

What makes a ‘good’ mathematical game?

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Most primary school teachers would concur that mathematical games are a valuable pedagogical tool to deploy in the primary classroom; however, not all mathematical games are likely to be equally valuable. How might teachers decide which games to introduce in their classrooms? In this paper we attempt to support teachers to address this question through presenting five principles of good mathematical games. Rather than operating as definitive criteria, our intention for presenting these principles is to stimulate critical discussion and to guide teacher decision-making. Examples of mathematical games that we believe appropriately capture each principle are provided.

INTRODUCTION

As primary mathematics educators, we both have a passion for designing mathematical games. Much of the time, this game design process can feel as much of an art as a science, as inspiration often strikes seemingly randomly, through casual conversation, playing with maths equipment, or doing a completely unrelated activity. However, as Gough (2004) notes, mathematical games do tend to cycle around specific mechanics and dynamics; so, whether we are consciously aware of it or not, most newly created games are likely to be derivative of games that we have previously played, read about, or used in our classrooms. Reflecting on this observation of Gough’s as well as our own process has led us to ask the question: What makes a “good” mathematical game?

To answer this question, we present five principles of good mathematical games. Under each principle, we present an example of a game we feel effectively illuminates the particular principle under consideration. The five principles are:

1. Students are engaged.
2. Skill and luck are balanced.
3. Mathematics is central.
4. Flexibility for learning and teaching.
5. Facilitates home-school connections.

Our intention in presenting these principles is to generate discussion, and to encourage teachers to reflect on the games they use in their classrooms and their rationale for doing so. You may have additional factors not discussed in the current paper which you may wish to consider for including a particular game. For example, perhaps a student in the classroom was responsible for developing the game, and therefore the class is particularly motivated to play it. Alternatively, you may have reasons for disregarding a particular principle. For example, when evaluating a particular game, you may decide to ignore Principle 5, reasoning that facilitating home-school connections is not relevant because the game you want to play requires specialised equipment only available in a classroom environment.

From our perspective, the most important consideration is that teachers are active when making choices about which games their students should play, and have given some thought as to why they believe the game chosen has particular value. We present these five principles as a starting point for this critical reflection. Interested readers may wish to refer to an article where these five principles are unpacked in more depth (see Russo, Russo, & Bragg, 2018).

PRINCIPLE 1: STUDENTS ARE ENGAGED

Some of the strongest evidence for the utility of mathematical games as a pedagogical tool stems from the fact that they tend to generate more on-task behaviour and mathematical dialogue than other non-game based activities (Bragg, 2012). Indeed, anecdotally, many teachers employ mathematical games largely because students enjoy playing them, and because they tend to support students to engage in learning mathematics (Rutherford, 2015).

We would suggest that Principle 1 is so central to any benefits potentially derivable from playing mathematical games that, if students are struggling to engage in the game and would rather be doing something else in the mathematics class, then, in fact, they should be doing something else. Games that students are not motivated to play are highly unlikely to generate positive learning outcomes compared to alternative activities.

PROBABILITY FOOTBALL

One example of a game that many students find highly engaging and enjoyable is *Probability Football*. Developed by the second author to explore conditional probability with his Year 5 and Year 6 students, the game has become a staple fixture in his classroom, particularly during September (see Figure 1).

How to Play

Materials: game board (an AFL football field), a ten-sided die and a counter.

This game is for 2 players.

1. Decide which player is red (kicking right) and which player is blue (kicking left). The counter (or “ball”) begins in the middle of the game board. The highest roll of the die determines which player starts in possession (simulating a “centre bounce”).
2. The controlling player (Player 1) must choose to which adjacent section they want to kick the ball: there is a red (or blue) arrow pointing to the adjacent sections and a matching probability for each (or “likelihood of success”).
3. To simulate a kick, Player 1 rolls the die. The number rolled determines if it was a successful play. For example, if the probability is 80%, a roll of 1-8 means the play is successful and they continue playing from the next section; however a roll of 0 or 9 means the play was unsuccessful and the other team (Player 2) takes possession from this point. Player 2 would then roll the die and follow the same process.
4. When a player is in a section with a box around the probabilities, they can choose to kick for goal. Success means they score a goal (and the ball begins in the middle with a “centre bounce”), failure means it is a behind (and the other team takes a kick in).
5. Play continues until the end of the given timeframe (e.g., 20 minutes), with the winner the player with the highest score (with a goal worth six points and a behind worth one point).

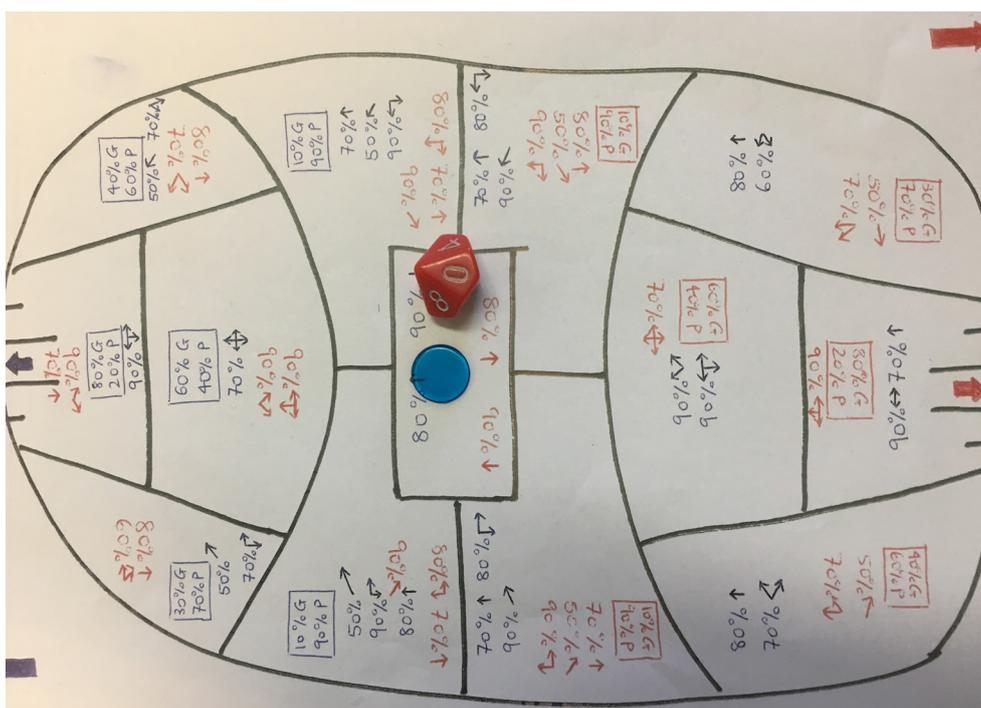


Figure 1. Probability Football game board.

PRINCIPLE 2: SKILL AND LUCK ARE BALANCED

Mathematical games should balance skill and luck.

In part to support prolonged student engagement in a mathematical game, we have found that it is important for the game to balance skill and luck. Without some component of luck, more mathematically-able students are at risk of dominating. This can be de-motivating for all learners. By contrast, without some component of skill, it is arguable that the activity even warrants being referred to as a game at all (Gough, 1999). Sometimes some form of turn-taking or role-reversal is central to ensuring that skill and luck are sufficiently balanced.

From an equity point of view, it is vital that all students are given opportunities to lose games, as well as to win. We would contend that losing games gracefully is integral to the developing characters of young people, and has powerful social consequences. Being a “good loser” or a “good winner” makes it more likely that other children will want to play games with you in the future, which is likely to be a latent objective for participants in any game. In addition, so-called failure provides a powerful opportunity for learning; a common sense claim increasingly substantiated by evidence (Kapur, 2014; Warshawer, 2015).

SKIP-COUNTING BINGO

The first author has often used a version of Bingo, called Skip-Counting Bingo, as an engaging game to play with his Year 2 students, partly because it effectively balances skill and luck to maintain student engagement.

How to Play

Materials: 100-chart, 6-sided die

This game is for 2 to 5 players.

1. To begin, children each choose three Bingo numbers in turn, and mark these numbers on their 100-chart. Note that players must choose numbers greater than 10 (or 20 to extend the range of numbers in the skip-counting sequence).
2. One of the children rolls the 6-sided die. Together, children begin counting by whatever the number rolled, using the 100-chart to keep track. For example, if they roll a four, they would begin counting by 4s from zero: 4, 8, 12, 16, 20, etc.
3. Children stop counting when they encounter a bingo number. In the game shown in Figure 2, children would stop counting at 28 if a four was rolled, as this is the first Bingo number encountered.
4. The die is rolled again, and a new counting sequence is explored. For example, if a 3 is rolled, the group would be counting by 3s from zero. They would again stop when they encountered a bingo number (42 in Figure 2).
5. Play continues until one of the players removes all their numbers and shouts “Bingo!”.

PRINCIPLE 3: MATHEMATICS IS CENTRAL

In order for a game to be of value in the mathematics classroom, it must have important mathematical ideas at its core. Badham (1997) suggested that the teacher should ensure that games are used purposefully, and that a chosen game matches the specific mathematical objective under focus.

The first author’s default approach when designing games for early primary school students is to use games to provide problem-based practice with a particular skill or concept (e.g., 10s facts; skip-counting sequences). The strategic aspect of the game serves to keep the game interesting through providing a compelling purpose for this practice. Conversely, Gough (1999) argues that games can operate as a context for learning a new concept, rather than simply practising a previously known skill. This is often the approach adopted by the second author when teaching upper primary students. For example, he has used *Colour in Fractions* (Clarke & Roche, 2010) to expose students to the notion of adding fractions with related denominators, prior to any formal instruction in the idea. Rather than being an explicit focus of game play, this concept emerges out of the game dynamics.



Figure 2. Three children begin a game of Skip-Counting Bingo.

HOW CLOSE CAN YOU GET?

We first came across this game in an older edition of *Australian Primary Mathematics Classroom*, where Vale (1999) was using it as a context for discussing mental computation strategies. We particularly like it because we have found it challenging to source worthwhile activities that focus on subtraction as the “difference between” two numbers.

How to Play

Materials: Playing cards.

This game is for 2 to 5 players. Remove the tens and picture cards from a deck of cards, leaving ace to 9 for the game.

1. Shuffle the cards. Deal each player 4 cards, face down.
2. Turn up 2 more cards. The first card goes in the tens place and the second in the ones place to form the target number. For example, a 6 then ace becomes 61.
3. Players turn up their four cards and arrange them into two 2-digit numbers, so that the difference between their two numbers is as close to the target number as possible.
4. To score, each player then finds the difference between his or her result and the target number. For example, if the target number was 61, and a player had A, 5, 3, & 9, the best she could do would be $95 - 31 = 64$. Her score would be $64 - 61 = 3$ for that round.

5. Note that you can go over or under the target number.
6. For the next round, turn up two new cards from the deck to form the next target number. Players can choose to use their same four cards or deal out four new cards.
7. At the end of five rounds the player with the lowest total score wins.

[Sourced from Vale (1999). Originally appeared in MAV/AAMT–Home Mathematics (1988).]

PRINCIPLE 4: FLEXIBILITY FOR LEARNING AND TEACHING

Good mathematical games should be flexible enough to cater for learners of different abilities. To some extent, this can be achieved through the balancing of skill and luck as a key aspect of the game dynamic. Consequently, many games have some degree of differentiation inherent in them, which allow learners of different abilities to play against each other, and to apply developmentally-appropriate strategies.

For example, consider the game Skip-Counting Bingo previously described. If this game was to be played by upper primary students, they would be more inclined to rely on their knowledge of multiples and factors, rather than on the skip-counting patterns used by lower primary students. This approach might allow such students to begin to calculate which numbers they are most likely to land on *before even playing the game*, rather than relying on insights that only emerge through repeated game play. Adopting this more sophisticated approach is likely to improve their chances of success in the game. However, the fact that the game literally relies on the “luck of the dice” still offers an opportunity for students who adopt less sophisticated strategies to be successful.

Other games more explicitly lend themselves to differentiation through directly modifying the game mechanics, including playing equipment (e.g., the dice used). Making such modifications may allow a teacher to group students with peers of similar-ability, thereby allowing the group to focus on a carefully targeted learning area. For example, if playing Multiple Mysteries (Russo & Russo, 2017), a teacher may group students together whom they determine would benefit from practice with a particular pattern of multiples (e.g., 3s, 7s, 8s).

MULTIPLE MYSTERIES

We like this game for its simplicity and versatility. It rewards both calculated risk-taking and an understanding of patterns and rules for identifying multiples.

How to Play

Materials: Playing cards, calculator.

This game is for 2 to 4 players. Remove the tens and picture cards from a deck of cards. You may choose to leave the Jacks and Jokers in the deck (representing zeroes).

1. The teacher, or the players, choose a target multiple (e.g., 3, 5, 8).
2. Five cards are dealt to each player, and the first player begins their turn.
3. The objective on any given turn is to use some or all of these five cards to make a multiple of the particular target that is the focus for that round. The more digits in the number you create, the more cards you use, and the more points you will score. For example, if the target multiple was 8, using the cards 4 and 8 to make the number 48 would earn you two points; whereas using the cards 2, 4 and 8 to make the number 248 would earn you three points.
4. The player then “banks” these cards in their own bank, and picks up some additional cards from the deck to ensure they have five cards in their hand at the beginning of the next turn.
5. On any given play, any opponent has the option of challenging the player’s multiple through using a calculator and dividing the created number by the target multiple (e.g., 248 divided by 8). If the challenger is correct, the cards used

to create the number are “banked” instead by the challenger. If the challenger is incorrect, they must pay a one-card penalty to the player who was challenged.

6. Play continues in turn. Once all the cards in the deck are used, the “banked” cards are counted up. The player who “banked” the most cards wins the game.

PRINCIPLE 5: FACILITATES HOME-SCHOOL CONNECTIONS

Mathematical games can be a tool for building positive connections between home and school environments (Rutherford, 2015). This can occur through bringing families into the school, as seen in family maths nights, but also by bringing maths “into the home.” The second author is currently piloting a program where students play games in class, watch an instructional video of the game when they get home, and then play the game together with a carer or sibling as “homework.”

There has been positive feedback about the program to date, and students have been highly motivated to play the games at home. They have taken on the role of “expert” when explaining the activities to family members, which has evidently helped to increase the confidence of some students. One student explained: *I taught the game to Mum without having to watch the video and then I had to teach her what multiples are!* The fun, competitive element of game play seemed to shift the perception around maths homework for a number of students. For example, another student, who is often a reluctant participant in maths activities and rarely completes homework tasks, spoke enthusiastically about playing Caught Red-Handed (Russo, 2017): *I had so much fun playing the red-handed game at home on the weekend. I beat my sister three times in a row and she couldn't work out how!*

CAUGHT RED-HANDED

Based on the concept of 11s, we have found this game to be useful for playing with students from Year 1 to Year 6. Children particularly enjoy the game because prior practice often results in them having a genuine competitive edge over novice adults.

How to Play

Materials: 20-sided die or spinner. Students can create their own game board by writing out the numbers 1 to 20. The game requires one red pen and a black/blue pen for each player.

This game is for 2 players.

1. Circle the number 20 with a red pen and roll a 20-sided die to determine the two other “red” numbers to circle.
2. A game of rock-paper-scissors decides who begins the game.
3. On their turn, each player is allowed to cross out 1 or 2 numbers. Players must cross out the numbers in order (i.e., 1, 2, 3, etc.).
4. The goal of the game is to cross out at least two of the three “red” numbers.
5. Variant: Play the game again (revenge match) with the red numbers in the same place, but this time allow players to cross out 1, 2 or 3 numbers. The loser of the previous game can decide whether they wish to play first or second.

CONCLUDING THOUGHTS

This paper has presented five principles of good mathematical games and provided some practical examples of games we believe illustrate these principles. As noted at the outset, rather than attempting to present definitive criteria of what denotes a good game, our intention is to stimulate critical discussion and to guide teacher decision-making. We encourage classroom teachers to use this paper as a springboard, and take the discussion of what constitutes a good mathematical game into the staffroom.

Finally, we do not want to leave readers with the impression that the only important role a teacher plays in a games-based mathematics lesson is in choosing an educationally-rich game. During both the exploration phase, when students are playing the game, and the post-game discussion, the teacher needs to play an active role in stimulating mathematical thinking (Badham, 1997). For example, the teacher may provide thoughtful prompts and questions that encourage students to:

- reason mathematically (Explain why that makes sense);
- justify decisions (Why did you make that move?);
- make predictions (What do you need to win?); and
- consider alternatives (What would you do differently next time?)

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