CURRICULUM DOCUMENTATION AND THE DEVELOPMENT OF EFFECTIVE SEQUENCES OF LEARNING EXPERIENCES

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In this article we focus on ways that the documented curriculum can inform the construction and implementation of planned sequences of experiences to support mathematics learning. We report on the early stages of a research project which is examining ways that thoughtfully created, cumulative, challenging and connected experiences can both initiate and consolidate mathematics learning. It is intended that through an iterative cycle of design-test-redesign-retest we will ultimately transform the documented curriculum into a set of refined and empirically developed sequences of learning experiences that are accessible by a diverse range of students.

THE RATIONALE FOR LEARNING SEQUENCES

The focus of this article is on ways that the documented curriculum might be transformed into planned sequences of learning that can inform teaching programs. The article reports on the early stages of a research project which is examining ways that thoughtfully created, cumulative, challenging and connected experiences can both initiate and consolidate mathematics learning.

The prompt for the project was earlier research by Sullivan, Borcek, Walker, and Rennie (2016), which found that cognitive activation is more likely when learning experiences are structured in particular ways, including:

- the ways tasks are posed in the introductory phase;
- actions taken to differentiate the task for students who might require additional support and those who finish quickly; and
- ways that the student activity on the task is reviewed emphasizing students reporting on their explorations and fostering classroom dialogue between students.

Sullivan et al. hypothesised that learning would be further enhanced if purposeful follow up experiences were posed to consolidate the learning. The nature and effectiveness of those follow up learning experiences is the focus of this new project. This process for consolidating learning is connected to considering sequences or trajectories of learning over a longer time frame than the single task and single lesson.

The notion of sequences of learning is informed by Variation Theory which was described by Kullberg, Runesson, and Mårtensson (2013) as follows:

> In order to understand or see a phenomenon or a situation in a particular way one must discern all the critical aspects of the object in question simultaneously. Since an aspect is noticeable only if it varies against a back-ground in invariance (emphasis in original), the experience of variation is a necessary condition for learning something in a specific way. (p. 611)

Similarly, Sinitsky and Ilany (2016) argued that considering both change and invariance illustrates not only the nature of the mathematics but also the process of constructing concepts. In the application of Variation Theory to the creation of sequences intended to consolidate learning prompted by an initial task, the intent is that some elements of the original experience remain invariant, and other aspects vary so that learners can focus on the concepts and not be misled by over-generalisation from a solution to a single example.

The hypothesised advantages of learning sequences are as follows:

Sequences can help students see the 'bigger picture'. One of the disadvantages of conventional approaches to mathematics and numeracy is that mathematics can seem to be broken into sets of micro skills rather than contributing to a coherent whole. Sequences may help students see connections by making the big ideas and progression of learning more obvious to the student.

Concepts are learned as much by what they are not, as from what they are (such as, for example, the attribute of length is different from volume). Carefully varied tasks within sequences can emphasise what the central ideas are (and what they are not) so allowing students to discern the essence of concepts.

Sequences of challenging tasks can prompt "light bulb" moments. But there are no light bulbs if students are told what to do. Students can benefit from working on tasks that are challenging, and progressively see meaning by experiencing connected tasks with success developing progressively especially where the insights or "aha" moments are the result of their own thinking.

Sequences can reduce the sense of risk experienced by some students. Many teachers report that some students do not embrace challenges possibly fearing the risk of failure. One of the goals of the sequences is for students to see that, even if they cannot do the current task, there is a similar task coming and they can learn how to do subsequent tasks by engagement in the current task, even if not successful yet.

The focus of this article is on ways that the documented curriculum can inform the construction and implementation of learning sequences.

Some characteristics of sequences

Part of the research focusses on validating the structure and principles that inform the design of the sequences. At this stage, our focus is on the first three years of formal school. The sequences are proposed to:

- represent one to two weeks of classroom mathematics lessons;
- facilitate movement from concrete to pictorial to symbolic/mental images;
- be challenging for students in that they will not initially know how to solve the problems;
- not only address important mathematics concepts and language as identified in the curriculum but also reflect the ways that young learners approach that mathematics;
- be applicable for Years F to 2 (although teachers of Foundation classes might spend more time on the initial suggestions and teachers of Year 2 class would extend the latter suggestions);

- allow students time to make choices on the type of answer and/or approaches to solution;
- be explicitly differentiable through a "low floor high ceiling" nature or enabling and extending prompts; and
- be structured similarly, especially identifying relevant curriculum focus and learning goals, presenting sequenced task suggestions, and assessment rubrics specific to the sequence.

In our project, the iterative cycle of design-test-redesign-retest will ultimately lead to a set of refined and empirically developed sequences of learning. While the sequences are intended for early years students (F to 2), the approach is applicable at all levels.

A SEQUENCE IN LENGTH

To illustrate the ways that the curriculum might inform a sequence, the following uses the example of a sequence of learning experiences focusing on the learning of length concepts. The section first presents a discussion of some earlier research on the development of length concepts then examines how length concepts are presented in the Australian Curriculum: Mathematics. The section also presents examples from a draft sequence.

A perspective on the learning of length

In describing the development of length concepts in the junior and middle primary levels, McDonough and Sullivan (2011, p. 34) wrote:

> Teachers of children in the first year of school can reasonably aim that nearly all students are able to compare the length of two objects, to order a third object even if not necessarily directly comparing it to the others, and begin to move towards quantifying lengths. It is relevant to note that … structured activities that provide experiences in (iteration) of length learning are important. For example, asking students to compare the lengths of two objects that cannot be placed next to each other.
>
> Teachers of children in the second year of school could emphasise activities that facilitate the movement of all students toward using informal units iteratively to quantify lengths, both using a single unit repeatedly and using multiple versions of the one unit. It is worth noting that approximately two thirds of the students at this level are either at or will become ready for using standard units during the year.
>
> Teachers of children in the third year of school should expect most children to be moving towards using standard units such as cm. Again it is noted that many students are ready for more sophisticated tasks involving measuring length.

The stages described above give a strong indication of the ways that these concepts develop. In summary, it seems that direct and indirect comparisons are followed by experiences in which a unit is used iteratively, which then lead into opportunities to use formal measurement units.

It is worth noting that McDonough and Sullivan found from an analysis of large data sets that students' responses to measurement items and their responses to number items were quite different and that facility with one did not imply facility with the other. This finding emphasizes another aspect of the sequences proposed in our current study in that they are designed for mixed achievement, whole class teaching.

Length in the Australian Curriculum

Of course, it is not expected that teachers will read research articles on the many topics that make up the mathematics curriculum. Therefore, the curriculum needs to communicate key ideas hopefully clearly and succinctly. The references to Length in the Australian Curriculum are as follows. Note that Foundation students are commonly aged 5.

Foundation Year

Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language (ACMMG006)

Year 1

Measure and compare the lengths and capacities of pairs of objects using uniform informal units (ACMMG019)

Year 2

Compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units (ACMMG037)

Year 3

Measure, order and compare objects using familiar metric units of length, mass and capacity (ACMMG061)

Year 4

Use scaled instruments to measure and compare lengths, masses, capacities and temperatures (ACMMG084)

Compare objects using familiar metric units of area and volume (ACMMG290)

Year 5

Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108)

Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109)

In other words, the same developmental sequence of key length concepts identified by McDonough and Sullivan (2011) are evident and appear in the Australian Curriculum. The descriptions of the content are succinct but the key terms – direct comparison, indirect comparisons, uniform informal units, familiar metric units, etc – are prominent.

A notional sequence

The following are the headings of five suggestions in a sequence that is intended to guide teachers in the first years of school. The sequence extract gives the title of each suggestion along with a possible learning focus to indicate the intention of matching experiences.

Suggestion 1: Direct comparisons

Learning focus: It is possible to compare lengths by putting one object against another

Suggestion 2: Indirect comparisons with lines

Learning focus: It is possible to compare one object against another using a third object

Suggestion 3: Indirect comparisons with different shapes

Learning focus: It is possible to compare one dimension against another using a third object

Suggestion 4: Using informal units iteratively

Learning focus: It is possible to compare lengths by using an object over and over again

Suggestion 5: Using informal units to compare different objects

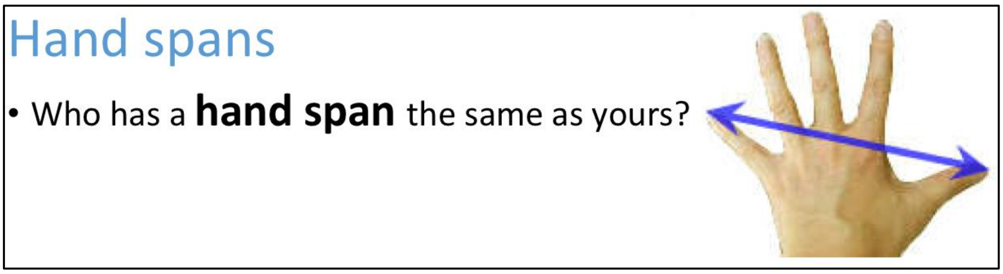
Learning focus: When comparing different informal units, the unit in each case must be constant

All of the experiences accompanying the respective suggestions are challenging for students. This allows Foundation students to engage with the tasks but also means that Year 2 students are required to think even in the initial suggestion. Our notion of 'challenging' experiences incorporates characteristics that require students to make connections between different aspect of mathematics, to devise solution strategies for themselves, to explore more than one solution pathway and to explain their strategies and justify their thinking.

We are currently working with early years teachers to test and refine the individual experiences, the sequences in which they are presented and accompanying support documentation.

A suggested sequence

The details of three connected learning experiences from the suggested length sequence intended for Foundation students moving from *Suggestion 1: Direct Comparison* to *Suggestion 2: Indirect comparisons with lines* are presented in this section. This sequence is representative of the type of connected experiences and accompanying support information provided to teachers. The first suggested experience is as follows:



A suggested connected experience following this, which still involves direct comparison, is:

> Find something that is longer than your hand span, but shorter than your foot.

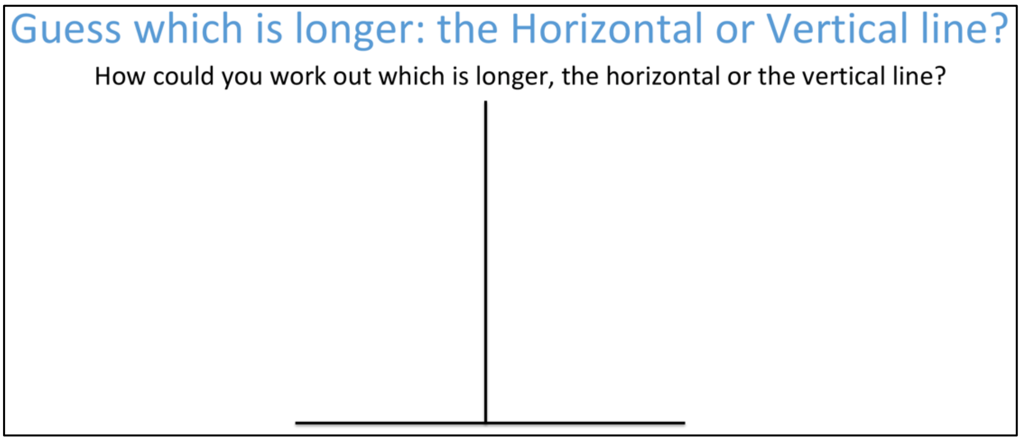
In the accompanying documentation for teachers, it is emphasized that key terms such as long, longer, longest, short, shorter, shortest, height, width are appropriate for use with students. A rationale for the suggested sequence and pedagogical considerations are also provided to teachers, including:

> The intention is to develop an intuitive sense of length so prompt students to estimate before measuring;
>
> The intention is to consolidate the concept before moving to formal units;
>
> All tasks allow students opportunity to explain their thinking; and
>
> The tasks are only illustrative – choose your own contexts.

Key ideas for measuring length described in the Australian Curriculum and by McDonough and Sullivan (2011) are evident in the sequence of experiences appropriate for early years students. The initial suggested experience introduces students to the concept of length using a familiar experience involving their own hands. The key ideas of 'longer', 'shorter' or the 'same' length are openly explored by direct comparison of just two objects. Students determine their own approach for finding someone with the same hand span as their own.

The connected consolidation task is intended to extend the learning by involving a third object ("your foot") and has an additional element that explicitly requires students to simultaneously consider "longer than" and "short than". This task is considered challenging because it allows students opportunities to devise their own strategies for directly comparing three objects that are not easily placed side-by-side to enable direct comparison. While multiple solutions are available, the range of possibilities is also constrained.

A third connected experience involving *Suggestion 2: Indirect comparisons with lines* is:



For indirect comparison experiences, teachers are advised to consider:

A third tool (string, streamer) is needed to compare the lengths; and

Some tasks involving the ordering of more than one length.

While this learning experience has a single correct answer, the intention is that the students will devise their own indirect comparison strategies involving a third object or tool. The task is considered challenging because multiple strategies are possible, the answer is not obvious, and students are required to explain their reasoning for their response. In terms of developing key concepts of length described by McDonough and Sullivan (2011), the task could involve unit iteration (e.g., measuring the length of each line with a paper clip rather than a single streamer), the length of two lines are compared when they cannot be placed next to each other and, therefore, requiring indirect comparison using a third object as a tool.

TEACHERS' RESPONSES TO SEQUENCES

The following data were from early years teachers responding to an online survey during a professional learning day to introduce them to the notion of sequences as a guide to planning. The teachers were asked to rate each statement on a 5-point Likert scale (strongly disagree to strongly agree). Table 1 presents the responses of 96 teachers.

As can be seen, around half of the teachers strongly agreed with the statements, with most of the rest agreeing. Most importantly, the majority of teachers indicate that they will implement the sequence as it was presented to them and feel confident teaching the challenging tasks to their students. Given that the length sequence is long and the time for discussion was brief, and that teachers came from schools of very diverse socio-economic backgrounds this is strong endorsement that the sequence suggestions would be welcomed by many teachers.

Statement	Strongly disagree (1)	2	3	4	Strongly agree (5)
The way the sequence builds is clear to me	0	0	7	41	52
The sequence is easy to follow and logical	0	0	6	40	54
I plan to use this sequence more or less as it is written	0	0	11	43	46
The tasks look interesting	0	1	6	25	67
I feel confident teaching this sequence of tasks	0	0	2	45	47

Table 1: Responses (%) of early years teachers to prompts about the length sequence (n = 96)

We also asked teachers to indicate which year level the sequence would be suited for. Table 2 presents their responses. Note that teachers could indicate more than one level.

Year level	Number of teachers (n = 96)
Foundation	69
1	67
2	73
3	42
4	32

Table 2: Numbers of teachers indicating particular levels at which the sequence is applicable

The majority of the teachers saw the suggestions as suitable for students aged 5 years old, there was also one third who felt the suggestions were applicable to students in their fifth year of school. Given that the experiences were explicitly designed with a "low floor high ceiling" nature in mind, it is affirming that the suggested sequences will be suitable for mixed achievement whole class teaching.

The teachers are currently teaching the sequence to their students and will complete a similar survey after they have taught the entire sequence.

CURRICULUM-INFORMED EMPIRICALLY DEVELOPED LEARNING SEQUENCES

In this article, we have presented an approach to curriculum development that is informed by the Australian Curriculum: Mathematics. The documented curriculum influences student learning through the sequences of experiences teachers plan and implement as part of their instructional program. It therefore makes sense that the potential of such sequences to improve student learning are empirically tested.

Guided by Variation Theory our research aim is to develop cumulative, challenging and connected sequences of experiences that can both initiate and consolidate students' learning of mathematics. Thoughtfully designed sequences of learning experiences are more likely to assist students see connections between mathematics concepts by making the big ideas and progression of learning more obvious. Carefully varied tasks within sequences can emphasise the big ideas to students, allowing them opportunities to distil the essence of concepts over time.

It is important to note that the suggested sequences do not disempower teachers from making professional judgements regarding appropriate experiences for their students. Teachers must still structure their lessons, orchestrate whole class strategy discussions, make 'in-the-moment' decisions about providing appropriate enabling and extending prompts, and assess student thinking to determine 'where to next'. Each suggested sequence of learning matches the development of concepts reflected in the Australian Curriculum, so teachers are free to adapt the experiences to reflect their contexts.

The sequences of learning experiences we are examining hold great promise for building student understanding of important mathematics concepts. It is the aim of our research to explore this potential.

References

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