Huygens-Fresnel acoustic interference and the development of robust time-averaged patterns from travelling surface acoustic waves

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The generation of periodic patterns using time-averaged acoustic forces conventionally requires the intersection of counter-propagating wave fields, where suspended micro-objects in a microfluidic system collect along force potential minimising nodal/antinodal lines. Whereas this effect typically requires either multiple transducer elements or whole channel resonance, we report the generation of scalable periodic patterning positions without either of these conditions. A single propagating surface acoustic wave interacts with the proximal channel wall to produce a knife-edge effect according to the Huygens-Fresnel principle, where these cylindrically propagating waves interfere with classical wavefronts emanating from the substrate. We simulate these conditions and describe a model that accurately predicts the lateral spacing of these positions in a robust and novel approach to acoustic patterning.

Ultrasonic acoustic fields are applicable to a number of industrial and clinical processes, where the resultant acoustic forces permit the non-contact manipulation of macro-scale objects, microparticles, and biological cells. Notable and diverse examples of acoustic manipulation include an acoustic tractor beam,[1, 2] acoustic levitation systems,[3–5] particle patterning and positioning [6–8] and continuous sorting according to size and mechanical properties.[9, 10] Acoustic fields are especially interesting for micro-scale biological processing, where acoustic wavelengths can be generated on the scale of tens of microns or smaller [11] for the long-term patterning and culture of cells.[12, 13] There are a number of acoustic actuation technologies at the microscale, using bulk acoustic waves (BAW) to excite channel resonances,[14–17] coupled substrate vibrations from surface acoustic waves (SAW),[18, 19] and thin-membrane oscillations[20, 21] to develop spatially periodic patterning. The patterning mechanism in each case, however, remains effectively the same, where the intersection of a propagating and counter-propagating or reflected acoustic waves results in time-averaged minimum pressure nodes to which suspended particles typically migrate, spaced 1/2 of an acoustic wavelength (\(\lambda_{ac}\)) apart.

While travelling waves can also be utilized for micro-scale manipulation and continuous sorting[22, 23] the time-averaged force arising from scattering is continuous in the propagation direction, albeit exponentially decaying, rather than periodic. Patterning of particles can however be achieved using holographic methods at the intersection of phase shifted wavefronts,[24] at either side of an surface acoustic wave generated beam[25] or with intersecting travelling waves. In the latter, the minimum pressure location(s) can be predicted from the rectilinear acoustic propagation model,[26] which is widely used when considering SAW systems.

However, there have been a limited number of observations of particle patterning with the imposition of travelling waves that are not accounted for by rectilinear travelling wavefronts.[7, 27] An alternative, less simplified, model for describing the propagation of acoustic energy is that according to the Hyugens-Fresnel principle, where every point on an advancing wavefront can be represented as a spherical wave source.[28] Considered in a 3-dimensional domain symmetric about the plane of interest (i.e., the x-z plane projected in the y-direction), this takes the form of a cylindrical wave source. While this model is necessary to describe interference patterns in the classical double-slit experiment, it has not been a common feature in describing microscale acoustofluidic phenomenon, where the small fluid acoustic attenuation values at even MHz frequencies generally lead to reflections and the resulting predominance of acoustic standing waves. A substrate-bound wave such as a Rayleigh-mode SAW, however, offers shorter acoustic attenuation lengths than other wavemodes[29] that can also be orders of magnitude smaller than the fluid one, \(\approx 10\lambda_{SAW}\) for common substrate materials,[22, 30] minimising reflections and allowing travelling wave dominant force fields.

In this work we describe the development of robust time-averaged spatial periodicity in acoustic force potential gradients from the imposition of a travelling wave incident with a water-filled microchannel (Fig. 1(a)). We present experimental results which demonstrate microfluidic particle patterning that can extend 10’s of SAW wavelengths along the substrate. In a system where this effect is predominant, this represents a novel mechanism for microfluidic patterning that permits utility not available with standing waves; the patterning extent scales with applied power, permitting particle alignment in a modifiable subset of a microfluidic channel.

This periodicity can be explained via numerical (Fig. 1(b)) and analytical models which combine the interaction of a planar wavefront emanating from the oscillating
substrate propagating through the fluid at the Rayleigh angle, \( \theta_R = \sin^{-1}(c_l/c_s) \) as shown in Fig. 1(c), where \( c_l \) and \( c_s \) are the sound speeds in the liquid and solid phases, respectively, with the cylindrically propagating wavefront emanating from the three-material contact line where the SAW enters the channel. This cylindrical wavefront is a manifestation of the Huygens-Fresnel principle, which states that each point across the transducer’s face emits a wavelet, and where a transducer edge serves as an effective point source of these wavelets.\[31, 32\] A common method of calculating the resulting interference pattern is via the convolution of the impulse response and excitation waveform.\[33\] The former is a geometrically calculated measure at each location related to the number of these wavelets which will arrive over time. An alternative approach examines the time differential of the impulse response. Here the impulse response changes most significantly when the first wavelet arrives, and when the number of wavelets first starts to decay. This is akin to the arrival of a plane wave from the face of the transducer, and when the pulse response. Here the impulse response changes most significantly when the first wavelet arrives, and when the number of wavelets first starts to decay. This is akin to the arrival of a plane wave from the face of the transducer followed by an edge wave from the periphery.\[34\] In our case, the plane wave emanates from the substrate at \( \theta_R \), and the edge wave from the three material contact point; having reduced the system to these two wavefronts, we numerically calculate the areas of pressure minima and maxima.

The SAW based acoustofluidic device consists of a 128° Y-cut X-propagating Lithium Niobate (LiNbO\(_3\), LN) piezoelectric crystal patterned with interdigital transducers (IDTs) and bonded to a PDMS microchannel (Fig. 1(a)). These transducers are 7 mm wide, and positioned approx. 2 mm from the channel edge, a combination that minimises the effect of diffraction induced amplitude distortions across the beam width.\[35\] To generate a travelling SAW (TSAW), a set of IDTs are excited with a harmonic signal at the resonant frequency \( f = c_s/\lambda_{SAW} \), resulting in a TSAW along the substrate.\[36\] To demonstrate the applicability of the effect for systems incorporating throughput, a particle solution flows past the transducer with a continuous velocity and is imaged in the middle of the SAW aperture. As modelling a 3-dimensional system would be unnecessarily computationally expensive, we propose a modelled domain that consists of the 2-dimensional fluid coupled to a piezoelectric, LN domain. The PDMS walls are replaced using a radiation boundary condition that ensures minimal reflections at the upper and distal channel wall boundaries. It should be noted; radiation boundary conditions are implemented with the sole purpose of clearly demonstrating the inherent Huygens-Fresnel interference pattern. Experimentally, we expect additional effects arising from wave reflections due to the acoustic impedance mismatch at the PDMS-fluid interface, discussed in more detail within the experimental section. The computational domain consists of a rectangular liquid-filled chamber, Fig. 1(b), and a varied liquid speed of sound, \( c_l \) with a constant substrate Rayleigh-mode sound speed, \( c_s = 3994 \text{ m/s} \).

An analytical description of the periodic field along the propagation direction can be found directly from geometric considerations, namely from the intersection of the cylindrical wavefronts propagating through the

\[ \text{FIG. 1. (a) Cross-sectional sketch of a typical TSAW driven acoustofluidic systems with PDMS confined microchannels. (b) Sketch of the computational domain with a given width, } W \text{ and height, } h, \text{ along with acoustic radiation boundaries, } BC_{\text{rad}}, \text{ a piezoelectric solid-acoustic coupled boundary condition, } BC_{\text{coupled}}. \text{ (c) Principle of acoustic self-interaction and resulting time-averaged near-field patterning. (d) The classical model predicts the emergence of unified wavefronts propagating from substrate vibrations with wavelength } \lambda_{SAW}, \text{ into the liquid domain at the Rayleigh angle, } \theta_R. \text{ (d) According to the Huygens-Fresnel principle, each point along a wavefront (both on the substrate and in the fluid) generates a cylindrically propagating wave with } \lambda_l, \text{ here shown from the PDMS/water/substrate contact point, which when superimposed result in the unified wavefronts in (c). (e) The asymmetric, one-sided source of these cylindrical waves (the knife-edge effect) and their interactions results time-averaged positions of maximum and minimum displacement velocities in the fluid, with periodic spacing } \lambda_{nf}, \text{ as } z \to 0. \]
fluid with those emanating from the oscillating substrate; these cylindrical waves can be approximated as horizontal wavefronts, appropriate for $z \to 0$ and for increasing $x$ values (see Supplementary Note 1). These traveling wavefronts create static interference patterns despite their continuous propagation through the fluid (see Supplementary Video 1) and results in time-averaged pressure maxima locations. Along a given height in the $x$-direction, this yields a near-field patterning distance in near-field in the plane of the substrate-fluid interface, given by

$$
\lambda_{nf} = \lambda_{l} \cot \left[ \frac{1}{2} \left( \pi - \theta_R \right) \right] \sec (\theta_R),
$$

where $\theta_R$ is the Rayleigh angle and $\lambda_{l}$ is the wavelength in the liquid, related to that of the SAW substrate by $\lambda_{l} = c_{l}/c_{SAW}$. Substituting $\theta_R = \sin^{-1} \left( c_{l}/c_{s} \right)$, we can re-cast Eqn. 1 in terms of the ratio of the liquid and substrate sound speed

$$
\lambda_{nf} = \lambda_{l} \left( 1 - \frac{c_{l}}{c_{s}} \right)^{-1},
$$

valid for wavefront propagation at $\theta_R$ where $c_{l} < c_{s}$; when $c_{l} > c_{s}$ an evanescent Scholte wave develops without penetration into the fluid bulk,[21] precluding the generation of wavefronts that intersect with the cylindrically propagating edge ones. The angle at which the maximum/minimum fluid displacement locations project into the fluid is given by

$$
\theta_{nf} = \frac{1}{2} \cos^{-1} \left( \frac{c_{l}}{c_{s}} \right).
$$

While the analytical model approach sets up this intersection condition explicitly, it is an emergent feature of the simulated one. As illustrated in Fig. 2 (a-e), we observe a time-averaged absolute pressure, $\langle |P| \rangle$ field with a spatial pressure distribution consisting of local minima and maxima locations arising from the edge effect at the leftmost channel extent and the SAW coupled along the substrate-fluid medium interface. The observed pressure distribution is established in accordance with the Huygens-Fresnel principle, analogous to the case of a multi-transducer ultrasonic beam,[37] where diffraction effects generate multi-lobed near-field amplitude patterns. This effect has been briefly shown in recent simulation results [7, 38]; here we attempt to note the key features and quantify this effect in light of the physical parameters that give rise to it. Noticeably, the amplitudes of the high pressure regions (in red) decrease as $x$ increases due to the inherent decay of leaky SAW as it couples energy into the fluid.[25] Furthermore, as the speed of sound of the fluid medium is increased from 500 ms$^{-1}$ to 2500 ms$^{-1}$ (as illustrated in Fig. 2(a)), the spacing between the pressure minima locations, $x_{\Delta P_{min}}$ (i.e. particle collection locations) similarly increases. These spacings normalised by the SAW wavelength, $x_{\Delta P_{min}}/\lambda_{SAW}$ along the $x$ propagating direction at a height near the substrate, $z = 5$ $\mu$m, are plotted in Fig. 2(b). These are in good agreement with the analytical predictions outlined in Eqn. 2 and Fig. 3, and represent a stark contrast to a standing wave system in which all wave speeds would lie along 0.5 $x_{\Delta P_{min}}/\lambda_{SAW}$. Notably, the amplitude decay of the pressure fields are more significant in mediums corresponding to a higher $c_{l}$. As $c_{l} \rightarrow c_{s}$ the coupling of energy into the fluid medium increases,[30, 39] resulting in increased SAW attenuation.

We confirm these predictions in an experimental setup that is a good match for the aspect ratio in the simulated system, where particles are vertically confined in a $h \approx 20$ $\mu$m high channel to limit the number of stable vertical positions particles are patterned. The effects of acoustic streaming are minimised by reducing the substrate...
displacement gradients that would give rise to streaming, namely by using transducer apertures that are wider than the channel length (e.g., a 9 mm wide 48 µm IDT in a 6 mm long channel). The small reflection coefficient at the water/PDMS interface (≈ 4%) results in a vertical time-averaged field component that is superimposed on the near-field one; a λ_{SAW} = 48 µm substrate wave results in vertical periodicity on the order of this channel height (Supplementary Video 2, 0.3 µl/min flow rate). Supplementary Video 3 (0.5 µl/min flow rate) and Fig. S2 (see Supplementary Note 2) demonstrates the same near-field patterning effect, however with a slightly higher channel height with respect to the acoustic wavelength (λ_{SAW} = 80 µm, h ≈ 42 µm), permitting multiple distinct heights at which particles are patterned. These multiple heights are present due to the reflection of the incident wave at the PDMS-fluid interface, which gives rise to a vertical periodicity in the resultant pressure field (see Supplementary Note 2). Interactions with the opposing channel wall are minimised by maximising the amount the SAW is attenuated at the point where it partially reflects off it. Here we use a channel much wider than the height dimension. In the case of a 2000 µm, corresponding to ≈3.4 SAW attenuation lengths, the SAW displacement is 4% at the opposing wall, and where only a fraction of that displacement will be reflected back to the proximal wall. Notably, this effect is capable of exerting appreciable forces on sub-micron particles; Supplementary Video 4 (0.2 µl/min flow rate, λ_{SAW} = 71 µm, h ≈ 25 µm) shows the long-term convergence of particles in the aforementioned patterns despite the increasing influence of streaming on particle trajectories with decreasing particle size.[40]

This periodic near-field is visualized in Fig. 4(a), which shows patterned 2 µm particles suspended in water with diminishing focussing behaviour with increasing distance from the channel wall, reflecting the decreasing acoustic force potential gradient magnitudes with decreasing SAW amplitude. Figs. 4(b,c) show the measured particle positions and spacing (image intensity averaged across the width of the frame) as compared to those predicted from Eqn. 3, with a measured average spacing of 0.63±0.3 λ_{SAW} as compared to the 0.61 λ_{SAW} spacing predicted from Eqn. 2, and distinctly larger than the 0.5 λ_{SAW} spacing in the case of a substrate-bound standing wave.[41] Fig. 4(d-f) demonstrates the scalability of this effect: by increasing the applied power, the region in which the acoustic field magnitude is sufficient to focus particles is (linearly) expanded. By using a narrower 1000 µm channel the effect of reflection off the opposing interface is observed: the particle spacing is much closer in the vicinity of the distal channel wall in Fig. 4(f) due to fluid wavefronts reflecting and interacting with incoming ones (standing waves) in the vicinity of the weakly reflective PDMS/water interface, with periodicity corresponding to 0.5 λ_1. This is only noted at the higher applied powers (here at 501 mW) required for particle translation at this location.

In summary, we have described and analysed a phenomenon where the imposition of a travelling wave can, through acoustic self interaction, result in the generation of a time-averaged spatially periodic field along the axis of acoustic propagation, as demonstrated here for a microfluidic SAW device. In the construction of a straightforward analytical model with confirming simulation and experimental results, we have demonstrated that this effect can be explained by the combination of the classical wavefronts emanating from a substrate with those arising from the edge effect, where a discontinuity along the oscillating substrate serves as the progenitor of cylindrically propagating waves. This effect is most pronounced in systems that are scaled to take advantage of it, namely those whose dimensions are on the order or exceed the attenuation lengths of the acoustic field driving the microfluidic phenomena. As acoustofluidic systems increasingly utilize frequencies approaching and exceeding 1 GHz for refined nanoscale manipulation,[11, 25] this characteristic will be increasingly represented. We envision that this new particle manipulation technique, namely the generation of spatially limited acoustic patterning locations using travelling waves emanating from a single transducer, will be a viable technique for microfluidic manipulation along with forces that result from conventionally applied standing waves, travelling waves and acoustic streaming. C.D. and D.J.C. contributed equally to this work.

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FIG. 4. Particle patterning due to near-field interactions. (a) A 48 µm, 81 MHz travelling SAW at 64 mW generates a near-field period force field whose acoustic force magnitude decays with increasing distance due to attenuation at the substrate/water interface. This force field is used to pattern suspended 2 µm polystyrene particles. Inset shows the macroscopic device (scale bar 200 µm). These patterning positions are measured across 143 separate frames (at 20 fps), where (b) shows a representative measurement, yielding (c) average particle positions corresponding to the predicted patterning spacing. (d-f) The penetration of this near-field effect scales with increasing applied power, though the effects of channel reflections, and thus the influence of standing waves, becomes apparent when the SAW amplitude is of sufficient magnitude to realize these effects at the distal channel wall. Scale bars are 100 µm.

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