INSIGHTS AND IMPLICATIONS ABOUT THE WHOLE NUMBER KNOWLEDGE OF GRADE 1 TO GRADE 4 CHILDREN

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Abstract

This paper provides a snapshot of the whole number knowledge of nearly 2000 Australian primary school children gained through a one-to-one assessment interview. The interview corresponds to a growth point framework that describes learning trajectories in counting, place value, addition and subtraction strategies, and multiplication and division strategies. The findings highlight the broad distribution of growth points in each domain for each grade level, and the wide distance between the lowest and highest growth points in each grade and domain. This demonstrates the complexity of classroom teaching and highlights the challenge of meeting each student's learning needs. Three issues related to children's Whole Number Arithmetic (WNA) emerged from the data. These suggest important themes for teacher professional learning and for refining mathematics curriculum and include: (1) interpreting 2-digit and 3-digit numbers; (2) using reasoning strategies as opposed to counting strategies in addition and subtraction; and (3) strategies for solving partially modelled and abstract problems in multiplication and division.

Key Words: arithmetic strategies, assessment interviews, teaching strategies

Introduction

Education outcomes for Australian children living in low Socio-Economic Status (SES) communities and Aboriginal and Torres Strait Islander communities are lower than for children not living in these communities (Commonwealth of Australia, 2008). An initiative launched by the previous Australian Government to address this issue in mathematics was a series of projects focused on how to close the numeracy gap for Australian children. This paper draws on the findings of one project, *Bridging the Numeracy Gap in low SES and Aboriginal Communities* (Gervasoni et al., 2011) that involved 44 schools in south-eastern Australia and Western Australia. Key approaches for improving mathematics learning in this study were: one-to-one interview-based mathematics assessments (Clarke et al., 2002; Gervasoni, Hadden and Turkenburg, 2007); using this data to guide instruction and curriculum development at individual, class and whole school levels (Gervasoni and Sullivan, 2007); and using the *Extending Mathematical Understanding* Program (Gervasoni, 2004) in the second year of school to provide intensive specialised instruction for those who were mathematically vulnerable. This paper reports on one aspect of this project; using the interview-based assessment and framework of growth points to gain insight about primary school children's whole number knowledge. Based on the insights gained, implications for WNA teaching and teacher professional learning will be discussed with the view to enhancing mathematics learning for all.

Using Frameworks and Interviews to Explore Whole Number Knowledge

Clinical assessment interviews are now widely used by teachers in Australia and New Zealand as a means of assessing children's mathematical knowledge. This is due to the experience of three large scale projects that informed assessment and curriculum policy formation in Victoria, NSW and New Zealand: *Count Me In Too* (Gould, 2000) in NSW, the Victorian *Early Numeracy Research Project* (Clarke, et al., 2002) and the *Numeracy Development Project* (Higgins, Parsons, and Hyland, 2003) in New Zealand. A common feature of each of these projects was the use of a one-to-one assessment interview and an associated research-based framework to describe progressions in mathematics learning (Bobis et al., 2005). A feature of assessment interviews is that they enable the teacher to observe children as they solve problems to determine the strategies they used and any misconceptions (Gervasoni and Sullivan, 2007). They also enable teachers to probe children's mathematical understanding through thoughtful questioning (Wright, Martland and Stafford, 2000) and observational listening (Mitchell and Horne, 2011). The insights gained through this type of assessment inform teachers about the particular instructional needs of each student more powerfully than scores from traditional pencil and paper tests, the disadvantages of which are well established (e.g., Clements and Ellerton, 1995). Because of the deep insight about children' mathematical knowledge gained through the use of one-to-one assessment interviews, the *Early Numeracy Interview* was chosen as the assessment tool for the *Bridging the Numeracy Gap* (BTNG) project. It was also anticipated that the data obtained would provide a rich snapshot of children's whole number knowledge.

The Early Numeracy Interview and Growth Points

The BTNG Project involved the assessment of primary school children's whole number knowledge at the beginning of each year using the *Early Numeracy Interview* (Department of Education Employment and Training, 2001) developed as part of the *Early Numeracy Research Project* (ENRP, Clarke et al., 2002). The data examined in this paper was drawn from this interview and the associated framework of growth points, so it is important that both are clearly understood. The principles underlying the construction and validation of the interview items and growth points, and the comparative achievement of children in the project and reference schools have been widely reported and are described in full in Clarke et al. (2002). A brief overview of the interview follows.

The *Early Numeracy Interview* is a clinical interview with a research-based framework of growth points that describe key stages of learning in nine mathematics domains, including the four whole number domains that are focused on in this paper: Counting, Place value, Addition and Subtraction, and Multiplication and Division. To illustrate the nature of the growth points, those for Addition and Subtraction follow. These describe strategies children use to solve problems. Each growth point represents substantial expansion in knowledge along paths to mathematical understanding (Clarke, 2001).

1. Counts all to find the total of two collections.
2. Counts on from one number to find the total of two collections.
3. Given subtraction situations, chooses appropriately from strategies including count back, count-down to and count up from.
4. Uses basic strategies for solving addition and subtraction problems (doubles, commutativity, adding 10, tens facts, other known facts).
5. Uses derived strategies for solving addition and subtraction problems (near doubles, adding 9, build to next ten, fact families, intuitive strategies).
6. Extending and applying. Given a range of tasks (including multi-digit numbers), can use basic, derived and intuitive strategies as appropriate.

The whole number tasks in the interview take between 15-25 minutes for each student and are administered by the classroom teacher. There are about 40 tasks in total. Given success with a task, the teacher continues with the next tasks in a domain (e.g., Place Value) for as long as the child is successful. The interview was refined during the BTNG project and renamed the *Mathematics Assessment Interview* (MAI) in 2010 (Gervasoni et al., 2011).

Gaining Insight about Children's Whole Number Knowledge

The data reported in this paper were collected in 44 Australian school communities in the States of Victoria and Western Australia. Participants included nearly 2000 Grade 1 to Grade 4 children (6-years to 9 years) who were assessed at the beginning of 2011 by their teachers using the MAI. Detailed interview record sheets were independently coded by research staff to determine the growth points children reached in each domain. This increased the validity and reliability of the data. The growth points for each student were entered into an SPSS database and analysed to determine the percentage of children on each growth point in each whole number domain and grade level.

Insights about Children's Whole Number Knowledge

Examination of the growth point distributions for nearly 2000 children gives a rich picture of whole number knowledge across the first five years of school. The following section explores the findings for Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies.

Counting Knowledge

The Counting growth point distributions for Grade 1 to Grade 4 children are shown in Fig. 1. The data highlights a wide distribution of growth points in each grade. This demonstrates the complexity and challenge of classroom teaching, and the need for activities and instruction to be customised for individuals. Apparent also is the growth that occurs from one grade to the next; the median Counting growth point increased by one growth in each grade.

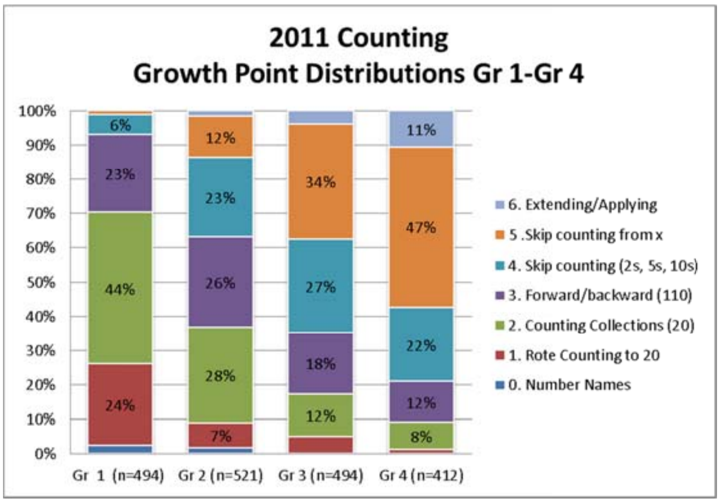

Fig. 1: Counting growth point distribution for Grade 1– 4 children

Examination of the growth point distribution for the Grade 4 children provides some insight about the growth of children's knowledge across the first four years of primary school. By the beginning of Grade 4 (fifth year at primary school), about 50% of children were working towards Growth Point 6 – extending and applying their counting knowledge. In contrast, about 10% of Grade 4 children could not yet count forwards and backwards by ones beyond 110, and another 20% could not yet skip count by 2s, 5s and 10s from zero. It is anticipated that these children (30%) would struggle with accessing some aspects of the Grade 4 curriculum and may benefit from customised instruction.

Place Value

Children's Place Value knowledge includes their abilities to read, write, order and interpret numbers. The growth point distributions are shown in Fig. 2.

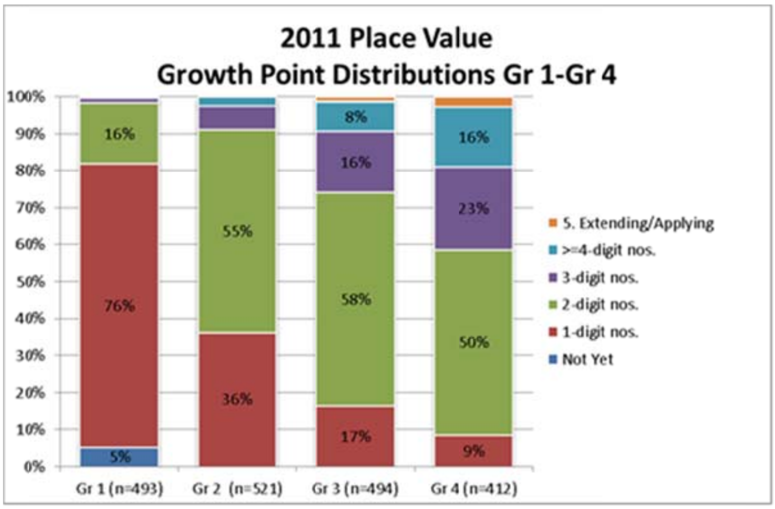

Fig. 2: Place Value growth point distribution for Grade 1-4 children

Again apparent is the wide distribution of growth points in each grade, but less growth is apparent from grade to grade compared with Counting. Indeed the median growth point remains GP2 (2-digit numbers) from Grade 2 to Grade 4. Reaching GP2 and GP3 (3-digit numbers) are significant challenges for young children. By Grade 4, 50% of children remained on GP2, and almost 10% on GP1. Thus the focus for 60% of these Grade 4 children was learning to interpret 2-digit and 3-digit numbers. This implied struggle with understanding 3-digit numbers well into Grade 4 is important for teachers to recognise. It was the interpretation of these quantities rather than the ability to read, write and order numerals that posed difficulty (Gervasoni et al., 2011). The complexity of teaching Place Value is highlighted further by the fact that 40% understand 3-digit numbers, while 15% reached GP4. This wide variation in the range of numbers children understand has implications for teaching arithmetic strategies.

Addition and Subtraction Strategies

A key learning focus in addition and subtraction is using reasoning strategies for calculating as opposed to counting strategies. The growth point distributions for Addition and Subtraction Strategies (Fig. 3) indicate that many children in all grades relied on counting strategies for calculating.

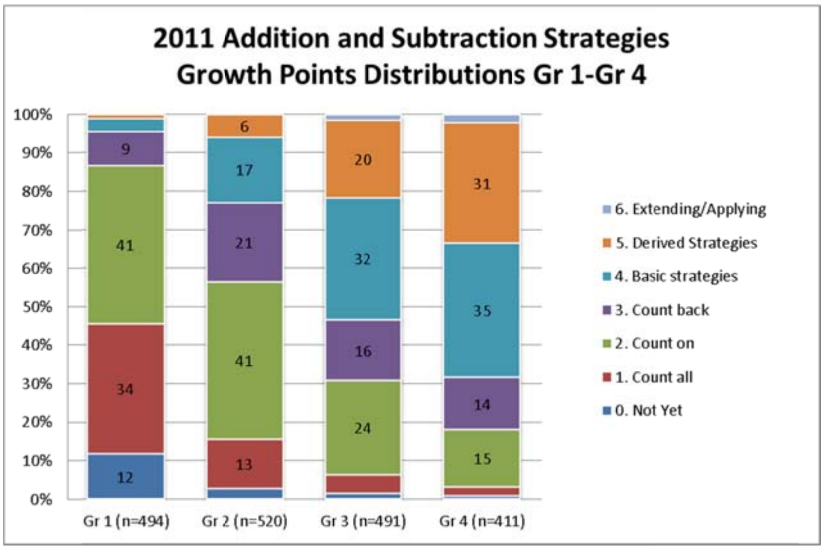

Fig. 3: Addition and Subtraction growth point distribution for Grade 1-4 children

The data in Fig. 3 indicate that 96% of Grade 1 children, 75 % of Grade 2 children, 46% of Grade 3 children and 30% of Grade 4 children used counting-based strategies for calculations, such as 4+4 and 10-3. The fact that so many Grade 4 children remain reliant on counting strategies for calculating, and that almost no Gr 4 students could solve mental calculations involving 2-digit and 3-digit numbers (GP6), is at odds with the tasks typically found in Grade 4 text books that involve calculations with much larger numbers.

Multiplication and Division Strategies

In Multiplication and Division, the key issue is children's ability to perform calculations without models being present, first in partial modelling situations and also when models are completely removed. Fig. 4 shows the growth point distributions for Multiplication and Division Strategies.

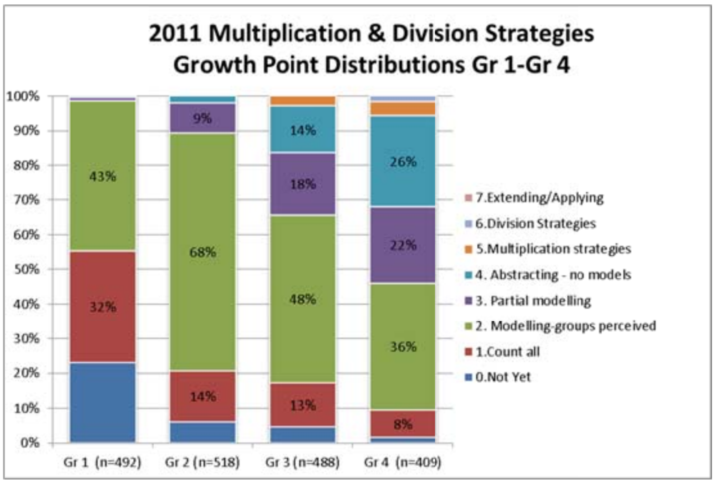

Fig. 4: Multiplication and Division growth point distribution for Grade 1-4 children

An outstanding feature of the growth point distributions shown in Fig. 4 is the large number of children in each grade on GP2 (models all objects to solve multiplicative and sharing situations). In Grade 3 only 35% of children were able to solve multiplicative problems in partial modelling or abstract situations. This increased to 50% of Grade 4 children. It is likely that children need more experience with an expanded range of multiplicative problems presented in partial modelling situations in order for their multiplicative reasoning to develop. Another feature of the data is that two-thirds of Grade 4 children were unable to solve multiplication problems without partial models present. This contrasts with the expectations implied in mathematics programs and text books.

Discussion and Conclusion

The findings presented in the previous section highlight several important issues related WNA learning. First is the wide distribution of growth points in each domain and each grade level, and the wide distance between the lowest and highest growth points in each grade level and domain. This highlights the complexity of teaching mathematics. The second issue is that progress through the growth points is more challenging for some growth points and in some domains. For example, progress from GP2 to GP3 in Place Value takes considerable time for many children. Knowledge about these challenging points and how to assist children to reach them is necessary to enable teachers to be most effective. This may be a useful focus for professional learning programs. The most challenging points identified in this study were: (1) interpreting 2-digit and 3-digit numbers; (2) using reasoning strategies as opposed to counting-based strategies in addition and subtraction; (3) and using strategies for solving partially modelled and abstract problems in multiplication and division. A third issue highlighted is the significant number of children on very low and very high growth points in most grade levels and domains. These children may be particularly vulnerable if teachers do not cater for their particular needs. These children are easily identified by referring to the growth point framework. It is also noteworthy that few children reached the highest growth points in each domain, even by Grade 4. This emphasises the importance of creating learning environments that enable all children to progress to the higher growth points.

In summary, the findings discussed in this paper suggest that there is no single ‘formula’ for describing children’s whole number knowledge or the instructional needs of children in a particular grade. Meeting the diverse learning needs of children requires teachers to be knowledgeable about how to identify each child’s current mathematical knowledge and customise instruction accordingly. This calls for rich assessment tools capable of revealing the extent of children’s knowledge in a range of domains, and an associated framework of growth points capable of guiding teachers’ curriculum and instructional decision-making. Assisting children to learn mathematics is complex, but teachers who are equipped with the pedagogical knowledge and actions necessary for responding to the diverse needs of individuals are able to provide children with the opportunities and experiences that will enable them to thrive mathematically.

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