLP) directly to tackle the problems through various relaxation and approximation techniques. To decouple the correlation between \(X\) and \(Z\), we relax constraints (1b) in (SLP) and add them to objective function (1a) with nonnegative Lagrangian multipliers \(\mu = \{\mu_{ij}, \forall i \in I, j \in J\}\). The relaxed problem becomes:

\[
\begin{align*}
\text{(RSLP)} \quad \min_{X,Y,Z,P} & \quad \sum_{j \in J} (f_j - \sum_{i \in I} \mu_{ij}) X_j - \alpha \sum_{i \in I} \sum_{k \in K} \sum_{r = 1}^{|J|} v_i e_{ik} P_{ikr} Y_{ikr} + \sum_{i \in I} \sum_{j \in J} \sum_{r = 1}^{|J|} \mu_{ij} Z_{ijr} \\
\text{s.t.} & \quad (1c) - (1k).
\end{align*}
\]

Given \(\mu\), the optimal solution of (5a) provides a lower bound to the original (SLP) problem. After the above relaxation, the (RSLP) reduces to two parts, which can be solved separately. The part involving \(X\),

\[
\min_{X_{j \in \{0,1\}}} \left( f_j - \sum_{i \in I} \mu_{ij} \right) X_j,
\]

can be solved by simple inspection; i.e., given any \(\{\mu_{ij}\}\), we can easily find the optimal \(X\) as follows:

\[
X_j = \begin{cases} 
1 & \text{if } f_j - \sum_{i \in I} \mu_{ij} < 0, \\
0 & \text{otherwise}.
\end{cases}
\]

The part involving \(Z\) and \(Y\) can be further separated into individual sub-problems, one for each neighborhood \(i\). For ease of notation, we omit the subscripts \(i\) in \(Z_{ijr}, Y_{ikr}\) and \(P_{ikr}\). The sub-problem (RSLP) with respect to neighborhood
i is:

\[
\text{(RSLP}_i\text{)}: \min -\alpha \sum_{k \in K} \sum_{r=N}^{\lfloor J \rfloor} v_{ij} e_{jr} Y_{kr} + \sum_{j \in J} \mu_j \sum_{r=1}^{\lfloor J \rfloor} Z_{jr}^n \tag{6a}
\]

s.t. \[
\sum_{r=1}^{\lfloor J \rfloor} Z_{jr}^n \leq 1, \forall j \in J, \tag{6b}
\]
\[
\sum_{r=1}^{\lfloor J \rfloor} Z_{jr}^n = 1, \forall j \in J, \tag{6c}
\]
\[
Z_{jr} = Z_{jr+1,r+1}, \forall r = 1, \cdots , |J| - 1, j \in J, j + |J| + N, \tag{6d}
\]
\[
\sum_{j \in J} Z_{jr}^n + \sum_{s=1}^{r} Z_{jr+1,s}^{n+1} = 1, \forall r = 1, \cdots , |J|, \tag{6e}
\]
\[
Y_{kr} = \frac{1}{N} \sum_{j \in J} \sum_{s=1}^{r} a_{kj} Z_{jr}^n, \forall k \in K, r = 1, \cdots , |J|, \tag{6f}
\]
\[
Y_{kr} \leq \sum_{j \in J} a_{kj} Z_{jr}^n, \forall k \in K, r = 1, \cdots , |J|, \tag{6g}
\]
\[
P_{kr} \leq \sum_{j \in J} P_{jr} P_{kr-1}, \forall k \in K, r = 1, \cdots , |J|, \tag{6h}
\]
\[
P_{kr} = \prod_{j \in J} (1 - p_j)^{a_{ij}}(p_j)^{-a_{ij}}, \forall k \in K, \tag{6i}
\]
\[
Z_{jr}, Y_{kr} \in \{0, 1\}, \forall k \in K, j \in J, r = 1, \cdots , |J|. \tag{6j}
\]

(RSLP) can be linearized the same way as (SLP) by adding (3a)-(3h). It is well-known that the optimal objective value of the above (RSLP) for any given $\mu$ provides a lower bound to the original (SLP) problem. According to Proposition 2, a nearer sensor must be assigned to an earlier level at optimum. Based on this property, we can find an upper bound to the original (SLP) quickly through fixing the optimal sensor location decisions $X$ obtained from the relaxed problem (RSLP) and assigning neighbourhoods accordingly. For each neighbourhood $i$, we sort all constructed sensors (i.e., $X_j = 1$) in ascending order of $d_{ij}$ and assign each sensor with a level $r$ equal to its rank in distance (i.e., $Z_{jr} = 1$ if sensor $j$ is installed to be the $r^{th}$ nearest sensor to neighborhood $i$). Based on the level assignment of the installed sensors (the value of $Z$), we enumerate all possible combinations $Y$ to get their total accuracy contribution.

In the remainder of the Lagrangian relaxation solution framework, we use standard sub-gradient technique (Fisher, 1981) to update the multipliers $\mu$; i.e.,

\[
\mu_{ij}^{n+1} = \mu_{ij}^n + s_j^n \left( \sum_{r=1}^{\lfloor J \rfloor} Z_{jr}^n \leq X_j^n \right), \tag{7}
\]
\[
s_j^n = \xi^n (g^* - g_P(\mu^n)) \left\| \sum_{r=1}^{\lfloor J \rfloor} Z_{jr}^n - X_j^n \right\|^2, \tag{8}
\]

where $\mu_{ij}^n$ represents a multiplier in the $n^{th}$ iteration, $s_j^n$ is the step size, $\xi^n$ is a scalar and $g^*$ and $g_P(\mu^n)$ are the best upper bound and the current lower bound, respectively. If the Lagrangian relaxation algorithm fails to find a solution with small enough gap in a certain number of iterations, we embed it into a branch-and-bound (B&B) framework to further close the gap.

However, solving the mixed integer program (RSLP) repeatedly for each neighborhood and across Lagrangian relaxation iterations could still be time-consuming. As such, an approximation approach is developed to quickly identify lower bounds to the relaxed sub-problems (RSLP).
3.2. Approximation of \( P_{kr} \)

Equations (6h) show that \( P_{kr} \) depends on \( P_{k-1} \) and \( Z_{jr} \), which builds connections across the decision variables and brings difficulties in solving the sub-problem. Similar to Cui et al. (2010), we approximate the variable probability \( P_k \) with fixed numbers. For each combination \( k \) with its highest level element sensor assigned at level \( r \), we select the regular sensors which are not in \( k \) and are closer to the monitored neighborhood than its most remote sensor in \( k \). Let the number of qualified regular sensors be \( \kappa \), where \( \kappa < |J| \). We rank those \( \kappa \) regular sensors based on their failure probabilities and let \( j_1, j_2, \cdots, j_\kappa \) be an ordering of the sensors such that \( p_{j_1} \geq p_{j_2} \geq \cdots \geq p_{j_\kappa} \). For \( N \leq r \leq N + \kappa \), we define one set of variables \( \beta_{kr} = \prod_{j \in J}(1 - p_{j})^{\kappa_j} \prod_{l=1}^{r} p_{j_l} \). While for \( r < N \) or \( r > N + \kappa \), we set \( \beta_{kr} = 0 \). Replacing \( P_k \) with \( \beta_{kr} \), we can modify the (RSLP\(_i\)) as:

\[
(\text{DRSLP}_{i}) \quad \min \quad -\alpha \sum_{k \in K} \sum_{r=N}^{J} v_i e_{ik} \beta_{kr} Y_{kr} + \sum_{j \in J} \mu_j \sum_{r=1}^{J} Z_{jr} \\
\text{s.t.} \quad (6b) - (6g), (6j).
\]

**Proposition 3.** The solution to (DRSLP\(_i\)) provides a lower bound to the relaxed subproblem (RSLP\(_i\)).

**Proof.** (DRSLP\(_i\)) is constructed through replacing \( P_k \) with \( \beta_{kr} \) and removing constraints (6h)-(6i). As removing constraints enlarges the feasible region of (RSLP\(_i\)), it will never increase the objective value of this minimization problem. The effect of replacing \( P_k \) with \( \beta_{kr} \) in the objective function is studied under two scenarios where \( Y_{kr} = 1 \) or \( Y_{kr} = 0 \). If \( Y_{kr} = 0 \), the value of \( \beta_{kr} \) won’t affect the optimal objective value as \( \beta_{kr} Y_{kr} = P_k Y_{kr} = 0 \). When \( r < N \), \( Y_{kr} = 0 \) must hold since the most remote sensor in \( k \) must be assigned at a higher level than \( N \). When \( r > N + \kappa \), we also have \( Y_{kr} = 0 \) since a sensor can’t be assigned to a level higher than the total number of sensors who are closer than it. When \( Y_{kr} = 1 \), the probability of using combination \( k \) is \( P_{kr} = \prod_{j \in J}(1 - p_{j})^{\kappa_j} \prod_{l \leq r} [\sum_{j \in J} Z_{jkr}(p_{j})^{1-\kappa_l}] \) according to the proof to Proposition 1. \( P_k \) calculates the probability to have the \( N \) sensors in \( k \) working and \( r - N \) regular sensors disrupted. Based on the construction of \( \beta_{kr} \) where \( N \leq r \leq N + \kappa \), \( \beta_{kr} \) provides an upper bound to \( P_{kr} \) if \( Y_{kr} = 1 \). Therefore, \( \beta_{kr} Y_{kr} \) must be an upper bound to \( P_k Y_{kr} \) for any \( k \in K, r = N, \cdots, |J| \) and the optimal objective value of (DRSLP\(_i\)) is a lower bound to the optimal objective value of (RSLP\(_i\)).

3.3. Approximation of \( Y_{kr} \)

In this section, constraints (6f) and (6g) are replaced by a simple equality formula to decouple the connection between \( Z_{jr} \) and \( Y_{kr} \). For a neighborhood \( i \), the Lagrangian multiplier \( \mu_j \) in (9a) can be interpreted as an extra installation cost of sensor \( j \) and the first term \( \sum_{k \in K} \sum_{r=N}^{J} v_i e_{ik} \beta_{kr} Y_{kr} \) represents the total accuracy contribution of the installed sensors to the system. Given that the \( N \) dummy sensors are always installed and assigned to the highest levels, we let the regular sensors be installed sequentially from level 1 to level \( |J| \). Let binary variables \( \{y_{kr} : k \in K, r \} \) be the combination assignments when \( t \) regular sensors are installed. \( y_{krt} = 1 \) if combination \( k \) is used \((Y_{kr} = 1)\) given \( t \) regular sensors are installed. As such, the total accuracy can be decomposed into \(|J|\) portions, one for each level \( t \). The \( t \)th portion calculates the additional benefits contributed by installing a sensor \( j \) at level \( t \).

The accuracy contribution of all sensors, i.e. the first term in (9a) omitting the constant \( v_i \), can be reformulated as:

\[
AC = \sum_{k \in K} \sum_{r=N}^{J} e_{ik} \beta_{kr} Y_{kr} = \sum_{k \in K} e_{ik} \beta_{kr} Y_{kr0} + \sum_{r=1}^{N} \sum_{k \in K} e_{ik} \beta_{kr} Y_{kr} - \sum_{r=1}^{N+1} \sum_{k \in K} e_{ik} \beta_{kr} Y_{kr,t-1},
\]

where \( \sum_{r=N}^{J} e_{ik} \beta_{kr} Y_{kr0} \) represents the accuracy contribution of the \( N \) dummy sensors, which is 0 by definition; \( \sum_{r=N}^{J} e_{ik} \beta_{kr} Y_{kr} \) states the accuracy level of the system with \( N \) dummy sensors and \( t \) regular sensors; difference of the two terms in the parentheses represents the accuracy improvement by adding one regular sensor at level \( t \) given that \( t - 1 \) regular sensors are already installed. If we expand the summation terms in (10), the intermediate accuracy level \( \sum_{r=N}^{J} \sum_{k \in K} e_{ik} \beta_{kr} Y_{kr} \) for any \( t \) where \( t < |J| \) will be cancelled out. As such, \( AC \) will be simplified as \( AC = \sum_{r=N}^{J} \sum_{k \in K} e_{ik} \beta_{kr} Y_{kr} = \sum_{k \in K} \sum_{r=N}^{J} e_{ik} \beta_{kr} Y_{kr} \), which mathematically proves the second equivalence in (10).

Adding one regular sensor \( j \) at level \( t \) is equivalent to replacing the dummy sensor \(|J| + 1\) with \( j \) and moving all the dummy sensors upward by one level. The resultant accuracy difference for the two systems with \( t \) or \( t - 1 \) regular
sensors can be calculated by updating the accuracy level of every combination relating to sensor \( j \). The total system accuracy is reformulated as:

\[
AC = \sum_{t=1}^{\lfloor J \rfloor} \sum_{r=\max(t,N)}^{N+t} \sum_{k \in K} \left( e_{ik} \beta_{kr} - (1 - p_j) e_{ik} \beta_{kr} \right) 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} Y_{krt},
\]

(11)

where \( J_k \) represents the set of regular sensors in combination \( k \) and parameter \( b_{kj} = 1 \) if \( j \) is the most remote regular sensor in \( k \) and 0 otherwise. In the indicator function \( 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} \) identifies the updated combination \( k \) who has one additional regular sensor \( j \) comparing with an existing combination \( k' \); \( b_{kj} = 1 \) forces this sensor \( j \) to be the most remote regular sensor in \( k \). Inserting \( j \) at level \( t \) brings new combinations – the term \( e_{ik} \beta_{kr} 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} \) calculates the accuracy contribution of a new combination \( k \) which uses \( j \) as its most remote regular sensor. Moreover, inserting \( j \) at level \( t \) changes the assignment level of all dummy sensors. An existing combination \( k' \) which contains dummy sensors in the old system (with \( t - 1 \) regular sensors) will be used in the new system (with \( t \) regular sensors) only when sensor \( j \) is disrupted – the term \( (1 - p_j) e_{ik} \beta_{kr} 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} \) in (11) calculates the contribution deduction due to the probability decrease of using combination \( k' \).

Fig. 2 illustrates the decomposition process in order to calculate the total accuracy level of the system with 3 dummy sensors and \( \lfloor J \rfloor \) regular sensors. Sensors \( a, b, c \), \( \cdots \), \( j \) are sequentially added to the system to calculate their contribution. For example, contribution of sensor \( c \) is equal to the difference in system accuracy when \( t = 3 \) or \( t = 2 \). Let the element sensors-combination index be defined as: \( abc-1, abD1-2, \cdots \). For a neighborhood \( \sum \beta y kr \sum k \), the probability of using combination \( K e ik \) is a lower bound to the optimal objective value of (RSLP 0 since a sensor can't be assigned to a level higher than the total number of sensors who are regular sensors. If we expand the summation terms in (10), the intermediate \( AC \in K e ik \) must be an upper bound to \( AC \sum k \). An, S. Xie, Y. Ouyang / Transportation Research Procedia 00 (2016) 000–000

The total system accuracy when \( t = 3 \) and \( t = 2 \) respectively are

\[
AC_{|t=3} = \sum_{r=N}^{N+3} \sum_{k \in K} e_{ik} \beta_{kr} 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} Y_{krt},
\]

\[
AC_{|t=2} = \sum_{r=N}^{N+2} \sum_{k \in K} e_{ik} \beta_{kr} 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} Y_{krt}.
\]

The contribution of inserting sensor \( c \) at level \( t = 3 \) is

\[
AC_{|t=3} - AC_{|t=2} = [e_{i1} 1_{\beta} + e_{i2} 2_{\beta} + e_{i3} 3_{\beta} + e_{i4} 4_{\beta} + e_{i5} 5_{\beta} + e_{i6} 6_{\beta} + e_{i7} 7_{\beta} + e_{i8} 8_{\beta}],
\]

(12)

where each combination \( k, k = 1, 3, 4 \) or 7, has element sensor \( c \) as its most remote regular sensor, namely \( b_{kc} = 1 \); the combinations paired up in brackets (1 and 2; 3 and 5; 4 and 6; 7 and 8) have the same regular sensors except for sensor \( c \), namely \( J_k \{J_r \& c = k \} \). According to the construction of \( b_{kr} \), the paired probabilities in the parentheses satisfy \( \beta 24 = p, \beta 23, \beta 55 = p, \beta 53, \beta 65 = p, \beta 64 \) and \( \beta 86 = p, \beta 85 \). Substituting \( \beta 8r+1 \) by \( p, \beta 8r \) in the parentheses, we can simplify (12) as follows:

\[
AC_{|t=3} - AC_{|t=2} = [e_{i1} 1_{\beta} - e_{i2} (2_{\beta} - 3_{\beta})] + [e_{i3} 3_{\beta} - e_{i4} (5_{\beta} - 5_{\beta})] + [e_{i5} 4_{\beta} - e_{i6} (6_{\beta} - 6_{\beta})] + [e_{i7} 7_{\beta} - e_{i8} (8_{\beta} - 8_{\beta})],
\]

(13)

For any \( t \) and \( r \) satisfying \( 1 \leq t \leq \lfloor J \rfloor, \max(t,N) \leq r \leq N + t \), a combination \( k \) fulfilling \( 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} Y_{krt} = 1 \) must have \( j \) as its most remote regular sensor, have its most remote sensor assigned at level \( r \) and thus have \( r - t \) dummy sensors. Hence we only need to choose \( N - 1 \) or \( t - r \) regular sensors from the \( t - 1 \) alternatives to get a qualified \( k \). We denote the maximum number of such updated combinations by \( n_{tr} = \binom{N}{N-1-r-t} \). We also let \( C_{i j r k} = (e_{ik} \beta_{kr} - (1 - p_j) e_{ik} \beta_{kr} 1_{\{J_i \cap J_r = j \& \& h_{ij} = 1\}} \) for each \( 1 \leq t \leq \lfloor J \rfloor, \max(t,N) \leq r \leq N + t, 1 \leq j \leq \lfloor J \rfloor, \) let \( k_1, k_2, \cdots, k_{|k|} \) be an ordering of the coefficients \( C_{i j r k} \) such that \( C_{i j r k_1} \geq C_{i j r k_2} \geq \cdots \geq C_{i j r k_{|k|}} \). We define \( \gamma_{ij} = \sum_{t=\max(t,N)}^{N+t} \sum_{r=1}^{t} C_{i j r k} \). Based on the construction of \( \gamma_{ij} \), \( \gamma_{ij} Z_{ij} \) provides an upper bound to the accuracy improvement from inserting sensor \( j \) at level \( t \). Replacing \( AC \) by its upper bound \( \sum_{t=1}^{\lfloor J \rfloor} \sum_{j \in J} \gamma_{ij} Z_{ij}, (DRSLP) \) further reduces to the following simple
assignment problem (TRSLP), which can be solved by the Hungarian algorithm.

\[(\text{TRSLP}) \quad \min - \sum_{r=1}^{\lfloor J \rfloor} \sum_{j \in J} v_{ij} j Z_{jr} + \sum_{r=1}^{\lfloor J \rfloor} \sum_{j \in J} \mu_{ij} j Z_{jr} \quad (14a)\]

\[\text{s.t.} \quad (6b) - (6e), \]

\[Z_{jr} \in \{0, 1\}, \forall j \in J, r = 1, \ldots, \lfloor J \rfloor. \quad (14b)\]

**Proposition 4.** The solution to (TRSLP) provides a lower bound to the relaxed sub-problem (RSLP).

**Proof.** (DRSLP) is constructed through replacing \(\sum_{k \in K} \sum_{r=1}^{\lfloor J \rfloor} v_{ik} \beta_{kr} Y_{kr}\) with \(\sum_{r=1}^{\lfloor J \rfloor} \sum_{j \in J} v_{ij} j Z_{jr}\) and removing constraints (6f)-(6g). As removing constraints enlarges the feasible region of (DRSLP), it will never increase the objective value of this minimization problem. Based on the construction of \(Y_{jr}, \sum_{r=1}^{\lfloor J \rfloor} \sum_{j \in J} v_{ij} j Z_{jr}\) provides an upper bound to \(\sum_{k \in K} \sum_{r=1}^{\lfloor J \rfloor} v_{ik} \beta_{kr} Y_{kr}\). Therefore, the optimal objective value of (TRSLP) is a lower bound to the optimal objective value of (DRSLP). Together with the result in Proposition 3, the solution of (TRSLP) is a lower bound to the relaxed sub-problem (RSLP).

\[\Box\]

4. **Case Study**

To demonstrate the applicability of the proposed models and algorithms, we apply them to a series of hypothetical grid networks as well as a more realistic Wi-Fi Access Point (AP) network in Terminal 5 of the Chicago O’Hare Airport. The proposed solution algorithms are programmed in C++ and run on a 64-bit Intel i7-3770 computer with 3.40 GHz CPU and 8G RAM. The linearized LSLP is tackled by commercial solver CPLEX 12.4 using up to 4 threads. We set the overall solution time limit to be 3600 seconds.

4.1. **Hypothetical grid networks**

A 2x3 rectangle grid network and six \(n \times n\) square grid networks for \(n \in \{3, 4, 5, 6, 7, 8\}\) are generated to represent various hypothetical study regions. In the square grid networks, each network contains \((n - 1)^2\) cells. The four corners of each cell represent the candidate sensor locations, adding to a total number of \(n^2\) candidate sensor locations. The centroid of each cell is constructed to be a surveillance neighborhood, adding to \((n - 1)^2\) neighborhoods. The network layouts are shown in Fig. 3. We omit the surveillance neighborhoods in some of the larger networks (i.e., from 5x5 to 8x8) for cleaner figure presentation. The edge length of each cell is set to 1. The customer demand
of each neighborhood $i$ is $v_i = 10$, the value of $\alpha$ is 1, and the fixed sensor installment cost is 10. The value of coverage is 1. The site-dependent failure probability of sensor location $j$ is assumed to vary from 0.1 to 0.2 based on its Euclidean distance to the center of the study region. The sensor(s) located nearest to the center have the highest failure probability of 0.2, the sensor(s) located farthest away have the lowest probability of 0.1. The failure probability of a sensor in the middle linearly decreases with the distance to the center. Each combination uses $N = 3$ sensors. Combination accuracy is computed based on $e_{ik} = \sum_{j \in J} \frac{d_{ij}}{d_{ij} + \epsilon}, \forall i, k \in K$, where $d_{ij}$ is the Euclidean distance and $\epsilon$ is a small positive number. The reliable sensor location problems are solved by two approaches: (i) CPLEX directly applied to the mixed-integer linear program LSLP and (ii) Lagrangian relaxation based branch-and-bound method with approximation algorithm (LR+B&B+Approx.). Table 1 summarizes and compares the results from the two approaches.

As one can observe from the table, the solution time and solution quality rapidly deteriorate with the network size, due to the significant increase in the number of integer variables $Y$ and $Z$. CPLEX could only find the optimal solution to the specifically constructed small rectangle network. In the second case, CPLEX identified a feasible solution but failed to find a lower bound, despite its rather small network size. For the other larger networks, CPLEX ran out of memory and could not provide a feasible solution or a lower bound. In contrast, optimal solutions to the first 6 cases were obtained by the LR+B&B+Approx. approach within 3 minutes. For the 8×8 network, there is a residue gap of 2.32% after 1 hour of computation.

<table>
<thead>
<tr>
<th>Sensor network size</th>
<th>Neighborhood network size</th>
<th>Number of installed sensors</th>
<th>Final UB</th>
<th>Final LB</th>
<th>Final gap (%)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 3</td>
<td>1 × 2</td>
<td>2</td>
<td>-1.31</td>
<td>-1.30</td>
<td>0</td>
<td>1.6</td>
</tr>
<tr>
<td>3 × 3</td>
<td>2 × 2</td>
<td>4</td>
<td>-14.01</td>
<td>fail</td>
<td>100</td>
<td>3600</td>
</tr>
<tr>
<td>4 × 4</td>
<td>3 × 3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>fail</td>
<td>3600</td>
</tr>
<tr>
<td>8 × 8</td>
<td>7 × 7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>fail</td>
<td>3600</td>
</tr>
<tr>
<td>2 × 3</td>
<td>1 × 2</td>
<td>2</td>
<td>-1.31</td>
<td>-1.31</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>3 × 3</td>
<td>2 × 2</td>
<td>4</td>
<td>-24.28</td>
<td>-24.28</td>
<td>0</td>
<td>0.1</td>
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<td>3 × 3</td>
<td>5</td>
<td>-77.38</td>
<td>-77.38</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>5 × 5</td>
<td>4 × 4</td>
<td>8</td>
<td>-150.78</td>
<td>-150.78</td>
<td>0</td>
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<tr>
<td>6 × 6</td>
<td>5 × 5</td>
<td>14</td>
<td>-243.48</td>
<td>-243.48</td>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>7 × 7</td>
<td>6 × 6</td>
<td>21</td>
<td>-360.99</td>
<td>-360.99</td>
<td>0</td>
<td>181</td>
</tr>
<tr>
<td>8 × 8</td>
<td>7 × 7</td>
<td>29</td>
<td>-489.07</td>
<td>-500.41</td>
<td>2.32</td>
<td>3600</td>
</tr>
</tbody>
</table>

In Fig. 3, the installed sensors in the best solutions from the LR+B&B+Approx. approach are marked green. We can observe that more sensors are installed in order to monitor a larger region. In the first three cases, the installed sensors are clustered in the center of the study region mainly owning to their short distances to all the surveillance neighborhoods, which provides better accuracy with a limited number of sensors. In the four larger cases, it is interesting to observe that the sensors are installed symmetrically along the diagonal lines. Moreover, no sensor is installed immediately next to the boundary, while all nearby candidate locations (e.g., slightly closer to the region center) are selected. Those properties indicate the possibility to decompose a larger yet symmetrical network into several smaller ones to obtain the sensor deployment effectively. Take the 8×8 network for example, if the sensor at coordinate (1, 1) is installed (assuming the bottom left sensor is located at the origin (0, 0)), then we can automatically install the sensor at (6, 6), which could significantly speed up solution process. As such, the proposed algorithm could possibly handle an even larger symmetrical network efficiently.

Fig. 4 illustrates how the sensor combinations are used by the customers in neighborhood $i = 1$ (i.e., indicated by the dark star in Fig. 3) in the 3-by-3 case. The installed sensors 4, 5, 6, 8 are assigned to levels 1 - 4 based on distance, while the dummy sensors are assigned at levels 5 - 7. Some representative combinations are illustrated in this figure. For example, the shaded combination ($k = u$) will be used to monitor this neighborhood if and only if sensors 5, 6 and 8 are functioning and sensor 4 has been disrupted. The most remote sensor in this combination is 8, which is
ranked at level $r = 4$. Hence combination $u$ corresponds to backup level $r = 4$ and it will be used with a probability of $P_{1u4} = p_4 (1 - p_5)(1 - p_6)(1 - p_8) = 0.15 \times 0.8 \times 0.75 \times 0.75 = 0.0675$ based on the sensor failure probability settings.

4.2. Wi-Fi Access Point Network for Chicago O’Hare Airport Terminal 5

The Chicago O’Hare International Airport is one of the busiest airports in the world. In June 2016 alone, a total of 7,329,084 travelers passed through the airport (CDA, 2016). Boingo, the O’Hare Airport’s Wi-Fi provider, has pioneered a new “S.M.A.R.T” network design (Secure, Multi-platform, Analytics-Driver, Responsive and Tiered) which allows increased access point density for location-based services like queue management, advertising, and passenger guidance. Such a system is expected to deliver valuable business intelligence and actionable insights to enable high-quality passenger service.

In this case study, we select the departure level of Terminal 5 to investigate Wi-Fi Access Point deployment for better location-based services. Terminal 5 contains Concourse M, which is used for all international arrivals and part of the international departures (those of most non-US carriers). We select 52 heavy-traffic venues inside the terminal, including 21 gates, 10 restaurants, 13 shops, 6 airline lounges and 1 security check point, as key surveillance
Average hourly surveillance demand at each neighborhood is assumed to be proportional to the local passenger flow per the monthly statistics report of the Chicago Department of Aviation (CDA, 2016). The terminal is further divided into square cells with an edge length of 10 meters\(^1\). The corners of every square cell are considered candidate sensor/AP locations. There are 222 candidate locations in total. Boingo uses Cisco’s AP systems with chipsets featuring 802.11ac standard, with an installation cost of about US$200 each (maintenance cost or other capital cost is not considered). The received signal strength (RSS) follows a logarithm function of distance (Shchekotov, 2014), and hence we assume a combination of sensors will yield an accuracy measure of 
\[ e_{ik} = \sum_{j=1}^{22} 2a_{kj} \log_{10} \left( \frac{40}{d_{ij} \alpha} \right), \] 
where \(d_{ij}\) is the Euclidean distance in meters, and 40 (meters) is the effective range of a Cisco AP. Each combination uses \(N = 3\) sensors.

We consider site-independent, yet low, median and high levels of sensor disruption probabilities; i.e., \(p_j = p \in \{0.01, 0.2, 0.5\}, \forall j\). The system performance measures under these scenarios are presented in Table 2. All results are obtained from the proposed LR+B&B+Approx algorithm within 3600 seconds. Overall, a higher sensor failure probability leads to a fewer number of installed sensors as well as a significant deterioration in the best objective value (i.e., the final UB). The residue gap also increases slightly with the failure probability. The value of \(\alpha\) reflects the tradeoff between the positioning accuracy \(e_{ik}\) and the unit sensor installation cost \(f_j\). Very often the value of \(\alpha\) may be subject to speculation and interpretation. We thus conduct sensitivity analysis over the formula for \(e_{ik}\), and when \(f_j = 200, p_j = 0.01\). When \(\alpha\) increases from 0.025 to 0.4, the number of installed sensors increases drastically from 17 to 111, and the objective function drops by about two orders of magnitude. These results indicate that the benefits of deploying more sensors far outweigh the installation costs in the O’Hare case study.

The optimal sensor locations for the three cases are shown in Fig. 6. The solid-line circles represent the surveillance neighborhoods, and their size indicates the volume of surveillance demand. The installed sensors are marked by shaded squares. They can be roughly clustered into groups, as shown by the dotted ellipses, in which the distances between any adjacent sensors do not exceed 15 meters – i.e., these sensors are likely to provide effective backups to each other. Under a low failure probability, the installed sensors can be clustered into 16 groups, and the sensor nearest to every surveillance neighborhood is always installed. This forms a rather dispersed sensor network overall.

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\(^1\) Generally, access points should be separated by at least 10 feet in order to reduce adjacent channel interference, and it is recommended that APs are mounted at 30-40 feet (or approximately 10 meters) from one another (https://supportforums.adtran.com/docs/D0C-7257).
Fig. 6. Optimal sensor locations under (a) low, (b) median, and (c) high sensor disruption probabilities.
Table 2. Performance measures for the O'Hare Airport case.

<table>
<thead>
<tr>
<th>Failure probability</th>
<th>$\alpha$</th>
<th>Number of candidate sensors</th>
<th>Number of neighborhoods</th>
<th>Number of installed sensors</th>
<th>Final UB</th>
<th>Final LB</th>
<th>Final gap(%)</th>
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In the two wings of the airport, 5 isolated sensors are installed in order to monitor their most adjacent neighborhoods, although these sensors only make marginal contributions to other neighborhoods.

When the sensor disruption probability increases to 0.2 and 0.5, the number of sensor groups drops to 12 and 10, respectively, and fewer isolated sensors are installed. Sensors within a group tend to become more clustered so as to better back each other up. This is clearly illustrated, for example, by the highlighted group (see the bold ellipse). Meanwhile, sensors also tend to cluster around the center of the concourse where demand is the heaviest. For example, 10 sensors are clustered within 20 meters from at the security checkpoint when $p = 0.5$, while there are only 7 when $p = 0.2$ and 5 when $p = 0.01$. In summary, under higher failure probability, the model tends to yield a higher degree of sensor clustering especially around the heavy-demand neighborhoods, while at the same time a smaller total number of sensors would be installed especially around the less crowded neighborhoods.

A closer look at the sensor deployment in the highlighted group (bold ellipse) reveals some interesting points. When $p$ increases from 0.01 to 0.2, sensor #29 is removed from the low demand neighborhood while sensor #34 is added to the high demand neighborhood. Such changes can be explained by the marginal costs and marginal benefits of these sensors. In the case of $p = 0.2$, if we add sensor #29 back, the marginal coverage benefit is $185.7$, which is lower than its installation cost $200$. On the other hand, if we remove sensor #34, the coverage accuracy loss is $252.1$ when $p = 0.2$, which is higher than $200$. This result can be generalized. When the disruption probability increases, the sensors become less reliable, and more sensors will be needed to maintain the same coverage accuracy. In high-demand neighborhoods, the net marginal benefit of installing an extra “back-up” sensor (e.g., to maintain the accuracy) may be high enough to outweigh the installation cost. We hence may observe an increase in the sensor number near those neighborhoods. In low-demand neighborhoods, however, the net marginal benefit of adding a sensor may not justify its cost, and we will therefore expect reduction of sensors. In other words, the spatial distribution of sensors tends to be more clustered near high-demand neighborhoods under high disruption probabilities, but at the same time more sparse near low-demand neighborhoods. The total number of sensors across all neighborhoods may not exhibit a monotonic relationship with the value of $p$.

5. Conclusion

This paper proposes a reliable sensor location model to maximize the accuracy and effectiveness of object positioning and surveillance. The model allows sensor failures to occur with site-dependent probabilities. We first formulate the reliable sensor deployment problem as a mixed-integer linear program. We then develop a customized Lagrangian relaxation and branch-and-bound algorithm (with approximation subroutine designed for subproblems) to effectively solve the mathematical model. A series of computational experiments with grid networks of varying sizes demonstrate that the proposed algorithm far outperforms CPLEX in terms of solution quality and computation time. In particular, the Lagrangian relaxation and branch-and-bound algorithm is able to solve median-size networks with up to 64 candidate sensor locations and 49 surveillance neighborhoods within 1 hour. A real-world application for the AP network design for Chicago O’Hare Airport Terminal 5 is presented to demonstrate the applicability of the model, the efficiency of the proposed algorithms, and draw practical insights.
This study can be further extended in several directions. First, the current model assumes that the sensor failures are independent. However, in the real world, the sensor failures may be correlated and this needs to be addressed in future work. Second, we assume that each object/receiver always uses $N$ sensors, and those with the lowest backup levels are always used for trilateration. We could relax this assumption by allowing each object to be positioned by any combination of surviving/available $N$ sensors in a sensor disruption scenario, and the choice of combination can be determined by the model. Next, our model suggests that sensors should not be installed near low-demand neighborhoods especially when the failure probability is high, e.g., most of the left and right wings are uncovered in Fig. 6(c). In real-world applications, access points may be deployed under other considerations; e.g., for equitable service everywhere, or for security coverage. It would be interesting to extend our model to seek a reliable sensor network design when a certain level of surveillance accuracy must be met for all surveillance neighborhoods. The surveillance neighborhoods or objects are represented by discrete spatial points in this paper. Similar sensor location problem can also be formulated in any continuous metric space where distributed demand is described by a density function. We leave this topic for future investigation. Finally, even though this paper focuses on sensor network planning, the proposed models and algorithms can be easily applied to a broader range of problems where similar resource allocation and positioning decisions must be made. Possible application contexts include: fire-tower placement to localize a forest fire, beacon deployment for the localization of firefighting robots, seismic sensor network configuration for earthquake epicenter calculations, and complex structure health monitoring for bridges and tunnels.

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**References**


