Optical Bloch oscillations and Zener tunneling of Airy beams in ionic-type photonic lattices

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Abstract: We report on the existence of optical Bloch oscillations (OBOs) and Zener tunneling (ZT) of Airy beams in ionic-type photonic lattices with a refractive index ramp. Different from their counterparts in uniform lattices, Airy beams undergoing OBOs show an alternatively switched concave and convex trajectory as well as a periodical revival of input beam profiles. Moreover, the ionic-type photonic lattice established in photorefractive crystal exhibits a reconfigurable lattice structure, which provides a flexible way to tune the amplitude and period of the OBOs. Remarkably, it is demonstrated that the band gap of the lattice can be readily controlled by rotating the lattice inducing beam, which forces the ZT rate to follow two significant different decay curves amidst decreasing index gradient. Our results open up new possibilities for all-optical switching, routing and manipulation of Airy beams.

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References and links

Theoretically, an Airy beam carries infinite energy and propagates invariantly along a parabolic trajectory. Owing to practical requirements, only finite energy Airy beams can be realized in laboratory, which requires exponential apodization of the Airy profile. These truncated Airy beams have properties very much similar to their theoretical counterparts but only within a limited diffraction-free zone. In fact, Airy beams with desirable non-diffracting paths beyond the parabolic law are highly desired from the points of view of fundamental physics and applications. Recently, tremendous research efforts have been devoted to the Airy beam trajectory control, providing a renewed focus and direction to this emerging field. In the linear regime, it has been demonstrated that an Airy beam can traverse any complex convex or concave trajectory that may consist of single or multi-paths by employing phase [4,13–15] or index potential modulations [16–18]. In the nonlinear regime, Airy beams can be manipulated through schemes that rely on second harmonic generation [19], wave mixing [20], Kerr and photorefractive effects [6,21,22], leading to tunable ballistic dynamics.

On the other hand, periodical optical potentials exemplified by photonic lattices possess a remarkable ability to harness flow of light, not only giving rise to peculiar phenomena such as negative refraction [23] and abnormal diffraction [24], but also promising applications including all-optical routing and logic gates [25]. Light propagation in periodical optical potentials behaves in a way that is analogous to electron travelling through a semiconductor crystal. Two renowned examples are optical Bloch oscillations (OBOs) [26,27] and Zener tunneling (ZT) [28,29] which are associated with electron propagating in a semiconductor superlattice under the action of an external driving force. Typically, OBOs can be realized in a photonic lattice with a refractive index ramp, where the beam follows a cosine-like oscillatory path due to the interplay between Bragg refractions and total internal reflections [26]. When the refractive index gradient is comparable with the band gap between adjacent Bloch bands, OBOs are damped by ZT that an interband transition occurs at the edge of the first Brillouin zone. Recently, it has been demonstrated that OBOs and ZT are capable of synthesizing arbitrary prespecified beam trajectories, lending themselves naturally to beam-steering applications [30]. Therefore, it is expected that OBOs and ZT can provide more effective ways for manipulating propagating paths of Airy beams, opening up many other applications.

Here, we investigate the propagations of Airy beams in ionic-type photonic lattices established in the photorefractive crystal under non-conventional biased (NCB) condition [31]. It is predicted that OBOs of Airy beam can occur by superimposing a transverse refractive index ramp on the lattices. The finite truncated Airy beam undergoing OBOs can retain almost perfect shape-preserving profile, and traverse convex and concave trajectories not restricted by parabolic path. Moreover, in an ionic-type photonic lattice, it is possible to easily reconfigure both the structure and the resulting band gap, enabling us to control OBOs of Airy beams via rotating the lattice inducing beam. Also, it is revealed that the ZT of Airy beam leads to beam intensity outburst from OBOs trajectory when the index gradient reaches the order of magnitude of the band gap. It is demonstrated that by rotating the lattice inducing beam, ZT rate follows two significantly different decay curves as the index gradient decreases, which is potentially beneficial for Airy beam based all optical switching and routing.

2. Theoretical model

As illustrated in Fig. 1(a), the ionic-type photonic lattices can be optically induced in a photorefractive crystal under NCB condition that the bias field \( E_0 \) is perpendicular to the crystal axis \( c \) [31]. We take the lattice inducing beam as \( I_0 = \cos[\pi(x\cos\theta - y\sin\theta)/3]^2\cos[\pi(x\sin\theta + y\cos\theta)/3]^2 \), where \( \theta \) is lattice beam orientation with respect to \( x \) axis. Then, two induced ionic-type photonic lattices named lattices 1 and 2 are displayed in Figs. 1(b) and 1(c), which correspond to \( \theta = 0 \) and \( \pi/4 \), respectively. It reveals a distinctive feature,
the alternately arranged positive and negative potentials, of these two lattices. The induced refractive index distributions of such lattices are governed by \[32\]

\[
\Delta n_0 = \frac{1}{2} n_e \gamma_{33} E_0 \frac{\partial}{\partial y} \varphi, \quad (1a)
\]

\[
\nabla^2 \varphi + \nabla \varphi \cdot \nabla \ln(1 + I_0) = E_0 \frac{\partial}{\partial y} \ln(1 + I_0). \quad (1b)
\]

Here, \(\Delta n_0\) is the induced index distribution of the lattice, \(n_e\) is the extraordinary refractive index, \(\gamma_{33}\) is the electro-optic coefficient, \(E_0\) is the applied electric field, and \(\varphi\) is the optically induced dimensionless electrostatic potential. It is worth noting that the ionic-type photonic lattice under NCB condition exhibits a high reconfigurability in period, potential depth and orientation, which can be controlled by merely rotating the lattice inducing beam. Also, obvious changes are found in the corresponding band gap structures and Brillouin zone spectra. Figures 1(d) and 1(e), respectively, show the dispersion relations (band diagrams) of lattices 1 and 2, which are derived with Floquet-Bloch approach \[33\]. It suggests that a nearly zero gap between the first two Bloch bands changes to an incomplete gap as \(\theta\) switching from 0 to \(\pi/4\). Owing to selective band gap closure and Bragg reflection suppression \[31\], these reconfigured photonic lattices result in the reshaped Brillouin zone spectra as depicted in the insets of Figs. 1(d) and 1(e), which are numerically obtained by using Brillouin zone mapping technique \[34\].

![Fig. 1.](image)

It was demonstrated that a linear transverse index gradient can be superimposed on the photonic lattice by introducing a transversely illuminating white light \[29\]. Therefore, we assume the photonic lattice with an index ramp is given by

\[
\Delta n = \Delta n_0 + (x \cos \phi + y \cos \phi) \rho, \quad (2)
\]
where $\rho$ is the refractive index gradient, and $\phi$ denotes the orientation of $\rho$ with respect to the $x$ axis. Then, the Airy beam propagating in this transverse index modulated lattice can be numerically analyzed by employing the paraxial equation of diffraction [35]

$$i \frac{\partial \psi}{\partial z} + \frac{1}{2} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \left[ -\frac{\partial \phi}{\partial z} + \alpha (x \cos \phi + y \sin \phi) \right] \psi = 0. \quad (3)$$

Here, $\psi = \text{Ai}((\zeta - \eta)/5) \text{Ai}((\zeta + \eta)/5) \exp(0.016 \zeta)$ is the exponentially truncated two-dimensional Airy wave-packet, $\zeta$, $\eta$ and $\zeta$ are the two normalized transverse coordinates and the propagation distance, which are respectively scaled by $x_0$, $y_0$ and $z_0$ from realistic coordinates, and $\alpha$ represents the normalized index gradient with scale factor of $\rho_0$. The evolution of Airy beam in the ionic-type photonic lattice with a refractive index ramp can be numerically derived by solving Eq. (3) with the split-step Fourier method. In the following simulations, we take parameters as $x_0 = y_0 = 1.8 \mu m$, $z_0 = 95 \mu m$, and $\rho_0 = 0.01/\mu m$ according to the typical experiment conditions [36].

3. Results and discussions

Light travelling in a photonic lattice is subjected to the dispersion relation or band gap structure of the lattice, and can consequently experience diffraction for a normal incident beam [31]. In Figs. 2 (a) and 2(b), we respectively examine the propagations of an Airy beam [shown in Fig. 2(c)] in two aforementioned uniform ionic-type photonic lattices (lattices 1 and 2). Owing to the linear coupling among adjacent waveguides, the normal incident Airy beam is strongly influenced by the discrete diffraction effect, which suppresses the non-diffracting property of an Airy beam [shown in Figs. 2(d) and 2(e)], and forces it to evolve to a rather broad pattern even lose its Airy-like intensity features [shown in Fig. 2(e)].

In the following, we consider the propagation dynamics of Airy beams in ionic-type photonic lattices with the transverse index gradient $\alpha = 0.008$. A typical OBOs occurs when Airy beam propagates through lattice 1, as shown in Fig. 3(a1) and Visualization 1. It exhibits a cosine-like beam trajectory that can be analytically described as [37]

$$r(z) = -\frac{2\kappa}{k_0 \alpha} \cos(Ak_0 \alpha z), \quad (4)$$
where $\kappa$ is the coupling coefficient between the adjacent waveguides, and $k_0$ is the wave number in vacuum. Different from those in uniform lattices, an Airy beam undergoing OBOs exhibits a periodic revival of input intensity that permits a fairly long propagation distance with negligible diffraction. This can be examined by comparing the beam intensity of the input [shown in the upper panel of Fig. 3(b1)] with that of the first recovery peak [shown in the upper panel of Fig. 3(d1)]. Intuitively, OBOs can be understood as the interplay between total internal reflections and Bragg reflections. The normal incident Airy beam has a transverse $k$ vector located at the center of the first Brillouin zone as shown in the lower panel of Fig. 3(b1). Under the action of the linear index gradient, the input Airy beam travels towards the high index region, accompanying with the transverse $k$ vector tilting away from the center of the Brillouin zone. When the tilted Airy beam arrives at the corner of the first Brillouin zone [shown in the lower panel of Fig. 3(c1)], Bragg reflection takes place, making the transverse $k$ vector reverse direction and deflect to the center of the Brillouin zone. Thereafter, the total internal reflection occurs and the next travel cycle is initiated, as the transverse $k$ vector reach the center of the Brillouin zone [shown in the lower panel of Fig. 3(d1)]. In Fig. 3(a2) and Visualization 2, we show OBOs of the Airy beam in lattice 2 with the same yet $\pi/4$ rotated index gradient. Although the Airy beam is rotated and deformed with propagation [shown in the upper panel of Fig. 3(c2)], it can recover at the oscillation peak of OBOs [shown in the upper panel of Fig. 3(d2)]. Notably, the OBOs in lattice 2 experience reduction in both amplitude and period, which are one fifth and a half of those in lattice 1, respectively. The underlying reason for this behavior is that when the transverse index pattern changes from lattice 1 to lattice 2, the index potential increases and the period $\lambda$ along the direction of index gradient is doubled. Meanwhile, the doubled lattice period and increased index potential [shown in Fig. 1(c)] result in the decrease of coupling coefficient $\kappa$. According to Eq. (4), the amplitude of OBOs is proportional to $\kappa$, and the oscillation period is inversely proportional to $\lambda$. Therefore, such a geometry change of the lattice leads to the decrease of amplitude and period of OBOs. In fact, it can be seen that the reconfigurability of ionic-type photonic lattice provides an effective way to control the trajectory of an Airy beam in terms of accelerating direction, maximal transverse displacement and recovery period.

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**Fig. 3.** Optical Bloch oscillations of Airy beam in (1) lattice 1 (see Visualization 1) and (2) lattice 2 (see Visualization 2) with index gradient $\alpha = 0.008$. (a)Trajectory of Airy beam at the longitudinal section. The solid arrows in (a) indicate the positions where Bragg reflection (BR) and total internal reflection (TIR) occur. The upper panels of (b)-(d) are the Airy beam intensity at the input and the sequent dashed-line labeled positions in (a). Here, the scale bar in (c1) is 80, the upper panels of (b1), (d1), (b2), (d2) and (c2) are magnified 4 and 2 times for clear identification, respectively. The lower panels of (b)-(d) are the corresponding Fourier spectra of the upper panels, where the solid lines indicate the boundaries of the first Brillouin zones, and the dashed-line arrows are the directions of the index gradients.
Another finding in Fig. 3 is that a portion of light escapes from the first Brillouin zone when Bragg reflection occurs [shown in the lower panels of Fig. 3(c1) and Fig. 3(c2)]. This can be considered as the ZT from the first to the second Bloch band at the band edge, where the band gap is minimal. We further examine the dependence of tunneling rate $\Gamma$, referred to the ratio between the tunneling power and the incident power, on the index gradient $\alpha$. The result is displayed in Fig. 4, where the red dots and yellow triangles represent the numerically calculated ZT rate $\Gamma$ in lattices 1 and 2, respectively. It shows an exponential relation between $\Gamma$ and $1/\alpha$, which is theoretically described by [38]

$$\Gamma \propto \exp\left(-A\Delta\beta / \alpha\right),$$

(5)

where $A$ is a constant, $\Delta\beta$ represents the band gap between the first two Bloch bands. The fitting parameter $A$ is 0.016 and 0.08 for lattices 1 and 2, respectively. Good agreements are achieved between numerical and theoretical results (solid curves in Fig. 4), further demonstrating that ZT is responsible for the leakage of Airy beam intensity from the first Brillouin zone in Fig. 3(c1) and Fig. 3(c2).

Fig. 4. Zener tunneling rate $\Gamma$ as a function of the inverse of the index grandniece $1/\alpha$. The red dots and yellow triangles are the numerically derived $\Gamma$ in lattices 1 and 2, respectively. The blue and green solid curves are to the theoretical predicted $\Gamma$ in lattices 1 and 2, respectively.

Fig. 5. Zener tunneling of Airy beam in (1) lattice 1 (see Visualization 3) and (2) lattice 2 (see Visualization 4) with index gradient $\alpha = 0.04$. The upper panels of (b) and (c) are the Airy beam intensity at the sequent dashed-line labeled positions in (a). Here, the scale bar in (c1) is 50, (b2) and (c2) are magnified 2 times for clear identification. The lower panels of (b) and (c) are the corresponding Fourier spectra of upper panels, where the solid lines indicate the boundaries of the first Brillouin zones, and the dashed-line arrows are the directions of the index gradients.
Due to the fact that there is a significant difference in the band gap between lattice 1 ($\Delta \beta_1 = 0.005$) and lattice 2 ($\Delta \beta_2 = 0.118$), $T$ exhibits two distinctive decay rates in two ionic-type photonic lattices. Especially, in the index gradient region from 0.02 to 0.045 (the shaded area in Fig. 4), the tunneling rate in lattice 1 can be three times higher than that in lattice 2. This unusual property can be utilized for steering the Airy beam by rotating the lattice inducing beam along with the index ramp. In Fig. 5, we show the ZTs in lattice 1 (see also in Visualization 3) and lattice 2 (see also in Visualization 4) with the refractive index gradient $\eta = 0.04$, whose directions are shown in the lower panels of Fig. 5(b1) and Fig. 5(b2). It is indicated that there is a considerable portion of beam energy escapes from the OBOs trajectory in lattice 1 [shown in Fig. 5(a1)], while in lattice 2 the escaped beam energy could be hardly found [shown in Fig. 5(a2)]. This is a direct consequence of band gap width mediated ZT rate. There is more beam energy tunnels at the band edge in the lattice with the smaller band gap [shown in the lower panels of Fig. 5(b1) and Fig. 5(b2)]. Interestingly, it is also found that the tunneling beam at the recovery position of OBOs in lattice 1 still remains an Airy-like pattern with a large deflection angle as illustrated in the upper panel of Fig. 5(c1). Therefore, the reconfigurability of ionic-type photonic lattice can also provide a strategy for switching and routing Airy beams or generating coherent Airy beams in periodic optical structures.

4. Conclusions

To summarize, we have demonstrated that OBOs and ZT of Airy beams can be realized in ionic-type photonic lattices with the transverse refractive index ramp. Our results show that Airy beams undergoing OBOs exhibit periodically switched accelerating direction and perfect recurrences of beam profiles. This unusual property permits Airy beams to propagate a fairly long distance without experiencing diffraction, which cannot be achieved in the uniform lattice. Interestingly, the reconfigurability of the ionic-type photonic lattice provides a flexible way to control the OBOs of Airy beam both in oscillation period and amplitude. Furthermore, it was shown that the dependence of ZT rate on index gradient can be controlled to follow two different decay curves by rotating the lattice inducing beam, which can be potentially exploited for all-optical routing and switching.

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