Electron Beams with a Twist

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Abstract: The properties and behaviour of electron beams possessing orbital angular momentum are considered. Methods of their creation are discussed, but we concentrate mainly on the method involving binarized holographic masks. It is shown how suitable masks can be modelled using Bessel functions as well as Laguerre-Gaussian functions. The theoretical far-field diffraction patterns of these holograms are simulated using Fourier transform techniques and compared with the recently reported experimental results of electron vortex beams. The limitations of the masks and the potential of the technique for future research are pointed out and discussed.

1. Introduction

1.1 Background Theory

The angular momentum of electrons is an intriguing subject as it draws together a wide array of key areas of physics within electromagnetism, optics and quantum mechanics in an unusual but elegant combination. From elementary quantum mechanics it is known that the total angular momentum of an electron can be separated into spin and orbital components. The spin angular momentum can be determined with established techniques such as the Stern-Gerlach experiment [1], whereas the orbital angular momentum (OAM) of electron beams has been significantly under studied until recently.

In the area of electron beams possessing OAM (known as electron vortex beams), the first paper of significance was published as recently as April 2010 [2]. It showed how, as for optical beams, an electron plane wave could be passed through a ramped phase plate within an electron microscope to impart OAM. However, there are a number of physical difficulties with this technique which led to results being approximate and limited to low orders of OAM. There is an alternative method, known as holographic reconstruction, the study and application of which forms the major focus of this paper. Both Laguerre-Gaussian (LG) and Bessel wavefunctions are considered as OAM-bearing entities, and some of their properties are discussed. This leads to the development of a series of hologram masks for creating electron vortex beams, the application and effectiveness of which are discussed with particular regard to their diffractive behaviour. They are compared with the theoretical and experimental results of electron vortex beam studies in published literature [3, 4].

1.2 Motivation and Objectives of the Study

It is imperative that the field of electron vortex beams is studied as there are many potential applications of this new development. An example that has already been demonstrated is electron energy-loss spectroscopy using such beams, allowing the properties of magnetic materials to be investigated on a very fine scale [3]. Additionally, there is potential for using such beams to rotate

particles of matter as an analogue of the optical screwdriver [5], and there are possible applications in the field of quantum information transfer [6].

With these potential applications in mind, this article explores electron vortex beams made from holographic masks using the Laguerre-Gaussian and Bessel wavefunction forms and aims at a comparative study of the holographic masks produced. The theoretical diffraction patterns of the generated holographic masks are evaluated and compared with the published experimental results [3]. We conclude with a summary and brief comments.

2. Methodology

2.1. Mathematical Basis of OAM beams

A beam possessing OAM expresses this property through its azimuthal phase dependence, $e^{il\varphi}$ where $\varphi = \arctan(y/x)$ and l is the OAM quantum number, as seen in figure 1. This reveals a helical wavefront structure whereby, at the beam axis, the phase must be undefined – this is the phase vortex. The two most common beams with this property are LG beams and Bessel beams.
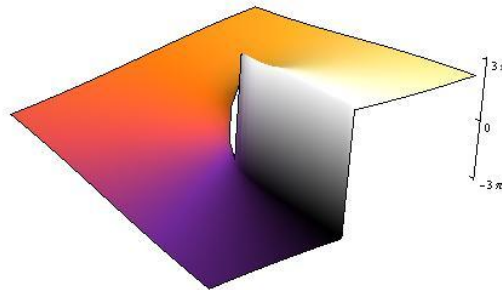

Figure 1: Wave front of a beam with $l = 3$, with the size of the phase ramp at 6π over the azimuth angular range of 2π

To describe beams possessing OAM, cylindrical polar coordinates may be conveniently used in solving the time-independent Schrödinger equation (TISE). Applying the paraxial approximation, i.e. when the diffraction and focussing occur over a distance much longer than the wavelength, one possible solution is the LG beam [7] in the long Rayleigh range limit:

$$\Psi_{LG} = C \left[\frac{\sqrt{2}.r}{W(0)}\right]^{|l|} L_p^{|l|} \left(\frac{2r^2}{W(0)^2}\right) e^{\frac{-r^2}{W(0)^2}} e^{ikz} e^{il\varphi} \qquad (1)$$

where C is an appropriate normalization constant, (r, φ, z) are the cylindrical polar coordinates and $W(0)$ is the beam waist. $L_p^{|l|}$ is the associated Laguerre function, with azimuthal index $l = \pm(0,1,2\ldots)$ and radial index $p = 0,1,2\ldots$. The wave vector is represented by k (considered here in the z direction).

Alternatively, Bessel beams are an exact solution of the TISE, as they are plane waves with a non-uniform intensity distribution. However, the infinite width of mathematical Bessel beams would require an infinite amount of energy, and thus they are not physically realisable in their true form. Good approximations are achievable, however, by applying a finite aperture to the function to produce a truncated Bessel function; a simpler alternative to the analytical Bessel-Gauss function [8]. This would allow the reduced diffraction properties of the Bessel beam to be employed [9] over a finite but still considerable distance. It is this non-diffractive property which makes electron Bessel beams an exciting prospect. The wavefunctions of a true Bessel beam take the form

$$\Psi_B = CJ_l(k_r r)e^{ik_z z}e^{il\varphi} \qquad (2)$$

where C is the normalization constant, J_l is a Bessel function of the first kind and is of the order l and a function of $(k_r r)$ where k_r is the radial component of the wave vector and k_z is the axial component of the wave vector. As for LG beams, (r, φ, z) are the cylindrical polar coordinates and l is the azimuthal index.

The similarities and differences between the Bessel and LG beams can be clearly seen in their radial distributions, displayed in figure 2.
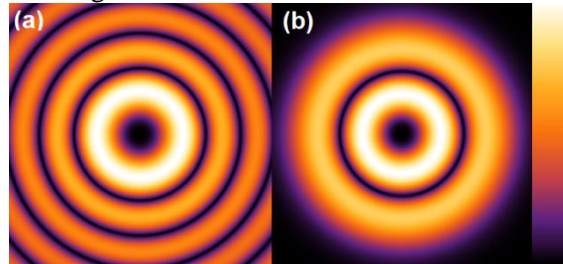
Figure 2: (Color Online) (a) the radial intensity distribution of an $l = 3$ Bessel beam,
(b) the radial intensity distribution of an $l = 3, p = 1$ Laguerre-Gaussian beam.
Maximum intensity represented as bright, minimum values as dark.

2.2. Holographic Reconstruction

Electron holography involves placing a diffraction grating in the path of a plane wave electron beam [10]. The OAM is imparted to the beam by the existence of a Y-shaped defect in the diffraction grating. Such holographic masks were first employed in the electron case by Verbeeck *et. al.*, and more recently by McMorran *et. al.* [3, 4]. Both groups produced electron vortex beams for a range of OAM.

Holographic masks represent the interference pattern of the intended beam (such as Ψ_{Bessel} or Ψ_{LG}) and a reference plane wave [10]. The interference of the desired beam with the reference wave allows the complex phase information of the desired beam to be combined with the amplitude, and retained in a real-valued array. This array is then binarized, by clipping away those areas below a threshold value, and setting those areas above the threshold to a maximum thickness. This array can be translated into the thickness of a foil to be placed in the beam path. This results in a mask resembling a typical diffraction grating, but with a Y-shaped defect appearing from the phase dependent term within the wavefunction. It is the passing of a beam around this defect which allows the transfer of OAM from the mask to the beam particles. This method was the technique first applied by Verbeeck *et. al.* to create electron vortex beams [3].

2.3 Fourier Transforms

The nature of the far-field diffraction pattern is one way to detect the form of the beam produced, and is described by a 2D Fourier transform of the aperture. Thus, theoretically expected results can be generated by assuming that the hologram is part of a periodic function, and performing a 2D Fourier transform on the mask. The modulus squared of the Fourier transform represents the expected intensity distribution created by the mask in the back focal plane of an electron microscope.
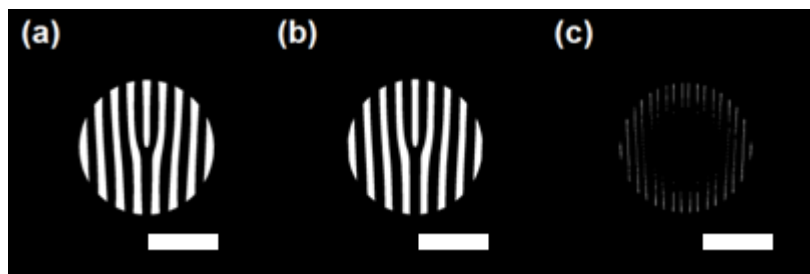
Figure 3: The binarized hologram masks made from $l = 1$ (a) a truncated Bessel wavefunction;
(b) a truncated Laguerre Gaussian wavefunction; (c) the difference between the masks shown in (a) and (b).
The scale bars represent the radial unit $r = j_1/k_r$, where $j_1 \approx 3.83$ is the first zero of the Bessel function.

3. Results and Discussion

The masks were simulated using Mathematica, following the process described above. The binarized masks produced using both the truncated Bessel and the LG wavefunctions are shown in figure 3.

3.1. Fourier Transforms

To analyse these masks, 2D Fourier transforms are applied to the arrays containing the mask data. This process allows for comparison with the experimental results in the literature. Figure 4 shows the Fourier transforms of the masks displayed in figure 3. It can be seen that both masks produce similar outputs. The binarized masks show the intended beams ($l = 0, \pm1$) and, additionally, higher orders are generated as by-products of the binarization process.
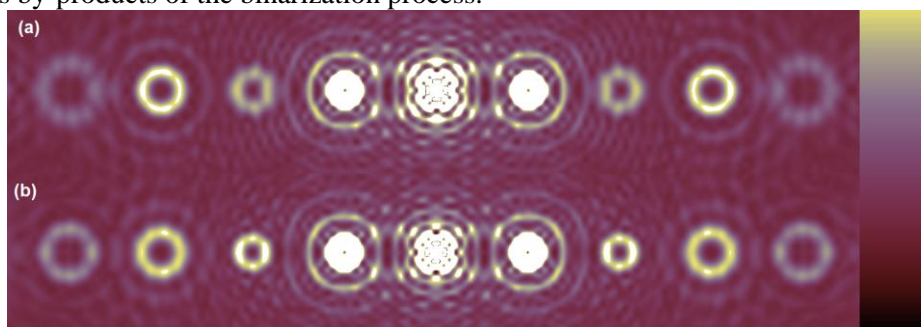

Figure 4: Fourier Transforms of the masks: (a) the Bessel case and (b) the LG case. Maximum intensity represented as bright, minimum values as dark.

These results agree closely with the experimental results for the beams displayed by Verbeeck *et. al.* [3]. However, the comparison of the masks (Figure 3, (c)) and the effect on the diffraction pattern (comparing Figure 4a and 4b), calls for a re-consideration of the effect of the chosen wavefunctions. The clearest difference is in the variation of the width of the fork tines as a function of the radius – as would be expected from a consideration of the beam profiles. However, this difference clearly has little effect on the far-field diffraction patterns. This suggests that to investigate the diffractive nature of electron vortex beams, a modification of this technique is required as the effect due to the approximation processes dominates over the differences between the these two wavefunctions.

4. Conclusions

We have considered the effect of differing radial profiles on electron vortex beams, by modelling holographic masks and the resulting diffraction patterns using Fourier transform techniques. We have shown that the radial differences of the Bessel and LG wavefunctions are small in the radially-truncated region from which holographic masks are produced. This implies that to investigate and control the spatial evolution of electron vortex beams, modifications to the current methods of producing the beams are required.

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