Teachers’ Perceptions of Students’ Development of Multiplicative Thinking

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Having an understanding of the key ideas underpinning multiplicative thinking is critical for future learning beyond the primary school years. The shift to multiplicative thinking can be challenging for both students and teachers due to its multifaceted nature. This paper reports on a pilot study of professional learning in schools that identified multiplicative thinking, an area of concern. We sought to explore in situ professional learning (school-based) within 14 primary schools across a six-month period. Our findings suggest that in situ professional learning had a positive impact on teachers’ mathematical content knowledge and pedagogical content knowledge.

In the current political climate, there is increased pressure on teachers to improve student-learning outcomes in mathematics education. In particular, there is a concern regarding the number of students in Years 5 to 8 who rely on additive thinking to solve proportional reasoning problems when multiplicative thinking is required and those who cannot distinguish whether a task requires additive thinking or multiplicative thinking (Van Doreen, De Bock, & Verschaffel, 2010). This may be attributed to an emphasis in the early and middle primary years on multiplication as repeated addition, equal groups and arrays. Alternatively teachers’ limited understanding of the complexity associated with the development of multiplicative thinking and their knowledge of the different multiplicative structures may be contributing factors.

Theoretical Framework

A recurring theme in the literature is that multiplicative thinking is a crucial stage in students’ mathematical understanding, the basis of proportional reasoning, and a necessary pre-requisite for understanding algebra, ratio, rate, scale, and interpreting statistical and probability situations (e.g., Hilton, Hilton, Dole, Goos, & O’Brien, 2012). Some scholars argue that the difficulties associated with students’ lack of proportional reasoning are related to their limited experiences of different multiplicative situations such as multiplicative comparison (times-as-many) and rate/ratio (e.g., Greer, 1988) or to their reliance on additive thinking when multiplicative thinking is required (e.g., Van Doreen et al., 2010). Greer (1988) suggests that students need to engage in multi-step contextual problems that include more complex numbers so that the appropriate operation cannot be intuitively grasped.

In relation to professional learning models, research suggests that professional learning for teachers needs to be situated in realistic contexts as part of the on-going work in schools, in contrast to one-off models of professional development (Bruce, Esmonde, Ross, Dookie, & Beatty, 2010). Teachers are seen as learners and schools as learning communities (Clarke & Hollingworth, 2002). Bruce et al., (2010) support Clarke and 2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia) pp. 63-66. Auckland: MERGA.
Hollingworth’s notion of professional learning (PL) being embedded in classroom experiences and practices within the school context, and argue that such professional learning is characterised as occurring in sustained and iterative cycles of planning, practice and reflecting. Dole, Clarke, Wright, and Roche (2008) engaged teachers in a focused professional learning program on teachers’ understanding of proportional reasoning. They found that although there were marginal differences in teachers’ proportional reasoning, teachers had the language to discuss proportional reasoning, and could articulate the difference between additive and multiplicative thinking.

Informed by the research literature, a pilot PL program focused on developing teachers’ knowledge of multiplicative thinking was situated within each participating school. The study aimed to address the research question: What is the impact of an in situ, spaced, professional learning on teachers’ pedagogical content knowledge for developing multiplicative thinking in their students?

Method

The purpose of this mixed methods study was to examine the perceived impact of an in situ PL program on teachers’ pedagogical content knowledge related to multiplicative thinking. We characterised this pilot study as an effectiveness study (Bruce et al., 2010) as it studied PL opportunities for classroom teachers within their own setting and measured their pedagogical content knowledge (PCK) through the use of an online survey, administered pre and post the PL.

The structure of the professional learning (PL) was informed by the abovementioned research. The research team, led by the first author, developed five 90-minute PL modules with co-researchers (Teaching Educators) from a New South Wales Catholic Education System. Each module focused on an aspect of multiplicative thinking and pedagogy, and included challenging tasks, professional readings and between session classroom tasks. The co-researchers facilitated the PL at participating schools across terms two to four, and provided in classroom support in Years 3 and 4, due to the identified need and high proportion of students still reliant on counting based strategies.

Fourteen primary schools (approximately 230 participants: classroom teachers, specialists, lead teachers, and leadership teams) across the diocese agreed to participate in this research, as multiplicative thinking was their PL priority. The data collection instruments included teacher online surveys, focus group interviews and teacher reflective journals. The data reported here pertain to one open response question from the online teacher survey: How do you believe students develop multiplicative thinking?

All responses were entered into a spreadsheet, coded and categorised through the analysis of data using a grounded theory approach (Strauss & Corbin, 1990). If a teacher wrote multiple ideas, each was coded as a separate response. The first two authors independently coded the teachers’ responses using open coding to identify key themes.

Results and Discussion

Table 1 shows seven themes developed from the data analysis and teachers’ illustrative responses to the aforementioned question. Pre PL 37% of respondents and post 25% believed that students develop multiplicative thinking by using some form of representation that leads to the development of abstract thinking. There is a noticeable shift in responses post the PL from a focus on aspects of general pedagogy (themes 1, 4, and 7) to focussing on aspects relating to multiplicative thinking (themes 2, 5, and 6).
Table 1

*Percentage of Responses Relating to How Students Develop Multiplicative Thinking*

<table>
<thead>
<tr>
<th>Theme</th>
<th>Pre (n=244)</th>
<th>Post (n=236)</th>
<th>Illustrative of comments written by teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Materials and representations moving to abstract thinking</td>
<td>37</td>
<td>25</td>
<td>Pre: By working with concrete materials, partial models, to abstract thinking.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: Build up multiplicative foundation, move from visualising arrays to abstract thinking and reasoning.</td>
</tr>
<tr>
<td>2. Moving from additive to multiplicative thinking</td>
<td>12</td>
<td>22</td>
<td>Pre: From additive thinking to applying known facts.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: Use of arrays, and times as many that encourage multiplicative thinking and reasoning strategies such as known and derived facts that shift their thinking.</td>
</tr>
<tr>
<td>3. Relationship: multiplication and division</td>
<td>4</td>
<td>10</td>
<td>Pre: Knowing link between multiplication &amp; division</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: When engaging with problems/tasks that require thinking about inverse operations.</td>
</tr>
<tr>
<td>4. Engage in real life problems and open tasks</td>
<td>30</td>
<td>12</td>
<td>Pre: Being exposed to real life problems.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: Engage in real life multiplicative tasks and multi step word problems and open tasks that encourage MT</td>
</tr>
<tr>
<td>5. Use of multiplicative language</td>
<td>4</td>
<td>11</td>
<td>Pre: Experience the language of ‘groups of’, ‘arrays’</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: Opportunities that expose them to multiplicative language such as commutativity, times as many.</td>
</tr>
<tr>
<td>6. Experiencing multiplicative structures</td>
<td>0</td>
<td>13</td>
<td>Pre: Provide ‘groups of’ and ‘arrays’ activities.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: Regular experience with challenging problems relating to arrays, times-as-many, allocation and rate.</td>
</tr>
<tr>
<td>7. Teacher demonstration and practice</td>
<td>13</td>
<td>7</td>
<td>Pre: Teacher modelling strategies, and practice times tables.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Post: Having strategies shared by students and reinforced by teachers and through practice of a variety of questions.</td>
</tr>
</tbody>
</table>

Prior to the PL 80% of responses related to general pedagogical approaches to mathematics, compare to 44% post the PL. In contrast, 56% of responses related to multiplicative thinking post the PL, which was more than double that of the pre PL (20%). This appears to suggest that the program challenged existing ideas about students’ development of multiplicative thinking and resulted in a shift in teachers’ perceptions.

We anticipated a reduction in a procedural approach to learning (Theme 7) and using materials (Theme 1). While there was some reduction as a result of the PL it is evident that these views are strongly held, particularly in relation to use of materials to support students’ shift to abstract thinking and teachers wanting to do explicit demonstration.

Teachers became increasingly aware that students’ development of multiplicative thinking is linked to shifting from additive thinking and counting based strategies (Theme 2, Table 1). Many responses indicated that some powerful and engaging tasks facilitated the transition from additive to multiplicative thinking. Nick, a Year 4 teacher, recorded the following in his reflective diary after exploring the carrot patch task with his students.
Having to imagine the missing carrots in the array was powerful and the kids were using distributive property and the language of arrays, partitioning, factors and multiples.

The biggest shifts related to themes four and six. While we were initially surprised that there was a decline in teachers’ focus on the importance of engaging students in real life problems and open tasks, we realised that teachers’ experience of the different multiplicative structures (rectangular array, rate, ratio, and times-as-many) had a major impact on their own learning. Sophie, a Year 3 teacher, recorded this entry in her diary.

The language of times-as-many was challenging for students initially but once they had more experience with tasks like this, I saw a shift in the strategies they used and they were using multiplicative language and making connections between multiplication and division.

Concluding Comments

The PL provided teachers with a range of rich and challenging tasks using everyday relevant content related to arrays, rate/ratio, and times-as-many that teachers then explored with their students in the classroom. Making links to proportional reasoning in the modules when exploring teachers’ and students’ solution strategies to rate/ratio was critical. Teachers realised that primary school students can engage in tasks such as these and do so using proportional reasoning and multiplicative thinking. They saw the tasks as a major source of their learning and understanding of the complexity of developing multiplicative thinking. Teachers also recognised that developing such tasks was their greatest challenge when planning for learning, and indicated they need further support in this area. The findings suggest that providing in situ targeted professional learning over a sustained period of time that requires teachers to implement the learning with their students improves their knowledge of multiplicative thinking and proportional reasoning. It equipped these teachers with ways to support their students’ development of multiplicative thinking using rich learning experiences relating arrays, rate/ratio and times-as-many. However, they still require on-going support and PL to embed the practices and deepen their understanding.

References


