Should Interpolation of Radar Reflectivity be Performed in $Z$ or dB$Z$?

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ABSTRACT

Interpolation of ground-based radar measurements is required when mapping data from their native spherical coordinates to a Cartesian grid. For reflectivity the question arises as to whether this processing should be performed in units of $Z$ (mm$^6$ m$^{-3}$) or dB$Z$. This study addresses this question using one year of data from three radars, operating in diverse climates across Australia. For each radar, a subset of 800 volume scans is processed to identify “triads”—groups of three consecutive gates with valid data—in each of the three coordinate directions: range, azimuth, and elevation. For every triad, the reflectivity at the central gate is estimated by linearly interpolating between the outer two gates in both $Z$ and dB$Z$. The resulting values are then compared with the true reflectivity at the central gate to quantify the interpolation errors. For all three sites and in all three coordinate directions, we find that interpolation in $Z$ is more accurate on average, especially in regions of high reflectivity and strong reflectivity gradient (i.e., convective cores). However, interpolation in dB$Z$ is better in regions of low and monotonically increasing/decreasing reflectivity. It is therefore recommended that reflectivities be converted from dB$Z$ to $Z$ prior to interpolation except when identifying echo-top height or other low-reflectivity boundaries.

1. Introduction

Interpolation of ground-based radar measurements is required when mapping data from their native spherical coordinates to a Cartesian grid and may also be applied when constructing constant-altitude plan position indicator (CAPPI) displays or identifying echo-top height (Lakshmanan et al. 2013). For the equivalent reflectivity factor (hereinafter, reflectivity) the question arises as to whether this processing should be performed in units of mm$^6$ m$^{-3}$, hereinafter $Z$, or in units of dB$Z$. Using $Z$ and $Z^*$ to represent reflectivity in $Z$ and dB$Z$, respectively, we have by definition

$$Z^* = 10 \log_{10} Z. \quad (1)$$

Lakshmanan (2012, hereinafter L12) noted that, in general, image processing is performed in dB$Z$, but offered anecdotal evidence of a “persistent belief” that processing in $Z$ is better since this is what the radar measures. The vast majority of methods for carrying out spatial interpolation implicitly assume that the data are locally linearly varying (Lakshmanan 2014, p. 33). This includes linear and bilinear interpolation (Mohr and Vaughan 1979; Zhang et al. 2005; Collis et al. 2010) but also methods that compute a weighted average over some radius of influence (ROI) (Cressman 1959; Barnes 1964; Askelson et al. 2000; Trapp and Doswell 2000). L12 rightly points out that the better field for such operations “has nothing to do with which quantity gets measured and everything to do with which quantity varies linearly within the substance being measured.”

L12 set out to examine the relative merits of interpolating reflectivity in $Z$ and dB$Z$. To do this he identified “triads”—groups of three consecutive range gates with valid data—in the lowest elevation scans of a single radar (the WSR-88D in Little Rock, Arkansas) for a

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1 This notation, while nonstandard, was chosen to provide a clear distinction between the two variables and between the variables and their units. Some authors use $Z$ and dB$Z$ to refer to both the variables and their corresponding units, which can lead to confusion. Others have suggested using $z$ to represent reflectivity in mm$^6$ m$^{-3}$ and $Z$ to represent reflectivity in dB$Z$, but this is problematic in atmospheric science where $z$ is already used for height.

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single year (2007). Estimates for reflectivity at the central gate of each triad were computed by linearly interpolating between the outer gates in both $Z$ and $d$B$Z$ and the resulting values were compared to the true reflectivity. Interpolation in dB$Z$ was found to consistently produce smaller errors and was therefore recommended as the standard going forward.

We note the following caveats with L12’s analysis:

1) Only triads in the range direction and for the lowest elevation scan are considered. This means that interpolation is always performed over a fixed distance of 2 km (i.e., 2 times the range gate spacing of 1 km) and on a quasi-horizontal plane at low altitude. It is possible that over larger distances, in the vertical direction, and/or at higher altitudes, spatial variations in $Z$ are more linear than those in $Z^*$. 

2) Data from only a single radar (and thus a single geographical location) are used. Given that the microphysical characteristics of precipitating systems vary geographically, it is conceivable that the spatial characteristics of reflectivity might also vary.

3) The dependence of interpolation error on reflectivity and reflectivity gradient is only partially explored. L12 subsets his result by the reflectivity of the two outer gates in each triad (one/both $>0$, 20, and 40 dB$Z$) and separately by the reflectivity gradient across them ($>2.5$ and $5$ dB$Z$ km$^{-1}$). He does not explore the full range of reflectivities and does not consider both of these factors in combination.

To address these points and provide a more definitive answer to the question posed in our title, we replicate and extend the analysis of L12 using full volume-scan data from three operational radars in Australia. We begin in section 2 with some theoretical considerations to illustrate why the choice of unit matters. Section 3 then provides details of our method. Results are presented in section 4, followed by a summary and discussion in section 5.

2. Theoretical considerations

To illustrate why the choice of unit matters, consider two proximate radar gates with reflectivities of $Z^*_1$ and $Z^*_2$. Say we want to estimate the reflectivity at a point $P$ that lies equidistant between these gates. We denote the estimates computed by interpolating in $Z$ and dB$Z$ with a bar ($\hat{Z}^*_p$) and a tilde ($\tilde{Z}^*_p$), respectively. Averaging in dB$Z$ we have

$$\hat{Z}^*_p = 0.5(Z^*_1 + Z^*_2) = 0.5(10 \log_{10}Z_1 + 10 \log_{10}Z_2) = 10 \log_{10}(Z_1/Z_2)^{1/2}. \quad (2)$$

Conversely, averaging in $Z$ and then converting to dB$Z$ we have

$$\tilde{Z}^*_p = 10 \log_{10}[0.5(Z_1 + Z_2)]. \quad (3)$$

Subtracting Eq. (2) from Eq. (3) we find that the difference in interpolated reflectivity at $P$ is

$$Z^*_p - \hat{Z}^*_p = 10 \log_{10}\left[\frac{Z_1 + Z_2}{2(Z_1/Z_2)^{1/2}}\right]. \quad (4)$$

We now let $Z^*_p = Z^*_1 + \Delta Z^*$, where $\Delta Z^* = 10 \log_{10}\Delta Z$, so that $Z_1 = Z_1\Delta Z$. Substituting for $Z_2$ in Eq. (4), the $Z_1$ terms cancel and we are left with

$$Z^*_p - \hat{Z}^*_p = 10 \log_{10}\left[1 + \frac{\Delta Z}{2(\Delta Z)^{1/2}}\right]. \quad (5)$$

This relationship is plotted in Fig. 1 for $\Delta Z^*$ values ranging from $-15$ to $+15$ dB. We note two things about this figure. First, the interpolation difference is non-negative for all $\Delta Z^*$. Thus, interpolating in $Z$ always gives a result that is greater than or equal to the result from interpolating in dB$Z$. This is because, as shown in Eq. (2), the arithmetic mean of two $Z^*$ values is equivalent to the geometric mean of the corresponding $Z$ values, and the arithmetic mean is always greater than or equal to the geometric mean for positive real numbers. The second thing we note from Fig. 1 is that the difference between interpolated $Z$ and $Z^*$ values can be large. For example, $\Delta Z^*$ of 6 dB gives an interpolation difference of 1 dB while for $\Delta Z^* = 10$ dB the difference is
2.4 dB. Differences of this magnitude can have a profound effect on estimations of rain rate or hail size and will also impact algorithms that apply reflectivity thresholds such as those for storm-cell identification (e.g., Dixon and Wiener 1993) and convective–stratiform separation (e.g., Steiner et al. 1995). For the data analyzed in this study, we find that reflectivity differences of 6–10 dB between adjacent bins are not uncommon, particularly in the azimuth and elevation directions where physical distances can be large at long range (not shown).

It is possible to generalize Eq. (5) to the case in which the reflectivity at \( P \) is computed as a weighted average across the \( n \) bins that fall within some ROI. In this case

\[
\tilde{Z}_k^w = 10 \log_{10}(Z_1^n Z_2^n \cdots Z_n^n)^{1/w} \quad \text{and} \quad \tilde{Z}_k = 10 \log_{10}[W^{-1}(w_1 Z_1 + w_2 Z_2 + \cdots + w_n Z_n)], \tag{6}
\]

where the weights \( w_k \) are positive and

\[
W = \sum_{k=1}^{n} w_k. \tag{8}
\]

We then let \( \tilde{Z}_k^w = Z_k^w + \Delta Z_k^w \) where \( \Delta Z_k^w = 10 \log_{10} \Delta Z_k \), so that \( \tilde{Z}_k = Z_k + \Delta Z_k \). Taking Eq. (7) minus Eq. (6) and simplifying, we obtain

\[
\tilde{Z}_k^w - \tilde{Z}_k = 10 \log_{10} \left[ \frac{w_1 \Delta Z_1 + w_2 \Delta Z_2 + \cdots + w_n \Delta Z_n}{W(w_1 \Delta Z_1 w_2 \Delta Z_2 \cdots w_n \Delta Z_n)^{1/w}} \right]. \tag{9}
\]

Unlike for the simple case of an unweighted average of two reflectivity values, we cannot easily plot this function as it depends on both the variations in reflectivity within the ROI and the weighting function. However, we can see that, as before, the interpolation difference 1) does not depend on the (mean) reflectivity, only its variability, and 2) is always positive. The latter result comes from the fact that the weighted arithmetic mean is always greater than or equal to the weighted geometric mean for positive real numbers. Thus, even when using ROI-based objective analysis methods, interpolating in \( Z \) will always give a larger value than interpolating in dBZ, except when all the reflectivities are equal, in which case the two interpolated values will also be equal.

It is emphasized that Eqs. (5) and (9) quantify the difference in interpolated values computed in \( Z \) and dBZ; they do not tell us anything directly about the errors associated with these approaches. For this we need to know \( Z_k^w \), the true reflectivity at \( P \). This leads naturally to L12’s triad-based approach, which is the focus of the remainder of this article.

### Table 1. Details of the radars used in this study. All three have an angular beamwidth of 1°, a pulse duration of 1 µs, a range gate spacing of 250 m, an azimuthal beam spacing of 1°, and a maximum range of 150 km. The Brisbane and Melbourne radars operate a volume coverage pattern (VCP) with 14 elevation tilts at 0.5°, 0.9°, 1.3°, 1.8°, 2.4°, 3.1°, 4.2°, 5.6°, 7.4°, 10.0°, 13.3°, 17.9°, 23.9°, and 32.0°. The Darwin VCP includes an additional tilt at 43.1°, but for consistency it was excluded from this analysis.

<table>
<thead>
<tr>
<th>Site</th>
<th>Coordinates</th>
<th>Radar</th>
<th>( \lambda ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brisbane</td>
<td>27.718°S, 153.240°E</td>
<td>Meteor-1500S</td>
<td>10.0</td>
</tr>
<tr>
<td>Darwin</td>
<td>12.457°S, 130.925°E</td>
<td>WSR-81C</td>
<td>5.3</td>
</tr>
<tr>
<td>Melbourne</td>
<td>37.855°S, 144.755°E</td>
<td>Meteor-1500S</td>
<td>10.0</td>
</tr>
</tbody>
</table>

### 3. Method

For our analysis we utilize one year of data from three operational radars located in the major Australian cities of Brisbane, Darwin, and Melbourne (Table 1). These sites cover a diverse range of climates and associated precipitation regimes, from tropical convection (Darwin) to midlatitude frontal systems (Melbourne). The year 2011 is selected as it was unusually wet across the country (Bureau of Meteorology 2012) and therefore provides a large sample of rain events. For each site, days with rainfall exceeding 1 mm over an area of at least 10 000 km² are first identified using gridded rain gauge data (Jones et al. 2009). From these, 100 days are randomly selected for subsequent analysis. To reduce computational expense, only eight scans are used for each day—specifically, those closest to the hours 0000, 0300, 0600, 0900, 1200, 1500, 1800, and 2100 UTC.

Following L12, triads of reflectivity are identified and stored for each radar scan. However, unlike L12, we consider all three coordinate directions: range, azimuth, and elevation. For every volume scan we consider the three directions separately, in each case searching for sets of three consecutive gates with valid data. Triads for which the central reflectivity is less than 10 dBZ are excluded to limit the influence of radar sensitivity on our results.² We also exclude all gates that are less than 1 km above radar level to try to eliminate residual clutter not removed during on-site quality control procedures (Rennie 2012). This precaution also limits potential effects associated with overly aggressive clutter filtering.

As part of the on-site processing, measured reflectivity values are quantized to reduce data transmission costs. For the three sites considered here, reflectivities are rounded down to the nearest of 160 quantization levels.

² Density plots of reflectivity versus range (not shown) indicate that, at the maximum range of 150 km, the minimum detectable signal is \(-5\) dBZ for the Brisbane radar, \(-8\) dBZ for the Darwin radar, and \(\leq -9\) dBZ for the Melbourne radar.
with increments of 1.5 dB from −9 to 0 dBZ, 1.0 dB from 0 to 10 dBZ, and 0.5 dB from 10 to 80.5 dBZ, plus additional levels of −32 and −30 dBZ. To mitigate systematic errors in our interpolation statistics, dithering is applied to the quantized reflectivity values in each triad. This involves adding a random number ranging from zero to the increment magnitude for that reflectivity value (e.g., 0–1 for a reflectivity of 0–9 dBZ). Triads comprising any reflectivity less than −9 dBZ are excluded from our analysis.

For each triad, the three reflectivity values are stored together with the range \( r \) (in meters), azimuth \( \phi \) (in degrees), and elevation \( \theta \) (in degrees) of the three gates. From these data, the interpolation errors can be computed, as illustrated in Fig. 2. Denoting the central bin of the triad with a subscript \( i \) and the neighboring bins with subscripts \( i - 1 \) and \( i + 1 \), the error \( \varepsilon \) (dB) associated with linearly interpolating in \( Z \) is

\[
\varepsilon = Z_i - Z_i = 10 \log_{10} [w(Z_{i-1} + (1 - w)Z_{i+1}) - Z_i^*],
\]

(10)

and the error associated with linearly interpolating in dBZ is

\[
\varepsilon_{\text{dBZ}} = \tilde{Z}_i - Z_i^* = [wZ_{i-1}^* + (1 - w)Z_{i+1}^*] - Z_i^*.
\]

(11)

Here \( w \) is the interpolation weight given by

\[
w_{\chi} = \frac{\chi_{i+1} - \chi_i}{\chi_{i+1} - \chi_{i-1}},
\]

(12)

where \( \chi = r, \phi, \) and \( \theta \) for triads in the range, azimuth, and elevation directions, respectively. The range gate and azimuthal spacing are constant so \( w_r = w_\phi = 0.5 \); however, since the elevation angles are unevenly spaced (Table 1), \( w_\theta \) varies between tilts, from 0.5 to 0.61.

In comparing interpolation errors for the two methods, we examine their distributions, characterized using box and whisker diagrams, together with the bias and root-mean-square error (RMSE), computed as

\[
\text{bias} = \frac{1}{N} \sum_{j=1}^{N} \varepsilon_j
\]

(13)

\[
\text{RMSE} = \left( \frac{1}{N} \sum_{j=1}^{N} \varepsilon_j^2 \right)^{1/2},
\]

(14)

where \( N \) is the number of triads. Following L12, we also compute the fraction of triads for which interpolation in dBZ is better (i.e., \(|\varepsilon_Z| > |\varepsilon_{\text{dBZ}}|\)) as well as the fraction of triads for which interpolation in \( Z \) is better (i.e., \(|\varepsilon_Z| < |\varepsilon_{\text{dBZ}}|\)). These two fractions do not necessarily sum to 1 because in some triads \( Z_{i-1}^* = Z_{i+1}^* \) and therefore \( \varepsilon_Z = \varepsilon_{\text{dBZ}} \).

In parts of our analysis, triads are divided into three types according to whether they represent a local maximum \((Z_i^* > Z_{i-1}^* \text{ and } Z_i^* > Z_{i+1}^*)\); hereinafter “max” triads), a local minimum \((Z_i^* < Z_{i-1}^* \text{ and } Z_i^* < Z_{i+1}^*)\); hereinafter “min” triads), or a region of monotonically increasing or decreasing reflectivity \((Z_i^* \leq Z_{i-1}^* \leq Z_{i+1}^*) \text{ or } Z_i^* \geq Z_{i-1}^* \geq Z_{i+1}^* \); hereinafter “mono” triads). This is relevant because, as previously stated, interpolation in dBZ will almost always produce a smaller value than interpolation in \( Z \) and thus will in general be more accurate for min triads but less accurate for max triads.

For mono triads the better variable for interpolation depends on the curvature (second derivative) of the reflectivity field when measured in \( Z \) versus dBZ; if one field has smaller curvature then by definition it varies more linearly and will have smaller interpolation errors.

The overall better variable for interpolation (i.e., across all triads) will depend on both the relative frequency of the three triad types and their associated error magnitudes. As we will see, both of these factors vary as a function of reflectivity.

For mono triads, it is desirable to be able to quantitatively compare the curvature of the reflectivity field when measured in \( Z \) and dBZ. To this end, we normalize the reflectivity and coordinate variable at each bin as follows:

\[
\hat{Z}_j = \frac{Z_j - Z_{j-1}}{Z_{i+1} - Z_{i-1}},
\]

(15)

\[
\hat{Z}_j^* = \frac{Z_j^* - Z_{j-1}^*}{Z_{i+1}^* - Z_{i-1}^*},
\]

(16)

and

\[
\hat{x}_j = \frac{x_j - x_{j-1}}{x_{i+1} - x_{i-1}},
\]

(17)
With these definitions, \( \hat{Z}_{r-1} = \hat{Z}_{r} = \hat{\chi}_{r-1} = 0, \hat{Z}_{r+1} = \hat{Z}_{r}^{*} = \hat{\chi}_{r+1} = 1 \), and \( \hat{Z}_{r+1} \), \( \hat{Z}_{r}^{*} \), and \( \hat{\chi}_{r} \) are between 0 and 1. We can then define the normalized curvature \( \hat{C} \) of the reflectivity in dB as

\[
\hat{C}_{Z} = \frac{1}{0.5(\hat{\chi}_{r+1} - \hat{\chi}_{r-1})} \left( \frac{\hat{Z}_{r+1} - \hat{Z}_{r}}{\hat{\chi}_{r+1} - \hat{\chi}_{r}} - \frac{\hat{Z}_{r} - \hat{Z}_{r-1}}{\hat{\chi}_{r} - \hat{\chi}_{r-1}} \right)
\]

\[
= 2 \left( \frac{1 - \hat{Z}_{r}}{1 - \hat{\chi}_{r}} \right), \quad (18)
\]

and the normalized curvature of the reflectivity in dBZ as

\[
\hat{C}_{anz} = \frac{1}{0.5(\hat{\chi}_{r+1} - \hat{\chi}_{r-1})} \left( \frac{\hat{Z}_{r+1}^{*} - \hat{Z}_{r}^{*}}{\hat{\chi}_{r+1} - \hat{\chi}_{r}} - \frac{\hat{Z}_{r}^{*} - \hat{Z}_{r-1}^{*}}{\hat{\chi}_{r} - \hat{\chi}_{r-1}} \right)
\]

\[
= 2 \left( \frac{1 - \hat{Z}_{r}}{1 - \hat{\chi}_{r}} \right). \quad (19)
\]

Since \( \hat{\chi}_{r} = 1 - w_{r} \) [cf. Eqs. (12) and (17)], we note that \( \hat{C} \) has limits of \( \pm 4 \) for triads in the range and azimuth directions, whereas in the elevation direction \( -5.1 \leq \hat{C} \leq 4 \).

Before presenting results we note two potential sources of error in our analysis: signal attenuation and radar miscalibration. Since the Darwin radar operates at C band, its measurements are subject to attenuation in the presence of large hydrometeors (i.e., heavy rain and hail). Consequently, in areas located behind intense storm cells (a common feature during the wet season) reflectivities are likely to be underestimated. For the S-band Brisbane and Melbourne radars, attenuation will be far less significant. Radar miscalibration, on the other hand, may affect all three radars. Using the Ku-band precipitation radars on board the TRMM and Global Precipitation Measurement mission (GPM) satellites as a reference, Warren et al. (2018) found calibration errors up to around 5 dB for three radars in the vicinity of Sydney, Australia. Real-time comparisons with GPM, which are now performed routinely across the entire Australian radar network, have revealed that errors of this magnitude are not uncommon. Warren et al. (2018) also found that sudden dramatic changes in calibration (>5 dB) can occur as a result of radar maintenance activities. Thus, reflectivities from all three sites may be underestimated and both the magnitude and sign of the errors may vary in time.

Both of these issues are relevant because, as we will show, interpolation errors vary as a function of reflectivity. Since no attempt was made to correct for attenuation or miscalibration we must be cognizant of their potential influence on our results. We note two possibilities. First, differences in calibration between the three sites may amplify or diminish true differences in the observed reflectivity distributions. In this case, any differences in interpolation error statistics between the sites would not be solely attributed to their different climates. For the Darwin site, this issue will be compounded by attenuation that will act to shift the reflectivity distribution to lower values. Second, changes in radar calibration over time may add additional scatter to the relationships between interpolation error and reflectivity. The former effect is more important for our conclusions. However, as we shall see, our results are remarkably consistent between the three radars, suggesting that the impacts of attenuation and miscalibration are minimal.

### 4. Results

#### a. Overall errors

We start by considering the interpolation error statistics computed across all triads for each radar and coordinate direction. These are summarized in Table 2. We note that there is a very robust sample size (between 65 and 110 million triads) for every radar–direction combination. On average, interpolation in Z produces small biases ranging from −0.1 to +0.2 dB, whereas interpolation in dBZ produces larger negative biases between −2.3 and −0.5 dB. RMSE values are comparable (within 0.1 dB) between the two approaches; however, considering pairwise comparisons, interpolation in Z is better in 53.0%–59.1% of the triads. This is a markedly different result from L12 who found interpolation in dBZ to be better for the majority (54%) of his 2.2 billion triads (his Table 1). We found that we were able to obtain results more in line with L12 by considering only mono triads. In this case, the dBZ-better fraction ranges from 53.2% to 56.4% and RMSE values are consistently higher for interpolation in Z (not shown). We initially speculated that L12 had filtered out max and min triads, perhaps inadvertently, during his data processing; however, inspection of the relevant code revealed that this was not the case (V. Lakshmanan 2018, personal communication). Further discussion of this discrepancy is reserved for section 5.

In addition to the error statistics, Table 2 also shows the relative proportion of max, min, and mono triads. Max triads make up around one-third (33.3%–36.0%) of

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3 Although L12 only used data from a single tilt, his sample size is still considerably larger because he analyzed all scans over the year whereas we have only taken eight scans per day for a subset of 100 days.
the total for all radars and coordinate directions, while the proportion of min and mono triads is more variable (10.9%–26.2% and 37.7%–54.0%, respectively). There are notably fewer min triads in the elevation direction, consistent with the fact that reflectivity generally decreases with height, except around the brightband, which is a local maximum. The overall higher frequency of max triads compared to min triads can in part explain the fact that interpolation in $Z$ is better on average. However, we must also consider differences in error magnitude between the different triad types.

Figures 3a–c shows the error distributions for each radar and coordinate direction, separated by triad type. As we would expect, errors are always negative for max triads and positive for min triads; however, on average the error magnitudes are larger for max triads. This implies that local maxima are more peaked than local minima. As previously noted, for mono triads, both positive and negative errors can occur, depending on the curvature of the reflectivity field. Positive errors tend to dominate for interpolation in $Z$, indicating that $Z$ is predominantly characterized by positive curvature. For interpolation in dBZ, the distributions are more symmetric around zero, but the mean error is slightly negative, indicating that $Z$ has weak negative curvature on average. In both cases, however, the error magnitudes are smaller than for the corresponding max and min triads. Taking all triads together (Fig. 3d) we find that the max, min, and mono errors largely cancel for interpolation in $Z$, while for interpolation in dBZ the mean error is negative, consistent with the results in Table 2.

The positive curvature of $Z$ and weak negative curvature of $Z'$ for mono triads is confirmed by Fig. 4, which shows distributions of the normalized curvature for each radar and coordinate direction. This figure also reveals that for both variables, but especially $Z$, the curvature is most pronounced in the elevation direction.

This helps to explain why interpolation errors are notably higher for mono triads in the elevation direction compared to those in the range and azimuth directions (Fig. 3c). Error magnitudes are also consistently higher in the elevation direction for max and min triads (Figs. 3a,b), suggesting that local reflectivity extrema tend to be more peaked in the vertical than in the horizontal. Despite these differences, the overall results are remarkably consistent across the three coordinate directions and for the three radars, suggesting that they reflect fundamental characteristics of reflectivity when measured in $Z$ and dBZ.

We note that the differences in mean interpolation error and mean normalized curvature between $Z$ and dBZ are all highly statistically significant according to a difference of means test for paired samples (e.g., Wilks 2011, chapter 5). Indeed, due to the very large sample sizes, the associated $p$ values are vanishingly small. This is true for all the paired $Z$ and dBZ distributions analyzed in this work and, as such, we shall not discuss statistical significance further.

b. Dependence on reflectivity

We now examine whether the above findings are consistent across all reflectivities. Figure 5 shows the triad and interpolation error statistics as a function of reflectivity for the Brisbane radar and the azimuth direction. Data are binned according to the true reflectivity at the central gate in increments of 5 dB. Note that in the final two panels, we only show the $Z$-better fraction and not the dBZ-better fraction. Given that the percentage of triads for which the two methods have equal errors is very small (<0.1%; cf. Table 2) we may reasonably assume that the dBZ-better fraction is simply equal to 1 minus the $Z$-better fraction.

As we would expect, the total number of triads decreases with increasing reflectivity (Fig. 5a) while the proportion of max triads increases (Fig. 5b). Taken

<table>
<thead>
<tr>
<th>Radar</th>
<th>Direction</th>
<th>$N$</th>
<th>$P_{\text{max}}$ (%</th>
<th>$P_{\text{min}}$ (%)</th>
<th>$P_{\text{mono}}$ (%)</th>
<th>Bias$_Z$ (dB)</th>
<th>Bias$_{\text{dBZ}}$ (dB)</th>
<th>RMSE$_Z$ (dB)</th>
<th>RMSE$_{\text{dBZ}}$ (dB)</th>
<th>$Z$ Better (%)</th>
<th>dBZ Better (%)</th>
</tr>
</thead>
</table>
alone, this would suggest that interpolation in $Z$ will be increasingly preferable as reflectivity increases; however, we must also consider how the errors for each triad type change. For max triads (Fig. 5c), the mean error shows little dependence on reflectivity, except around 40 dB$Z$ where it decreases slightly (becoming more negative). We speculate that this represents the transition from predominantly stratiform to predominantly convective precipitation, with the latter characterized by more peaked reflectivity maxima (e.g., Steiner et al. 1995) and thus larger interpolation errors. For min triads (Fig. 5d), errors decrease with increasing reflectivity, indicating that local minima become less peaked. The mono triads (Fig. 5e) show the most interesting behavior. For both interpolation approaches, mean error decreases with increasing reflectivity; however, since it is initially positive for $Z$ and negative for dB$Z$, this means that the absolute error decreases with increasing reflectivity for $Z$ but increases with increasing reflectivity for dB$Z$. As a consequence, interpolation in dB$Z$ is better at reflectivities below 30 dB$Z$, but interpolation in $Z$ is better at higher reflectivities (Fig. 5g). Taking all triads together (Figs. 5f,h), we see that at reflectivities below about 20 dB$Z$, the two approaches have comparable errors; however, at higher reflectivities interpolation in $Z$ is emphatically better. As shown in Fig. 6, this finding is consistent across all radars and coordinate directions. This again contradicts the results of L12 who found that while the dB$Z$-better fraction decreased with increasing reflectivity, it remained greater than 50% even for a threshold of 40 dB$Z$.

Figure 6 shows that the transition from dB$Z$ better to $Z$ better for mono triads is also consistent across all radars and coordinate directions. To understand this behavior we must again consider the curvature of the reflectivity field. Figure 7 shows the mean normalized curvature as a function of reflectivity for every radar and coordinate direction. We see that at low reflectivities (below 25 dB$Z$), the reflectivity field has pronounced positive curvature when measured in $Z$ and slight negative curvature when measured in dB$Z$. Thus, the reflectivity at the central volume is overestimated when we interpolate in $Z$ and slightly underestimated when we interpolate in dB$Z$ (cf. Fig. 5e). Variations in reflectivity are clearly more linear in dB$Z$ in this case, so interpolating in these units is better. As reflectivity
increases, the positive curvature in $Z$ diminishes while the negative curvature in dBZ increases. Consequently, at high reflectivities (above 40 dBZ), interpolation in $Z$ is quite accurate while interpolation in dBZ produces significant negative errors (cf. Fig. 5c). Variations in reflectivity are now more linear in $Z$ so this is the better field for interpolation. For intermediate reflectivities (25–40 dBZ), the curvature in $Z$ and dBZ, and thus the interpolation errors, are comparable in magnitude. 

As an aside, we note that for the C-band Darwin radar, in all three coordinate directions, the normalized curvature shows a strong downward trajectory at the highest reflectivities (above 50 dBZ) and actually goes negative in $Z$. Since this behavior is not observed for the two S-band radars, we speculate that it may be a consequence of attenuation. Other subtle differences between the three radars (e.g., the exact reflectivity at which mono triads transition from dBZ better to $Z$ better) may be attributable to differences in calibration.

c. Dependence on reflectivity gradient

For mono triads, it is also worth considering whether interpolation errors vary with the reflectivity gradient. L12 found a pronounced increase in the percentage of triads for which interpolation in dBZ is better as the reflectivity gradient increased from 0 to 10 dB km$^{-1}$ (his Fig. 1). However, he did not explore whether this relationship held across different reflectivities. In Fig. 8 we again show the $Z$-better percentage for mono triads as a function of reflectivity (cf. Fig. 6), but this time with the data binned according to the absolute reflectivity gradient $G$, computed as

$$G = \frac{Z_{i+1}^* - Z_{i-1}^*}{\alpha \chi_{i+1} - \chi_{i-1}}. \tag{20}$$

Here $\alpha_i$ is a factor that converts $\chi_{i+1} - \chi_{i-1}$ to a physical distance ($\alpha = 1; \alpha_d = \alpha = r\pi/180$). Three gradient bins are considered: $0 \leq G < 5$, $5 \leq G < 10$, and $G \geq 10$ dB km$^{-1}$.

We first note that the observed positive trend (i.e., with interpolation in dBZ better at low reflectivities and interpolation in $Z$ better at high reflectivities) is maintained across all gradient bins, although it is very weak in the range direction for the lowest gradient bin (because the reflectivity differences in this case are very small). In general, the trend becomes more pronounced with increasing reflectivity gradient, although the relationship is nonmonotonic in some cases (e.g., for the Darwin elevation triads at low reflectivities; Fig. 8f). Considering only the range direction and taking the average over all reflectivities (i.e., the intersection of each curve with the corresponding vertical line) we obtain qualitatively similar results to L12, although the increase in dBZ better percentage is less pronounced, presumably due to our smaller range gate spacing (250 m vs 1 km) and associated smaller reflectivity differences.

d. Dependence on range

Due to beam spreading, the volume sampled by each radar gate increases with range from the radar. At the same time, the distance between gates in the azimuth and elevation directions also increases. Given that these changes can be dramatic across the radar’s field of view, it is worth considering whether interpolation errors vary with range. Figure 9 shows similar information to Fig. 5, but with data binned by range (in increments of 15 km) instead of reflectivity. In this case, we see little change in either the errors or the $Z$-better fractions across the parameter space. There is an increase in the proportion of mono triads, and a corresponding decrease in the proportion of max and min triads, which is consistent with the increase in sample volume (i.e., greater smoothing of the reflectivity field). However, since max triads still dominate in terms of their errors, and occur more frequently than min triads, the effect of this change in triad proportion is minimal. Very similar results are obtained for the other radars and coordinate directions (not shown).

5. Summary and discussion

Following the triad-based method of L12, we have investigated the relative accuracy of performing interpolation of reflectivity in units of $Z$ (mm$^3$ m$^{-3}$) and dBZ. While that study considered only a single radar site...
FIG. 5. Interpolation statistics, plotted as a function of reflectivity, for the Brisbane azimuth triads: (a) number of triads, on a logarithmic scale; (b) relative frequency of max (red), min (blue), and mono (yellow) triads; error distributions (as in Fig. 3) for interpolation in $Z$ (blue) and dB$Z$ (red) computed across (c) max, (d) min, (e) mono, and (f) all triads; percentage of (g) mono and (h) all triads for which interpolation in $Z$ is better. Dashed vertical lines in (g) and (h) show the mean reflectivity for mono and all triads, respectively.
(Little Rock) and coordinate direction (range), we have analyzed observations from three radars operating in diverse climates (tropical, subtropical, and temperate) and in all three coordinate directions (range, azimuth, and elevation). We have also extended the study of L12 by more fully characterizing the dependence of interpolation error on reflectivity and reflectivity gradient, as well as exploring the dependence on range.

We find that, on average, interpolation in $Z$ is more accurate in around 53%–59% of cases, with percentages being slightly higher in the elevation direction but very consistent between the three radars. Furthermore, as reflectivity increases, interpolation in $Z$ becomes increasingly preferable. For example, above 50 dBZ interpolation in $Z$ is better more than 70% of the time (Fig. 6). There are two reasons for this. First, as one might expect, with increasing reflectivity the relative frequency of local maxima (for which interpolation in $Z$ is better) increases and the relative frequency of local minima (for which interpolation in dBZ is better) decreases. The second reason is less intuitive. It turns out that for regions of monotonic reflectivity change, interpolation errors vary strongly with reflectivity. At low reflectivities (below 25 dBZ) absolute errors are smaller for interpolation in $Z$ while at high reflectivities (above 40 dBZ) the opposite is true. These characteristics are indicative of the curvature of the reflectivity field. When measured in $Z$, reflectivity displays predominantly positive curvature that is most pronounced at low reflectivities. In contrast, reflectivity in dBZ displays predominantly negative curvature that is most pronounced at high reflectivities. In general, the degree of curvature increases with the reflectivity gradient. As a result, the differences between interpolation in $Z$ and interpolation in dBZ are typically more pronounced in regions of strong reflectivity gradient. On the other hand, there appear to be no significant variations in interpolation error with range.

Fig. 6. Percentage of all (purple) and mono (yellow) triads for which interpolation in $Z$ is better, plotted as a function of reflectivity, for each radar and coordinate direction. Dashed vertical lines show the corresponding mean reflectivities.
In summary, we find that interpolation of reflectivity is more accurate in $Z$ than in dBZ, especially in regions of high reflectivity and strong reflectivity gradient, that is, convective cores. Preservation of detail in these areas is critical since they are often associated with large rain rates and hazards such as hail, lightning, and tornadoes. We therefore recommend that radar gridding procedures convert reflectivities to $Z$ before carrying out any interpolation or spatial smoothing, and then convert back to dBZ for display. One instance where we would recommend interpolating in dBZ is the identification of echo-top height, since this is a region of low reflectivity and monotonic reflectivity change. Our results show that if echo-top height is defined using a threshold of 18 dBZ (cf. Lakshmanan et al. 2013), then interpolation in dBZ will produce lower errors in around 55% of cases (Fig. 6). For a lower reflectivity threshold the percentage will likely be higher.

Our results stand in stark contrast to those of L12 who found interpolation in dBZ to be better on average, even when considering moderately high reflectivities (>40 dBZ). We see two possible explanations for this inconsistency: differences in our data processing/analysis methods and differences in study region. One potentially important difference in data processing is our exclusion of triads where the central volume has a reflectivity less than 10 dBZ. Our analysis clearly shows that the benefit of interpolating in $Z$ diminishes as reflectivity decreases. Extrapolating backward following the trends shown in Fig. 6 suggests that interpolation in dBZ would be more accurate for reflectivities below 10 dBZ. Given that low reflectivities occur much more frequently (Fig. 5a), these values could dominate L12's average across all triads, giving a dBZ-better fraction above 50%. However, as noted above, L12 also found interpolation in dBZ to be more accurate when considering only high-reflectivity triads. A crucial difference in this case may be the way this filtering was applied. While we binned our triads according to the reflectivity of the central volume, L12 used the
reflectivities of the two outer volumes. Both approaches are valid but ours is likely to have favored local maxima over local minima while the opposite is true for L12’s approach. For a given reflectivity threshold we would therefore expect our results to show a lower dBZ-better fraction. It should be emphasized that L12 did find a decrease in this fraction with increasing reflectivity, with values of 55%, 54%, and 52% for thresholds of 0, 20, and 40 dBZ, respectively.

The consistency of our results across different sites (climates) and coordinate directions suggests that they represent fundamental characteristics of reflectivity when measured in units of $Z$ and dBZ. However, given the disagreement between this study and L12 we must entertain the possibility that our conclusions do not hold globally. To address this uncertainty we plan to apply L12’s method to several years of three-dimensional reflectivity observations from the dual-frequency precipitation radar on board the GPM Core Observatory satellite. Results from this analysis will be presented in a future study.

One criticism that could be levelled at this study, and that of L12, is that the triad-based method uses linear interpolation, whereas real objective analysis of radar data typically involves more sophisticated methods, such as distance weighting over an ROI. However, as mentioned in the introduction, these methods still assume that the data are locally linearly varying. As such, we anticipate that our findings will carry over to other interpolation approaches, at least qualitatively. No doubt real interpolation errors will differ in magnitude from those found here (e.g., in Table 2), particularly when data from multiple radars are combined in a mosaic. One could quantify these differences by extending the analysis performed here to one or more ROI-based interpolation schemes (e.g., Cressman 1959; Barnes 1964). In this case the true reflectivity at a given gate would be compared with an estimate computed by

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**FIG. 8.** As in Fig. 6, but for mono triads with reflectivity gradients of 0–5 (green), 5–10 (cyan), and ≥10 (blue) dB km$^{-1}$. Numbers in the bottom-right corner of each panel show the percentage of triads that fall in each of these categories.
averaging (using the appropriate distance weighting) over a radius around the gate. One could even incorporate data from multiple radars in the interpolation. However, we believe that it would be more fruitful to investigate the “real” impacts of interpolating in $Z$ versus $\text{dB}Z$, by comparing objectively analyzed reflectivity grids, and a variety of derived products, computed using these two approaches. Work on this topic is already under way, using recently reprocessed data from the C-band polarimetric research radar (CPOL) in Darwin (Louf et al. 2019), and will form the subject of a future paper.

**Fig. 9.** As in Fig. 5, but plotted as a function of range.
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