Unidirectional scattering exploited transverse displacement sensor with tunable measuring range

WUYUN SHANG,1 FAJUN XIAO,1,4 WEIREN ZHU,2,3 LEI HAN,1 MALIN PREMARATNE,3 TING MEI,1 AND JIANLIN ZHAO1,5

1MOE Key Laboratory of Material Physics and Chemistry under Extraordinary Conditions, and Shaanxi Key Laboratory of Optical Information Technology, School of Natural and Applied Sciences, Northwestern Polytechnical University, Xi’an 710129, China
2Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
3Advanced Computing and Simulation Laboratory (AχL), Department of Electrical and Computer Systems Engineering, Monash University, Clayton, Victoria 3800, Australia
4fjxiao@nwpu.edu.cn
5jlzhao@nwpu.edu.cn

Abstract: We propose a scheme to extend the measuring range of a transverse displacement sensor by exploiting the interaction of an azimuthally polarized beam (APB) with a single metal-dielectric core-shell nanoparticle. The focused APB illumination induces a longitudinal magnetic dipole (MD) in the core-shell nanoparticle, which interferes with the induced transverse electric dipole (ED) to bring forth a transverse unidirectional scattering at a specific position within the focal plane. Emphatically, the rapidly varying electromagnetic field within the focal plane of an APB leads to a remarkable sensitivity of the far-field scattering directivity to nanoscale displacements as the nanoparticle moves away from the optical axis. Moreover, the scattering directivity of the APB illuminated core-shell nanoparticle is also a function of structure-dependent Mie scattering coefficients, rendering the measuring range of the transverse displacement sensor widely tunable. The culmination of all these features enables the continuous tuning of the displacement measuring range from several nanometers to a few micrometers. Thus, we envision the proposed scheme is of high value for modern optical nanometrology.

© 2019 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

1. Introduction

Optical nanoantennas are fundamental elements for efficient interconversion between far-field light radiation and near-field localized energy [1,2]. These properties stem from the fact that they have the ability to tailor and control light radiation at subwavelength scale [2,3]. In particular, the configurations of nanoantennas have been extensively studied to engineer the directivity of the far-field radiation pattern. Examples include the well-known optical Yagi-Uda type nanoantennas that can direct the scattering light into a specific direction relying on phase matching between several appropriately spaced antenna elements [4,5]. Another alternative is to use subwavelength cubic antenna cascades to improve the directivity as shown in [2]. However, such designs are generally complex owing to many detrimental effects as they not only need to satisfy rigorous phase matching and retardation requirements but also need to overcome unavoidable intrinsic losses of metals, which largely affect their overall scattering performance and limit their scalability in practical applications. Recently, all-dielectric nanoantennas have received much interest as a viable alternative to their plasmonic counterparts, mainly due to low optical losses and excellent compatibility with semiconductor device technologies [6–8]. Specifically, it has been theoretically and experimentally demonstrated that dielectric nanoparticles with high refractive index can also exhibit unidirectional scattering for plane wave illumination, owing to the interference of
simultaneously excited electric (ED) and magnetic (MD) dipole modes with comparable strength [9–11]. This is similar to the scattering behavior of a magnetic sphere predicted by Kerker et al. in 1983, known as Kerker conditions [12]. That is, under certain conditions of the relative electric permittivity and magnetic permeability, equal amplitudes of ED and MD induced in the magnetic sphere can be reached where in-phase oscillation of the dipoles results in zero-backward radiation (first Kerker condition) and out-of-phase oscillation leads to near-zero-forward radiation (second Kerker condition) [11,13]. Such Kerker type nanoantennas possess robust unidirectional scattering with even single nanoparticle and have flexible tunability by introducing metallic core or shape engineering to realize broadband unidirectional scattering [14,15].

As nanoscience and nanotechnology mature, nanometrology has become increasingly important for localization of nanoparticles (e.g. single molecules) in subwavelength regions. The value proposition for these techniques stems from the applications such as super-resolution microscopy [16,17]. For accurate nanometrology, optical nanoantennas are preferred owing to their correlated near-field and far-field properties. For example, minute changes in the near-field can cause dramatic changes in the far-field radiation (e.g. scattering directivity [18,19]). Kerker type nanoantennas are tailor-made candidates for nanometrology as their scattering can be controlled by the illumination field via induced ED and MD. Current stage of the technology is such that the electric and the magnetic resonant modes in single dielectric nanoparticles can be selectively excited and the broadband unidirectional scattering can be actively tuned by cylindrical vector beam (CVB) illumination [20–24]. The CVBs, including radially (RPBs) and azimuthally (APBs) polarized beams, have been exploited for various applications in linear and nonlinear optical processes, basing on their singularity features with unique amplitude, polarization and phase distributions [25–38]. Moreover, the rapidly varying electromagnetic field within the focal plane of these beams enables the transverse Kerker condition to be met at a specific position and makes the scattering directivity of the nanoantenna position dependent [39]. Especially, based on the remarkable sensitivity of far-field scattering directivity to nanoscale displacements, the nanoscopic position sensing with the resolution of single Angstrom were demonstrated [39,40]. Besides the high precision of nanometrology, extended measuring range is also desired in applications [41,42]. However, the measuring range of such a displacement sensor, an essential parameter of nanometrology, has rarely been considered in previous studies.

In this paper, we analytically and numerically investigate a novel scheme to widely tune the measuring range of a transverse displacement sensor based on the interaction of an APB with a single metal-dielectric core-shell nanoparticle. The APB illumination makes the core-shell nanoparticle perform a transverse unidirectional scattering at a specific position within the focal plane where the transverse Kerker condition is fulfilled. When the nanoparticle moves away from the optical axis, the scattering directivity of the far-field radiation changes dramatically with respect to the transverse displacements, which can be cleverly exploited to make high-precision distance measurements in a transverse displacement sensor. Theoretical analysis indicates that the measuring range of the displacement sensor is set by the first two Mie scattering coefficients of the core-shell nanoparticle for a specific focused APB illumination. The amplitude ratio and phase difference between the Mie scattering coefficients can be adaptably tuned by adjusting the ratio between outer and inner radii of the core-shell nanoparticle and the corresponding measuring range can be continuously tuned from several nanometers to a few micrometers.

2. Unidirectional scattering of a metal-dielectric core-shell nanoparticle under an azimuthally polarized beam illumination

As schematically illustrated in Fig. 1(a), light scattered by a spherical multilayered nanoparticle in free space can be analytically solved with Mie theory [43,44]. The core-shell nanoparticle under consideration has a silver core and a dielectric shell with inner radius of,
$R_{in}$, and outer radius of, $R_{out}$, as the cross-sectional view shown in Fig. 1(b). The permittivity of silver is fitted from experimental data [45] and the refractive index of the dielectric shell is kept as a constant for $n = 3.4$ (e.g., Si, GaAs) [14] in our analysis.

![Fig. 1. (a) Schematic diagram of light scattered by a spherical nanoparticle. (b) Cross-sectional view of the studied spherical core-shell nanoparticle with inner radius of $R_{in}$ and outer radius of $R_{out}$.](image)

It has been shown that the scattering of a spherical nanoparticle depends only on local electromagnetic fields and field-independent Mie scattering coefficients, when illuminated by an inhomogeneous beam [21]. Given that the nanoparticle is small enough compared to the wavelength of the illumination light, the far-field scattering response can be described by the radiation of the dominant induced ED and MD resonances, $p = \varepsilon_0 \alpha_e^r E_{inc}(r_p)$ and $m = \alpha_m^r H_{inc}(r_p)$, where $\varepsilon_0$ is the permittivity of free space, $E_{inc}(r_p)$ and $H_{inc}(r_p)$ are the incident illumination electric and magnetic fields at the center of the nanoparticle [46]. According to Mie theory, the electric and magnetic dipolar polarizabilities can be written as $\alpha_e^r = i6\pi a_1/k^3$ and $\alpha_m^r = i6\pi b_1/k^3$, where $k$ is the wavenumber, $a_1$ and $b_1$ are the first two Mie scattering coefficients [43]. The corresponding far-field scattering field composed of these induced ED and MD is [46]

$$E_{scat}(r) = \frac{1}{4\pi\varepsilon_0} k^2 e^{\omega r} \left[ (i \times p) \times i - i \times m / c \right] ,$$

(1)

where $c$ is the speed of light, $r$ is the position of far-field scattering field detection point and $i$ is the standard radial unit vector of the spherical coordinates. When we closely look at the expressions of induced dipole moments ($p$ and $m$), we note that they are dependent on the polarizabilities ($\alpha_e^r$ and $\alpha_m^r$) and local electromagnetic field components ($E_{inc}$ and $H_{inc}$). The polarizabilities are dependent on the inner and outer radii of a core-shell nanoparticle, whilst the local electromagnetic field components can be varied by the illumination fields [14,20–22]. In particular, we note that CVBs can selectively excite the ED and MD within a spherical nanoparticle [20,21]. Figures 2(a)-2(c) show the electromagnetic field intensity and corresponding phase distributions at the focal plane of the focused APB (NA = 0.3, $\lambda = 1.55 \mu m$), which are calculated by the vector diffraction theory [47]. The electric field is purely transverse and azimuthally polarized. The magnetic field is composed of radially and longitudinally polarized components. It is worth noting that the transverse and longitudinal magnetic fields are $\pi$ and $\pi/2$ out-of-phase with respect to the transverse electric field, respectively. Figure 2(d) depicts the corresponding amplitudes of electromagnetic field components $E_y$, $ZH_x$ and $ZH_y$, as well as $|E_y|/|ZH_x|$, along the $x$ axis. Here, $Z = \sqrt{\mu / \varepsilon}$, is the impedance of the background medium, where $\mu$ and $\varepsilon$ being the permeability and permittivity of the medium. It is shown that $|E_y|/|ZH_x|$ varies approximately linearly within the region up to 1.1 $\mu m$ away from the optical axis.
When a core-shell nanoparticle is subjected to a focused APB illumination, a transverse ED, a transverse MD and a longitudinal MD could be induced. Without loss of generality, considering the nanoparticle has a transverse displacement away from the optical axis and along +x direction, the far-field scattering field superimposed by an induced electric dipole moment ($p_y$ along y axis) and two magnetic dipole moments ($m_x$ along x axis and $m_z$ along z axis) can be deduced from Eq. (1)

$$E_{sc}(r) = \frac{1}{4\pi\varepsilon_0} \frac{k^2 e^{i\theta}}{r} \left[ p_y \left( \cos \theta \sin \varphi_i + \cos \varphi_i \right) - m_x \left( \sin \varphi_i + \cos \theta \cos \varphi_i \right) + m_z \sin \theta \right], \quad (2)$$

where $\varphi$ and $\theta$ are the azimuthal and polar angles, $\mathbf{i}_\varphi$ and $\mathbf{i}_\theta$ are the standard azimuthal and polar unit vectors of the spherical coordinates, respectively. The far-field radiation pattern of the core-shell nanoparticle can be determined by the differential scattering cross section ($\sigma_d$), which can be expressed as $\sigma_d = r^2 |E_{sc}|^2 / |E_{inc}|^2$ [46,48]. Due to the existence of field singularity of the APB with zero electric field on the optical axis when calculating the differential scattering cross section, it is reasonable to replace $|E_{inc}|^2$ with $I_{total} = |E_{inc}|^2 + |Z_{H_{inc}}|^2$, which is the total field intensity of the incident field at the center of nanoparticle. The resulting differential scattering cross section is given by

$$\sigma_d(\theta, \varphi) = \frac{r^2 |E_{sc}|^2}{I_{total}}$$

$$= \frac{9}{4k^2 I_{total}} \left( a_x E_x \cos \theta \sin \varphi - Z_h H_z \sin \varphi \right)^2$$

$$+ \left( a_y E_y \cos \varphi - Z_h H_z \cos \theta \cos \varphi + Z_h H_z \sin \theta \right)^2. \quad (3)$$
As the existence of a longitudinal magnetic dipole moment, the APB illuminated nanoparticle could produce a lateral scattering, which is unattainable with the illumination of a plane wave. The leftward and rightward radiations, defined as the far-field scattering fields in the \(-x\) direction and \(+x\) direction, are

\[
\sigma_{d,l} = \sigma_d \left( \frac{\pi}{2}, \pi \right) = \frac{9}{4 k^2 I_{\text{total}}} \left| a_1 E_y - Z b_1 H_z \right|^2,
\]

\[
\sigma_{d,r} = \sigma_d \left( \frac{\pi}{2}, 0 \right) = \frac{9}{4 k^2 I_{\text{total}}} \left| a_1 E_y + Z b_1 H_z \right|^2.
\]

If the amplitudes and phases of the illumination field \(E_y\) and \(Z H_z\), as well as Mie scattering coefficients \(a_1\) and \(b_1\), fulfill the condition

\[
\frac{|E_y|}{|Z H_z|} = \frac{|b_1|}{|a_1|},
\]

\[
\phi_{E_y} - \phi_{Z H_z} = \phi_{a_1} - \phi_{b_1},
\]

the rightward unidirectional scattering could be realized. Similarly, the leftward unidirectional scattering can be achieved when

\[
\frac{|E_y|}{|Z H_z|} = \frac{|b_1|}{|a_1|},
\]

\[
\left( \phi_{E_y} - \phi_{Z H_z} \right) - \left( \phi_{a_1} - \phi_{b_1} \right) = \pi.
\]

These leftward and rightward unidirectional scattering conditions are referred to as the transverse Kerker condition for the \(x\) directional scattering [39]. Due to the rapidly varying electromagnetic field within the focal plane of an APB and the \(\pi/2\) out-of-phase between \(E_y\) and \(Z H_z\) (see Fig. 2), a core-shell nanoparticle fulfilled the condition of Eq. (5) or Eq. (6) may scatter field rightward or leftward at a specific position of the \(x\) axis. For simplicity and without loss of generality, we focus our attention on the scattering properties of an APB illuminated core-shell nanoparticle under the leftward unidirectional scattering condition throughout this paper. Figure 3 shows the amplitude ratio \((|b_1|/|a_1|)\) and phase difference \((\phi_{b_1} - \phi_{a_1})\) between \(b_1\) and \(a_1\) of a core-shell nanoparticle with \(R_{in} = 106\) nm and \(R_{out} = 206\) nm. \(|b_1|/|a_1| = 0.149\) and \(\phi_{b_1} - \phi_{a_1} = -1.56\) rad are obtained at the wavelength of 1.55 \(\mu\)m. As examining the values of \(|E_y|/|Z H_z|\) in Fig. 2(d), it can be predicted that the nanoparticle may perform a leftward unidirectional scattering at the position of \(x = 73\) nm.

![Fig. 3](image_url)

Fig. 3. (a) Amplitudes of \(a_1\) and \(b_1\), the ratio \((|b_1|/|a_1|)\) and (b) phase difference \((\phi_{b_1} - \phi_{a_1})\) between \(b_1\) and \(a_1\) of a core-shell nanoparticle with \(R_{in} = 106\) nm and \(R_{out} = 206\) nm.
To demonstrate the scattering properties of the core-shell nanoparticle illuminated by an APB, corresponding scattering efficiencies, near-field intensity distributions and far-field radiation patterns are performed. The scattering efficiency of the nanoparticle is [43]

\[ Q_{\text{sca}} = \frac{\sigma_{\text{sca}}}{\pi R_{\text{out}}^2}, \]  

(7)

with the scattering cross section \( \sigma_{\text{sca}} = \frac{P_{\text{sca}}}{I_{\text{inc}}} \), where \( I_{\text{inc}} \) is the incident field intensity and \( P_{\text{sca}} \) is the total power scattered via ED and MD modes giving by [21]

\[ P_{\text{sca}} = \frac{3\pi}{k \alpha_0} \left[ |a|^2 |E_{\text{inc}}(r_p)|^2 + |Z\alpha|^2 |H_{\text{inc}}(r_p)|^2 \right], \]  

(8)

where \( \omega \) is the angular frequency. Figures 4(a) and 4(d) show the scattering efficiencies of the core-shell nanoparticle (\( R_{\text{in}} = 106 \text{ nm}, R_{\text{out}} = 206 \text{ nm} \)) located within the focal plane at the optical axis (\( x = 0 \text{ nm} \)) and the position of \( x = 73 \text{ nm} \) on the \( x \) axis, when illuminated by a focused APB. When situated at the optical axis, only the longitudinal magnetic field polarizes the nanoparticle, a pure MD resonance at the wavelength of 1.263 \( \mu \text{m} \) is induced. As moving away from the optical axis, \( p_{\text{y}} \) and \( m_{\text{x}} \) are induced, finally, \( p_{\text{y}} \) and \( m_{\text{z}} \) become equal in strength at \( x = 73 \text{ nm} \) for the wavelength of 1.55 \( \mu \text{m} \). Here, the contribution of \( m_{\text{x}} \) can be ignored due to its low strength resulting from the weak local illumination field and the non-resonant state. The induced ED and MD are further confirmed by the electric near-field intensity distributions in \( xy \) cut plane through the center of nanoparticle, as shown in Figs. 4(b) and 4(e). The pure longitudinal MD has a ring-like electric field distribution inside the dielectric shell at the wavelength of 1.263 \( \mu \text{m} \). The corresponding far-field radiation pattern shows a characteristic donut-like shape of a dipole oriented along the \( z \) axis, as shown in Fig. 4(c). The typical two-lobe electric field distribution in Fig. 4(e) corresponds to an ED oriented along the \( y \) axis. The superposition of these transverse ED and longitudinal MD with equal strength leads to a strongly leftward unidirectional scattering, as shown in Fig. 4(f). To quantitatively determine the scattering directivity, the left-to-right ratio (LRR) in dB defined as

\[ 10 \log_{10} \left( \frac{\sigma_{\text{L}}}{\sigma_{\text{R}}} \right) \]  

is introduced. The LRR with 45 dB (numerically calculated result: 40.4 dB) is obtained for the core-shell nanoparticle located at \( x = 73 \text{ nm} \).

To confirm the analytical results, optical responses of the core-shell nanoparticle are also numerically calculated by using a commercial software (Lumerical Solutions Inc., Canada) based on the finite-difference time-domain (FDTD) method. All simulations are performed with a minimum mesh size of 1 nm and perfectly matched boundary conditions are employed to suppress any spurious reflections and mimic fields scattered to infinity. The scattered fields are recorded with a closed box monitor centered on the core-shell nanoparticle. The numerically calculated scattering efficiencies and far-field radiation patterns are in good agreement with the analytical results, as shown by the red dot markers in Figs. 4(a), 4(c), 4(d) and 4(f).
3. Transverse displacement sensing with tunable measuring range

The LRR (in dB) of far-field radiation from a core-shell nanoparticle illuminated by an APB can be deduced from Eq. (4)

$$LRR = 10 \log_{10} \left( \frac{E_y}{ZH_z} \right)$$

which is the function of illumination determined electromagnetic field at the particle center \((E_y/ZH_z)\) and structure dependent Mie scattering coefficients \((b_1/a_1)\). The APB illumination makes the core-shell nanoparticle perform a leftward unidirectional scattering at a specific position on the \(x\) axis, when the condition described by Eq. (6) is fulfilled. Due to the rapidly varying electromagnetic field within the focal plane of an APB, LRR of the core-shell nanoparticle becomes position dependent as the nanoparticle moves away from the optical axis. Therefore, it provides an opportunity to detect the transverse displacement of a single core-shell nanoparticle for an APB illumination. Figures 5(a) and 5(b) give the amplitude ratio and phase difference between \(b_1\) and \(a_1\) of a core-shell nanoparticle with \(R_{in} = 113\) nm.
and $R_{\text{out}} = 206$ nm. We adopt $|b_1|/|a_1| = 0.114$ and $\phi_1 - \phi_a = -1.403$ rad to make the LRR vary approximately linearly with respect to the transverse displacement at the wavelength of 1.55 μm. It is found that the far-field radiation changes dramatically from a symmetric pattern to a highly asymmetric unidirectional one, when the nanoparticle moves away from the optical axis to $x = 50$ nm for an APB illumination, as shown in Fig. 5(c). As can be seen, the corresponding LRR shown in Fig. 5(d) changes rather linearly as the nanoparticle moved from the optical axis to a displacement of 50 nm. Meanwhile, the actual ratio between leftward and rightward radiances varies from 1 to 100.

As seen in Figs. 5(a) and 5(b), the amplitude ratio and phase difference between $b_1$ and $a_1$ of a core-shell nanoparticle with $R_a = 113$ nm and $R_{\text{out}} = 206$ nm. Changes of APB excited (c) far-field radiation patterns and corresponding (d) LRR as the nanoparticle moved from the optical axis to a displacement of 50 nm along the $x$ axis at the wavelength of 1.55 μm. (e) LRR map of the focused APB illuminated nanoparticle as the function of wavelength and lateral displacement ($\Delta x$). (f) LRRs change with respect to lateral displacement for the wavelengths of 1.55 μm, 1.523 μm, 1.518 μm and 1.514 μm. The corresponding measuring ranges are 50 nm, 55 nm, 60 nm and 65 nm. The numerically calculated results based on FDTD method are given in (c), (d) and (f) as hollow and solid dots.

As seen in Figs. 5(a) and 5(b), the amplitude ratio and phase difference between $b_1$ and $a_1$ of the nanoparticle are very sensitive to the wavelength. As a result, the LRR is also wavelength dependent. Figure 5(e) shows the LRR of the APB illuminated nanoparticle changed with respect to the wavelength and lateral displacement ($\Delta x$). The maximum LRR is achieved at the wavelength of 1.533 μm with $\Delta x = 61$ nm, where the leftward unidirectional scattering condition of Eq. (6) is met. In addition, the LRR reduces quickly as the wavelength deviated from the leftward unidirectional scattering condition. Here, to ensure high detection
accuracy, the relatively linear change of LRR with respect to the transverse displacement is taken into account. The maximum displacement is defined as the measuring range of the displacement sensor, for which the LRR reaches 20 dB as the actual experimental measurements are taken into account [49]. As illustrated in Fig. 5(f), the measuring range from 50 nm to 65 nm is realized when the wavelength changes from 1.55 μm to 1.514 μm.

Fig. 6. (a) Amplitude ratio (|b_1|/|a_1|) and phase difference (\(\phi_1 - \phi_2\)) between \(b_1\) and \(a_1\) for a core-shell nanoparticle with \(R_{in} = 78.5\) nm and \(R_{out} = 220.5\) nm. (b) Changes of far-field radiation pattern as the nanoparticle moved from the optical axis to a displacement of 500 nm along the x axis within the APB focal plane, where the radiation for the displacements of 200 nm, 300 nm, 400 nm and 500 nm are multiplied by 1.1, 1.3, 1.6 and 1.9 for clear comparison. (c) LRR map of an APB illuminated (NA = 0.3, \(\lambda = 1.55\) μm) core-shell nanoparticle as a function of |\(b_1|/|a_1|\) and lateral displacement when \(\phi_1 - \phi_2 = -\pi/2\). (d) LRRs and (e) leftward differential scattering cross sections (\(\sigma_d\)) change with respect to lateral displacement for |\(b_1|/|a_1|\) (as well as \(\phi_1 - \phi_2\)) with the values of 0.022 (−1.384 rad), 0.114 (−1.403 rad), 0.631 (−1.401 rad), 1.145 (−1.388 rad), 1.941 (−1.4 rad) and 2.776 (−1.385 rad). The corresponding inner and outer radii (\(R_{in}, R_{out}\)) of the nanoparticles are (176 nm, 244 nm), (113 nm, 206 nm), (79 nm, 215 nm), (78.5 nm, 220.5 nm), (80.5 nm, 224.5 nm) and (82.5 nm, 227 nm), with the measuring ranges tuned from 10 nm, 50 nm, 280 nm, 500 nm, 800 nm to 1.1 μm. The numerically calculated results based on FDTD method are given in (b) and (d) as hollow dots.

Besides the illumination field, the measuring range of the displacement sensor can also be tuned by changing the Mie scattering coefficients (\(b_1/a_1\)), which relies on the ratio between outer and inner radii of the core-shell nanoparticle. As depicted in Fig. 6(a), |\(b_1|/|a_1|\) = 1.145 and \(\phi_1 - \phi_2 = -1.388\) rad of a core-shell nanoparticle (\(R_{in} = 78.5\) nm, \(R_{out} = 220.5\) nm) are obtained at the wavelength of 1.55 μm. According to the distribution of |\(E_y/ZH_z\) in Fig. 2(d), the nanoparticle may perform a leftward unidirectional scattering at the position around \(x = 500\) nm when illuminated by a focused APB. Figure 6(b) shows the far-field radiation patterns changed with respect to the lateral displacements for the nanoparticle (\(R_{in} = 78.5\) nm, \(R_{out} = 220.5\) nm) moved from \(x = 0\) nm to \(x = 500\) nm within the focal plane of an APB. The far-field radiation changes dramatically from a symmetric pattern to a highly leftward unidirectional one and the LRR reaches 20 dB for the displacement of 500 nm. Figure 6(c) shows the LRR of the APB illuminated nanoparticle as a function of |\(b_1|/|a_1|\) and lateral...
displacement at the wavelength of 1.55 μm. Here, the phase difference between \( b_1 \) and \( a_1 \) is kept at \(-\pi/2\) to satisfy the leftward unidirectional scattering condition. It is found that the corresponding lateral displacement increases monotonically as the ratio \(|b_1|/|a_1|\) vary from 0 to 2.8. Specifically, the measuring range from 10 nm to 1.1 μm is achieved by changing \(|b_1|/|a_1|\) from 0.022 to 2.776, with the inner and outer radii of the core-shell nanoparticles shown in Fig. 6(d). The corresponding leftward differential scattering cross sections (\(\sigma_{ld}\)) for these nanoparticles that changed with respect to lateral displacement are given in Fig. 6(e). This observation, which is the ability for lateral displacement sensing with widely tunable measuring range is also confirmed by the numerical calculation based on FDTD method, as shown by the graphs with hollow and solid dot markers in Figs. 5 and 6.

4. Conclusions

In summary, we have proposed a novel scheme to extend the measuring range of a transverse displacement sensor. The scheme is based on the interaction between an azimuthally polarized beam (APB) illumination and a single metal-dielectric core-shell nanoparticle. When subjected to a focused APB illumination, the induced longitudinal magnetic dipole (MD) superimposes with the transverse electric dipole (ED), leading to a transverse unidirectional scattering at a specific position within the focal plane, where the transverse Kerker condition is fulfilled. As the nanoparticle moves away from the optical axis, the scattering directivity becomes position encoded, which can be utilized for transverse displacement sensing. Without loss of generality, we consider the nanoparticle moves along the \(x\) axis and analytically derive the leftward and rightward unidirectional scattering conditions, which indicate that the lateral scattering directivity is completely determined by the amplitudes and phases of the illuminating electromagnetic fields and the associated first two Mie scattering coefficients of the spherical nanoparticle. Furthermore, the Mie scattering coefficients of the core-shell nanoparticle are sensitive to the wavelength and dependent on the ratio between outer and inner radii. Owing to these dependencies, it provides with us an abundant opportunity to efficiently control the scattering directivity of an APB illuminated core-shell nanoparticle with respect to the lateral displacements. Strikingly, based on the leftward unidirectional scattering condition, the measuring range of the displacement sensor can be continuously tuned from several nanometers to a few micrometers by adjusting the ratio between outer and inner radii of the core-shell nanoparticle. In addition, to confirm the analytical results, we have carried out numerical calculations using the finite-difference time-domain (FDTD) method. The numerical results are in good agreement with the corresponding analytical values. Although our present work is based on the interaction of an APB illumination with a spherical nanoparticle, the fundamental principles that we have exploited are equally valid for other singular beams, such as a radially polarized beam (RPB), and different particle shapes, such as nanodisks. Thus, we envision wide applicability of our scheme. In particular, the proposed scheme is essentially important for modern optical nanometrology and may promote applications in large-scope and high-precision nanoparticles localization and single molecules tracking in nanoscale region.

Funding

National Key R&D Program of China (2017YFA0303800); National Natural Science Foundation of China (NSFC) (11634010, 61675170, 11874050 and 61675171); Natural Science Basic Research Plan in Shaanxi Province (2017JM6022); Fundamental Research Funds for the Central Universities (3102017zy017).

References

43. C. F. Bohren and D. R. Huffman, Absorption and scattering of light by small particles (John Wiley & Sons, 1983).