An SVD-based Transparent Watermarking Method

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***Abstract*—A new watermarking method on the basis of Singular Value Decomposition is proposed in this paper. The method is blind and modifies one of the orthonormal matrices of the decomposition of 4x4 block to enclose a bit of a watermark. A procedure for minimization of embedding distortions is considered. Two embedding rules have been proposed for watermarking which provide different robustness and transparency. Popular attacks have been applied and experimental results have been illustrated. According to the results the robustness of the proposed watermarking method toward some attacks is more than 36% better compared to other known blind watermarking methods.**

Keywords—Watermarking, Singular Value Decomposition, Robustness, Transparency

I. INTRODUCTION

Protection of digital rights is one of the most important tasks of cyber security. These kinds of problems are addressed by Digital Image Watermarking (DIW), where a digital image is being protected by enclosing a digital watermark. Three important characteristics for a particular watermarking method are robustness, transparency and data payload. The tradeoff between robustness and transparency depends on a kind of transform which provides coefficients for modification as well as embedding rule that interprets values of coefficients.

In most cases robustness against noise, some kinds of filtering and geometric attacks is required [1]. The most approved index of robustness is Bit Error Rate (BER) of a watermark upon extraction. Transparency is an ability to preserve original image by watermarking it and many measures could be used to define it quantitatively [2]. Though, the most popular measure is Peak Signal to Noise Ratio (PSNR). Data payload is a number of watermark bits embedded into an image.

The original image can be modified in many different ways to embed a watermark. Modification of the Least Significant Bit is a simplest example of watermarking in spatial domain [3]. Some suitable examples of frequency domain watermarking are the methods on the basis of Discrete Cosine Transform (DCT) [4] and Discrete Wavelet Transform (DWT) [5]. Usually modification of some spectral coefficients is more favorable as it is more robust against noise and image processing attacks. Singular Value Decomposition (SVD) is a unique kind of transform [6]. It separates an image fragment on several independent layers which number is much less than that, for example, for DCT. Therefore the most important layer is quite stable to various attacks.

An efficiency of watermarking also depends on a rule exploited for embedding. Each embedding rule could have several parameters that influence robustness-transparency tradeoff.

In this paper we propose new SVD-based blind watermarking method which minimizes embedding distortions. It uses an orthonormal matrix obtained by SVD of 4x4 block to embed a bit of a watermark. The proposed method provides good robustness-transparency tradeoff and high data payload.

The rest of the paper is organized as following: a short review of relevant watermarking methods exploiting SVD is given in the Section II; Section III bears our own approach which is described in detail; then, some experimental results are represented in Section IV followed by a discussion of their importance in Section V; finally, Section VI concludes the paper.

II. SVD-BASED WATERMARKING

A. SVD Transform

An image fragment I_k of size $n \times n$ is being decomposed according to SVD [6] in the following way:

$$I_k = USV^T = \begin{pmatrix} U_{1,1} \cdots U_{1,n} \\ U_{2,1} \cdots U_{2,n} \\ \vdots \\ U_{n,1} \cdots U_{n,n} \end{pmatrix} \times$$

$$\times \begin{pmatrix} S_{1,1} & 0 & \cdots & 0 \\ 0 & S_{2,2} & \cdots & 0 \\ & & \vdots & \\ 0 & 0 & \cdots & S_{n,n} \end{pmatrix} \times \begin{pmatrix} V_{1,1} \cdots V_{1,n} \\ V_{2,1} \cdots V_{2,n} \\ \vdots \\ V_{n,1} \cdots V_{n,n} \end{pmatrix}^T, \quad (1)$$

where U and V are some orthonormal matrices and S is a diagonal matrix of singular values.

An alternative representation demonstrates that fragment I_k is being decomposed on n independent layers where geometry of i-th layer is defined by a pair of i-th columns (one from matrix U and one from V) and a luminance component $S_{i,i}$:

$$I_k = \sum_i S_{i,i} U(1 \ldots n, i) V(1 \ldots n, i)^T \quad (2)$$

The luminance $S_{1,1}$ has the biggest value and for most fragments of natural images this value is much bigger than the

This paper was supported by Tampere Program in Information Science and Engineering (TISE).

other. Therefore the first layer is the most substantial and provides the best robustness for watermarking.

B. Review of SVD-based Watermarking Methods

Popular strategy for SVD-based methods that modify singular values is to quantize the biggest value of a block depending on the corresponding bit of a watermark. The first SVD-based blind method for DIW was proposed in [7]. It features high data payload and uses color RGB images. Another pioneering noninvertible non-blind scheme exploiting SVD was introduced in [8]. However, later in [9] it has been shown that the scheme is vulnerable for the counterfeit attack and is invertible. In the paper [10] the first in the literature DWT-SVD non-blind watermarking method was proposed which demonstrates good robustness and high data payload. The downside is that embedding distortions are quite considerable. The semi-fragile method proposed in [11] applies adaptive quantization to DWT-SVD. The disadvantage is that additional information should be transmitted and the method causes considerable degradation of original image. Recently a quite robust method with adaptive quantization of SV has appeared in [12], but additional information is required for extraction. One of the most robust blind watermarking schemes based on SVD-DCT transform was proposed in [13].

Modification of orthogonal matrices is a very rear approach of SVD watermarking. The advantage of such kind of watermarking is that more elements are available for modification. The paper [14] proposes a blind watermarking method that modifies the second and the third elements in the first column of the left orthogonal matrix of SVD of a 4x4 block. The quality of watermarked images is quite high, but robustness toward common distortions like JPEG-compression, Gaussian noise and cropping is not sufficient. Another paper developing further previous approach is [15] where authors proposed to adjust a threshold in a way that PSNR of each modified blocks is higher than 42 dB whenever it is possible. As a result robustness-invisibility tradeoff is better than for the method proposed in [14].

There are several shortcomings in the latter two methods. First, an orthonormal matrix where two elements are modified becomes non-orthonormal which could cause an embedded bit to be lost even without influence of the third person or noise. Second, only two elements of the left orthogonal matrix are used while utilization of all the four elements of the first column could improve robustness-transparency trade-off.

III. Proposed Watermarking Method

The method described below resolves mentioned shortcomings of the methods proposed in [14] and [15] and minimizes embedding distortions. Further we consider orthonormal matrices of SVD of particular image block I_k which size is 4×4. A watermark embedding rule should be used to embed and extract a bit. The watermarking method proposed in this paper modifies one of the orthonormal matrices which is either U or V to embed a bit of a watermark.

A. Watermark Embedding Task

Let us assume that matrix U is being modified: $U \to U'$. Singular values can be modified as well: $S \to S'$. Different embedding rules can be used, but in order to provide high robustness we suggest that the first column $\boldsymbol{u}'^T_1$ of the modified orthonormal matrix U' interprets a bit, $\boldsymbol{u}'_1 = [U'_{1,1}, U'_{2,1}, U'_{3,1}, U'_{4,1}]$. Optionally the first column $\boldsymbol{v}^T_1$, $\boldsymbol{v}_1 = [V_{1,1}, V_{2,1}, V_{3,1}, V_{4,1}]$, of the other matrix V can also be used by embedding rule to interpret a bit, however, only one orthonormal matrix is being modified. Each new watermarked image fragment I'_k that carries corresponding bit is composed from two orthonormal matrices $\{U', V\}$ and a diagonal matrix of singular values S':

$$I'_k = U'S'(V)^T. \quad (3)$$

Matrix U' should be orthonormal in order to provide proper decoding of a bit of a watermark. Otherwise the result of SVD of I'_k might be different and a left orthonormal matrix U'' might be obtained such that $U' \neq U''$. In that case a bit of a watermark might be inverted.

We propose to modify matrix U according to the model of rotations in 4D space, which assures that the resulting matrix U' is orthonormal:

$$U' = R_U U \quad (4)$$

Rotational matrix R_U in four dimensional space can be fully described according to Van Elfrinkhof's formulae [16]:

$$R_U = R_U^L R_U^R, \quad (5)$$

$$R_U^L = \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix}, \quad (6)$$

$$R_U^R = \begin{pmatrix} p & -q & -r & -s \\ q & p & s & -r \\ r & -s & p & q \\ s & r & -q & p \end{pmatrix}, \quad (7)$$

where a, b, c, d, p, q, r, s are reals and $a^2 + b^2 + c^2 + d^2 = 1$, $p^2 + q^2 + r^2 + s^2 = 1$. Rotational matrices R_U^L and R_U^R are left-isoclinic and right-isoclinic respectively [16]. According to the mentioned rotational model any orthonormal matrix U can be transformed to any other orthonormal matrix U'. Thus, using (4) and (5) we can express modified block I'_k in general case as following:

$$I'_k = R_U^L R_U^R U S'(V)^T. \quad (8)$$

In order to provide good transparency a watermark should be embedded with minimal distortions. The embedding distortion for the particular block is:

$$G = \|I'_k - I_k\|_2^2 = \|R_U^L R_U^R U S'(V)^T - I_k\|_2^2. \quad (9)$$

Parameters R_U^L, R_U^R and S' should be adjusted to minimize goal function G taking into account constraints that are given by an embedding rule. This is obviously non-linear Least Squares (LS) task which solving would require considerable computational costs and a global minimum is not guaranteed.

B. Simplified Solution for Embedding Task

With the aim to simplify embedding and make it more sufficient in computational sense we propose to alter the first column of orthogonal matrix $\boldsymbol{u}^T_1 \to \boldsymbol{u}'^T_1$ independently from the other columns. The rest columns of orthogonal matrix U' should be adjusted after.

The first layer $S'_{1,1}\boldsymbol{u}'^T_1\boldsymbol{v}_1$ is the most essential as it interprets watermark's bit and the first singular value is the largest which corresponds to the highest visual importance. Hence, the task of the first phase of embedding is to satisfy a condition given by a rule and to minimize the term $\left\|I_k - S'_{1,1}\boldsymbol{u}'^T_1\boldsymbol{v}_1\right\|_2^2$. In order to make it simple we propose: a) to rotate in 4D space vector $\boldsymbol{u}_1^T$ on a minimal angle to satisfy an embedding rule, that is $\boldsymbol{u}_1^T \to \boldsymbol{u}'^T_1$; b) to adjust $S_{1,1} \to S'_{1,1}$ and minimize $\left\|I_k - S'_{1,1}\boldsymbol{u}'^T_1\boldsymbol{v}_1\right\|_2^2$. The action a) depends on a rule, but we will show further that in our case it is simple. The action b) is ordinary Least Squares (LS) task.

The rest columns of matrix U' and the rest singular values in S' should be defined to minimize embedding distortions $\|I_k - I'_k\|_2^2$ during the second phase, but $\boldsymbol{u}'^T_1$ and $S'_{1,1}$ have to remain unchanged. For that purpose we have proposed a special procedure that guarantees orthonormality of the resulting U' and requires ordinary LS solver on each step.

In the proposed procedure each column of U' starting from the second is being defined on a separate step. A special rotational matrix is used on each step that does not change columns defined on previous steps.

The steps of the proposed procedure are:

1. Calculate rotation matrix R^L that transforms $\boldsymbol{u}_1^T$ to $\boldsymbol{u}'^T_1$;
2. Calculate new orthonormal matrix $U'^{(1)} = R^L U$;
3. Calculate rotation matrix $R''_{d=0}$ that does not change the first column of $U'^{(1)}$ but minimizes $\left\|I_k - \sum_{g=1}^{2} S'_{g,g}\boldsymbol{u}'^T_g\boldsymbol{v}_g\right\|_2^2$ by modifying the second column $\boldsymbol{u}'^T_2$, find $S'_{2,2}$;
4. Calculate new orthonormal matrix $U'^{(2)} = U'^{(1)}R''_{d=0}$;
5. Calculate rotation matrix $R'_{c=d=0}$ that does not change the first two columns of $U'^{(2)}$ but minimizes $\left\|I_k - \sum_{g=1}^{3} S'_{g,g}\boldsymbol{u}'^T_g\boldsymbol{v}_g\right\|_2^2$ by modifying the third column $\boldsymbol{u}'^T_3$, find $S'_{3,3}$;
6. Calculate final orthonormal matrix $U' = U'^{(2)}R'_{c=d=0}$, find $S'_{4,4}$ and compose I'_k.

As can be seen from the steps the final matrix U' can be expressed in terms of original U and calculated rotational matrices: $U' = R^L U R''_{d=0}R'_{c=d=0}$. More detailed description of each step is provided further.

Left- or right-isoclinic rotation model can be used on step 1. According to left-isoclinic rotation model:

$$R^L\boldsymbol{u}_1^T = \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix} \times \begin{pmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \\ U_{4,1} \end{pmatrix} = \boldsymbol{u}'^T_1. \quad (10)$$

Parameters a, b, c, d of R^L are calculated as following:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} U_{1,1} & -U_{2,1} & -U_{3,1} & -U_{4,1} \\ U_{2,1} & U_{1,1} & U_{4,1} & -U_{3,1} \\ U_{3,1} & -U_{4,1} & U_{1,1} & U_{2,1} \\ U_{4,1} & U_{3,1} & -U_{2,1} & U_{4,1} \end{pmatrix}^T \boldsymbol{u}'^T_1. \quad (11)$$

New matrix $U'^{(1)}$ can be calculated on step 2:

$$U'^{(1)} = R^L U. \quad (12)$$

A rotation matrix that does not change the first column of $U'^{(1)}$ should be calculated on the next step 3. A suitable model for that kind of changes is Euler-Rodrigues rotation matrix [16] with parameter $d = 0$:

$$R'_{d=0} = \\ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a^2+b^2-c^2 & 2bc & 2ac \\ 0 & 2bc & a^2-b^2+c^2 & -2ab \\ 0 & -2ac & 2ab & a^2-b^2-c^2 \end{pmatrix}. \quad (13)$$

The matrix can also be represented in a different way using elements $x = a^2 + b^2 - c^2$, $y = 2bc$, $z = -2ac$, which provides $x^2 + y^2 + z^2 = 1$. The result is:

$$R''_{d=0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & x & y & -z \\ 0 & y & \frac{z^2-y^2+(1-x)^2}{2(1-x)} & \frac{yz}{1-x} \\ 0 & z & \frac{-yz}{1-x} & \frac{z^2-y^2-(1-x)^2}{2(1-x)} \end{pmatrix}. \quad (14)$$

Matrix $U'^{(2)}$ can be defined as:

$$U'^{(2)} = U'^{(1)}R''_{d=0}. \quad (15)$$

Therefore the second column of $U'^{(2)}$ is defined as:

$$\boldsymbol{u}'^T_2 = U'^{(1)}\begin{pmatrix} 0 \\ x \\ y \\ z \end{pmatrix}. \quad (16)$$

In order to find parameters x, y, z it is necessary to solve:

$$\left\|I_k - S'_{1,1}\boldsymbol{u}'^T_1\boldsymbol{v}_1 - S'_{2,2}\boldsymbol{u}'^T_2\boldsymbol{v}_2\right\|_2^2 \to min. \quad (17)$$

Here everything except the term $S'_{2,2}\boldsymbol{u}'^T_2$ is known.

After parameters $S'_{2,2}$, x, y, z are found matrix $U'^{(2)}$ can be determined on step 4 according to (15).

A rotation matrix that does not change the first two columns of $U'^{(2)}$ should be calculated on the next step 5. The model for that kind of rotation is even more simplified Euler-Rodrigues matrix with parameters $c = d = 0$, $a^2 - b^2 = x$, $2ab = y$:

$$R'_{c=d=0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & x & -y \\ 0 & 0 & y & x \end{pmatrix}. \quad (18)$$

Matrix U' is defined as

$$U' = U'^{(2)}R'_{c=d=0}. \quad (19)$$

Therefore the third column of U' is defined as:

$$\boldsymbol{u}'^T_3 = U'^{(2)} \begin{pmatrix} 0 \\ 0 \\ x \\ y \end{pmatrix}. \quad (20)$$

The third column should satisfy:

$$\left\| I_k - S'_{1,1}\boldsymbol{u}'^T_1\boldsymbol{v}_1 - S'_{2,2}\boldsymbol{u}'^T_2\boldsymbol{v}_2 - S'_{3,3}\boldsymbol{u}'^T_3\boldsymbol{v}_3 \right\|_2^2 \rightarrow min, \quad (21)$$

where everything except the term $S'_{3,3}\boldsymbol{u}'^T_3$ is known.

After parameters $S'_{3,3}$, x, y are found final matrix U' can be defined on step 6 according to (19). The last remained undefined parameter $S'_{4,4}$ should minimize the following:

$$\left\| I_k - S'_{1,1}\boldsymbol{u}'^T_1\boldsymbol{v}_1 - S'_{2,2}\boldsymbol{u}'^T_2\boldsymbol{v}_2 - \right.$$
$$\left. -S'_{3,3}\boldsymbol{u}'^T_3\boldsymbol{v}_3 - S'_{4,4}\boldsymbol{u}'^T_4\boldsymbol{v}_4 \right\|_2^2 \rightarrow min \quad (22)$$

After $S'_{4,4}$ is found watermarked image fragment I'_k can be composed. Every optimization step in the described procedure requires just ordinary LS solver which does not consume much computational sources.

C. Steps of Watermarking

The proposed method of watermark embedding can be described as following:

1) Split the whole image I on fragments of size 4×4;
2) Select image fragments for watermark embedding according to some secret key, assign to each fragment a bit of a watermark;
3) Perform SVD of a particular selected image fragment I_k. Check if a corresponding bit of a watermark is interpreted correctly according to a watermarking rule. In case interpretation is correct set $I'_k \leftarrow I_k$ and proceed to 6), else proceed to 4);
4) Depending on the rule modify the first column of either U or V to satisfy the rule, calculate $S'_{1,1}$;
5) Apply the described simplified procedure of embedding to the rest columns of the chosen matrix (U or V) and the rest singular values in S. Compose watermarked image fragment I'_k;
6) Substitute original fragment I_k by I'_k. If $k+1$ is less or equal to the length of a watermark set $k \leftarrow k+1$ and proceed to 3), else finalize embedding.

Watermark extraction can be specified by the steps:

1) Split the whole watermarked image I' on fragments of size 4×4;
2) Select image fragments for watermark extraction according to the key;
3) Apply SVD to the selected fragment I'_k and obtain $\{U', V'\}$;
4) Interpret a bit according to a watermarking rule using $\{U', V'\}$. If $k+1$ is less or equal to the length of a watermark set $k \leftarrow k+1$ and proceed to 3), else finalize extraction and output a watermark.

IV. Experimental Results

A. Embedding Rules

Orthogonal matrices should satisfy some requirements that can be expressed in a rule. This is necessary for proper embedding and extraction of a bit of a watermark. Main components of a rule are 4×1 reference vectors $\boldsymbol{ref}_1$ and $\boldsymbol{ref}_2$ and a scalar threshold Th. Two different embedding rules are proposed.

The first rule considers the both orthogonal matrices of SVD of a block I_k. Matrix U or matrix V should be modified to embed a bit and the choice depends on properties of a block. However, the both matrices are important for extraction as the person who extracts does not have additional information about embedding conditions. We further use notation $\{U', V'\}$ because during extraction the both matrices should be treated equally.

Let us use the following notations. For original block I_k we define two indexes: $Y_{k,1} = \boldsymbol{u}_1\boldsymbol{ref}_1 - m'$, $Y_{k,2} = \boldsymbol{v}_1\boldsymbol{ref}_2 - m''$. For modified block I'_k we define another two indexes: $Y'_{k,1} = \boldsymbol{u}'_1\boldsymbol{ref}_1 - m'$, $Y'_{k,2} = \boldsymbol{v}'_1\boldsymbol{ref}_2 - m''$. Here m' and m'' are corresponding mean values. The signs $sign(Y'_{k,1})$ and $sign(Y'_{k,2})$ should be computed in order to extract a bit. Sign pairs $\{-,-\}$ and $\{+,+\}$ are interpreted as 0. Sign pairs $\{-,+\}$ and $\{+,-\}$ are interpreted as 1.

If an embedding rule is not satisfied for an original block I_k and there is a need of modification of one of orthogonal matrices we propose to check inequality $|Y_{k,1}| \leq |Y_{k,2}|$. Matrix U should be modified if the inequality is true. Otherwise matrix V should be modified. Such guidance reduces embedding distortions as the level of total distortion for I'_k depends mostly on distortion of the first column. A column that provides smaller absolute value of dot product with reference vector requires lower distortion to change the sign of the product.

In order to maintain good robustness-transparency tradeoff we consider two parameters in the rule that limit embedding distortions. The first parameter is empirically defined positive real threshold Th, which is necessary to regulate robustness-transparency tradeoff. The second parameter Max depends on the properties of a block: $Max = max(|Y_{k,1}|, |Y_{k,2}|)$. The first embedding rule is:

$$Rule\#1: \begin{cases} Embedding: \ (-1)^{bit}\, Y'_{k,1}Y'_{k,2} = Max * min(Th, Max); \\ Extraction: bit = \left(2 + sign(Y'_{k,1}Y'_{k,2})\right) mod\ 3. \end{cases}$$

The second rule interprets the first column of U only and only Th limits embedding distortions:

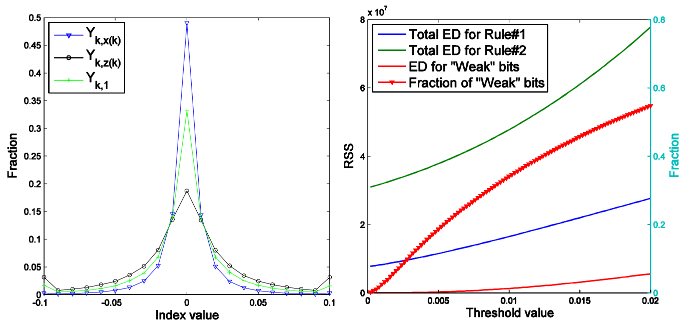
$$Rule\#2: \begin{cases} Embedding: \ (-1)^{bit}\, Y'_{k,1} = Th; \\ Extraction: bit = \left(2 + sign(Y'_{k,1})\right) mod\ 3. \end{cases}$$

For the both proposed rules the task of adjustment of the first column of corresponding matrix is easy. Let us assume that the $Rule\#1$ is used and for a particular block I_k the rule is not satisfied and $|Y_{k,1}| \leq |Y_{k,2}|$. Thus, it is necessary to find $\boldsymbol{u}'^T_1$ which belongs to the plane determined by $\boldsymbol{u}^T_1$, $\boldsymbol{ref}_1$ and the origin. This new column $\boldsymbol{u}'^T_1$ should satisfy $\boldsymbol{u}'_1\boldsymbol{ref}_1 = (-1)^{bit}min(Th, |Y_{k,2}|) + m'$.

B. Characteristics of Embedding Rules

In order to analyze watermarking properties of the proposed embedding rules we first compared histograms of corresponding indices that are being interpreted by rules. Sixteen grayscale images with resolution 512x512 were used, which provided 262144 different blocks of size 4x4. The following sets of indices were considered: $\{Y_{k,1}\}$, $\{Y_{k,x_k}\}$, $\{Y_{k,z_k}\}$, where $k = 1 \dots 262144$, $x_k = arg\min(|Y_{k,1}|, |Y_{k,2}|)$, $z_k = arg\max(|Y_{k,1}|, |Y_{k,2}|)$. The sets $\{Y_{k,x_k}\}$ and $\{Y_{k,1}\}$ are potentially being modified if $Rule\#1$ or $Rule\#2$ are used respectively. The variances of set $\{Y_{k,x_k}\}$ and set $\{Y_{k,1}\}$ are good estimates to compare embedding distortions for each rule. Higher variance of set $\{Y_{k,z_k}\}$ corresponds to higher variance of Max and less equal ability to withstand disturbances by different blocks if $Rule\#1$ is used. The following reference vectors were used by the rules: $\boldsymbol{ref}_1^T = \boldsymbol{ref}_2^T = (0.5 \quad -0.5 \quad 0.5 \quad -0.5)$.

The histograms of all the mentioned above sets are shown on Fig.1a.



(a) (b)

Fig. 1. a) Histograms of indices $Y_{k,1}$, Y_{k,x_k}, Y_{k,z_k}; b) Graphs of embedding distortions for $Rule\#1$ and $Rule\#2$.

It can be seen that $Var\{Y_{k,z_k}\} > Var\{Y_{k,1}\} > Var\{Y_{k,x_k}\}$. From comparison $Var\{Y_{k,1}\} > Var\{Y_{k,x_k}\}$ one could derive that the $Rule\#1$ causes lower embedding distortions. Furthermore the higher $Var\{Y_{k,z_k}\}$ the more advantageous $Rule\#1$ compared to $Rule\#2$ in terms of transparency and less advantageous in terms of robustness.

We have used 16 test images to embed the same watermark using the both rules. The dependencies between different values of Th and Residual Sum of Squares (RSS) for each rule are represented graphically on Fig.1b. Two main graphs to compare transparency are Embedding Distortions (ED) for $Rule\#1$ and $Rule\#2$. Another two graphs ED for “weak” bits and Fraction of “weak” bits provide details about robustness of a watermark embedded using $Rule\#1$. The definition of “weak” bit is used for a bit embedded using $Rule\#1$ in a block where $Max < Th$. The Fraction of such “weak” bits shows how big the disadvantage in robustness is compared to $Rule\#2$. Additionally the graph ED for “weak” bits demonstrates how ED is distributed between “weak” and “typical” bits embedded with $Max \geq Th$. For example, for $Th = 0.02$ cumulative ED for more than a half of all the blocks is less than 20% of total ED. This is quite unequal distribution of ED which worsens robustness in general when compared to $Rule\#2$. On the other hand $Rule\#2$ introduces much higher distortions during embedding.

C. Watermarking Results

The proposed watermarking method was used with the both embedding rules. Sixteen different grayscale images of size 512x512 were used for watermarking. The watermarking performance was compared with the performance of other blind SVD-based methods proposed by Tehrani [15], Li [13] and Gorodetski [7] (adopted for grayscale images). Selected methods provide quite different robustness-transparency tradeoffs and different data payloads. With the aim to make comparison fair we used the same watermark for all the methods and all the images. We also tried to adjust methods in a way that quality of the watermarked images is comparable. For that purpose different methods performed embedding with different redundancy. The watermark payload was 512 bits per image as this is the maximum payload of Li method. The redundancy rates were: 12 for the proposed method and $Rule\#1$, 8 for the proposed method and $Rule\#2$, 6 for Tehrani’s method, 1 for Li’s method, 6 for Gorodetski’s method. All the watermarked images undergo attacks simulated by StirMark Benchmark 4. For all the methods Bit Error Rates (BERs) of extracted watermarks were averaged among all the 16 watermarked images. The results are represented in Table I.

TABLE I. RESULTS OF WATERMARK EXTRACTION

Method, PSNR	**GN, PSNR 35dB**	**Salt & Pepper, 3%**	**JPEG, Q=50**	**3x3 Median Filter**	**Cropping, 75%**	**Rotation, 0.25**
Tehrani, 43.47dB	5.42	6.55	5.04	11.32	8.92	10.32
Li, 42.13dB	**2.19**	5.11	**2.35**	**1.83**	38.12	20.81
Gorodetski, 42.85dB	4.89	**4.41**	7.57	10.58	8.83	9.42
Proposed, *Rule*1, 44.13dB	3.74	4.61	3.16	6.24	**1.62**	**8.30**
Proposed, *Rule*2, 43.87dB	5.35	6.73	3.79	8.61	5.15	9.53

In our experiment we set $Th = 0.002$ for both proposed rules. The quality of images watermarked according to the proposed method and $Rule\#1$ remains quite acceptable. This can be witnessed by Fig.2 where the watermarked Livingroom image is depicted.

Fig. 2. Watermarked Livingroom image with PSNR=44.29dB.

V. DISCUSSION

The proposed watermarking method provides good robustness-transparency tradeoff in conjunction with both proposed embedding rules. Considerable advantages are achieved in comparison with the methods proposed by Gorodetski and Tehrani. Another point is that our method maintains much higher robustness toward geometrical attacks in comparison with the method proposed by Li.

High robustness for the proposed method and the both proposed rules in comparison with the methods proposed by Gorodetski and Tehrani is mainly due to higher redundancy of embedding. Higher redundancy is possible thanks to low distortions of embedding of a bit in a single 4x4 block. Embedding according to $Rule\#1$ provides better results in terms of the both robustness and transparency in comparison with that for $Rule\#2$. It can be explained by recalling that $Rule\#1$ causes much lower distortions while fraction of “weak” bits for threshold $Th = 0.002$ is also small (Fig. 1b).

Comparison with the method proposed by Li leads to different conclusions depending on a kind of attack. For signal processing and noise attacks (except Salt&Pepper) the method of Li demonstrates better robustness than the proposed method with $Rule\#1$. On the other hand robustness toward geometric attacks is much worse for Li’s method. This is because such factors as smaller blocks and higher redundancy are more beneficial in such kind of attacks (especially cropping). Another advantage of the proposed method is considerably better transparency of the watermarked images.

The proposed method does not provide the best robustness in all the cases of different attacks. However, its robustness is sufficient and embedding distortions are low. Maximum data payload is the same as for the method of Tehrani and Gorodetski and is equal to 16384 bits per 512x512 image, which is quite large. All the mentioned characteristics make the proposed method favorable in most of watermarking applications.

VI. Conclusions

The watermarking method proposed in this paper is blind. It uses SVD of a 4x4 image block in order to embed a bit of a watermark by modifying an orthonormal matrix which is either left or right. The modification is done according to one of the proposed embedding rules and the procedure for minimization of embedding distortions is considered. Redundant embedding is applied with the aim to maintain good robustness-transparency tradeoff.

Modification of one of the orthonormal matrices of SVD of 4x4 image block is done according to the model of rotations in four dimensional space. Optimization tasks are being solved with the aim to minimize distortions. The modification is split on two phases that simplifies each separate optimization task and reduces computation load.

Two different embedding rules were proposed which are $Rule\#1$ and $Rule\#2$. The $Rule\#1$ interprets both orthonormal matrices of SVD, while the $Rule\#2$ interprets only the left one. Each rule can be used in conjunction with the proposed method which leads to different embedding distortions and robustness.

Redundant embedding is beneficial in a combination with the proposed watermarking method as the latter introduces low distortions, but maintains considerable payload. Utilization of different redundancy rates for various applications yields to different robustness-transparency tradeoffs.

The efficiency of the proposed method was confirmed experimentally. For some kinds of popular attacks BER of watermark extraction is 36% lower compared to other known methods.

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