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An effective technique for the conformable space-time fractional EW and modified EW equations

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Abstract: The current work deals with the fractional forms of EW and modified EW equations in the conformable sense and their exact solutions. In this respect, by utilizing a traveling wave transformation, the governing space-time fractional models are converted to the nonlinear ordinary differential equations (NLODEs); and then, the resulting NLODEs are solved through an effective method called the $\exp(-\phi(\epsilon))$-expansion method. As a consequence, a number of exact solutions to the fractional forms of EW and modified EW equations are generated.

Keywords: Fractional forms of EW and modified EW equations; Conformable sense; $\exp(-\phi(\epsilon))$-expansion method; Exact solutions

1 Introduction

These days, the fractional differential equations (FDEs) have been the subject of a lot of research, owing to their frequent appearance in various areas from physics and chemistry to biology and engineering. A variety of useful methods, such as sub-equation method [1–4], modified trial equation method [5–8], (G'/G)-expansion method [9–12], exp-function method [13–16], Kudryashov method [17–20], and first integral method [21–24] have been applied to find the exact solutions of FDEs. One capable technique which has newly gained special interest is the $\exp(-\phi(\epsilon))$ method. For instance, Hosseini et al. [25] exerted the $\exp(-\phi(\epsilon))$ method to produce the exact solutions of the density-dependent conformable fractional diffusion-reaction equation and Raza et al. [26] adopted the $\exp(-\phi(\epsilon))$ method to extract the explicit solutions of higher dimensional equations with fractional temporal evolution. For more articles, see [27–40]. In this paper, the exact solutions of the fractional forms of EW and modified EW equations in the conformable sense are achieved by means of the $\exp(-\phi(\epsilon))$ method. The mathematical modelings of these fractional differential equations are presented below:

- The fractional EW equation [41]

$$D_t^{\alpha} u(x,t) + \sigma D_x^{\alpha} u^2(x,t) - \delta D_{xxt}^{3\alpha} u(x,t) = 0,\quad t>0,\ \ 0<\alpha\leqslant 1. \tag{1}$$

- The fractional modified EW equation [41]

$$D_t^{\alpha} u(x,t) + \sigma D_x^{\alpha} u^3(x,t) - \delta D_{xxt}^{3\alpha} u(x,t) = 0,\quad t>0,\ \ 0<\alpha\leqslant 1. \tag{2}$$

The EW equations describe the propagation of the wave in nonlinear media [42]. Several schemes have already been exerted for studying the above models; for example, Kudryashov method [17], ansatz method [41], and Fan sub-equation method [43]. This paper is organized as below: In Section 2, we will introduce the conformable derivative and its properties. In Section 3, we will explain the ideas of the $\exp(-\phi(\epsilon))$ method. In Section 4, we will employ the $\exp(-\phi(\epsilon))$ method to solve the fractional forms of EW and modified EW equations; and at the end, we will provide the results.

2 Conformable fractional derivative

Recently, a new version of fractional derivatives "the conformable fractional derivative" was proposed in [44] which obeys some classical properties that cannot be satisfied by the other definitions [45]. The conformable fractional

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derivative of f of order α is defined as [44]

$$T_\alpha(f)(t) = \lim_{\tau\to 0}\frac{f(t+\tau t^{1-\alpha})-f(t)}{\tau},$$

where $f:(0,\infty)\to R$, $t>0$, and $\alpha\in(0,1]$. Some worthwhile features of the conformable derivative are as follows

(i)
$$T_\alpha(af+bg)=aT_\alpha(f)+bT_\alpha(g), \forall a,b\in R.$$

(ii)
$$T_\alpha(t^\mu)=\mu t^{\mu-\alpha}, \forall\mu\in R.$$

(iii)
$$T_\alpha(fog)(t)=t^{1-\alpha}g'(t)f'(g(t)).$$

3 Basic ideas of exp($-\phi(\epsilon)$)-expansion method

Let's consider a nonlinear space-time FDE in the conformable sense as follows

$$F(u, D_t^{\alpha_1}u, D_x^{\alpha_2}u, D_t^{2\alpha_1}u, D_t^{\alpha_1}D_x^{\alpha_2}u, D_x^{2\alpha_2}u,\ldots)=0, \quad 0<\alpha_1,\alpha_2<1. \tag{3}$$

By using the transformation

$$u(x,t)=f(\epsilon), \quad \epsilon=k\frac{x^{\alpha_2}}{\alpha_2}-l\frac{t^{\alpha_1}}{\alpha_1},$$

Eq. (3) changes into an ODE of integer order as

$$G(f,f',f'',\ldots)=0, \tag{4}$$

where G is a function in the unknown function f and its derivatives.

Let present the solution of Eq. (4) as below

$$f(\epsilon)=\sum_{n=0}^{N}a_n(\exp(-\phi(\epsilon)))^n, \tag{5}$$

where $a_n, n=0,1,\ldots,N$ ($a_N\neq 0$) are unknown constants and $\phi(\epsilon)$ satisfies a nonlinear ordinary differential equation as

$$\phi'(\epsilon)=\exp(-\phi(\epsilon))+\mu\exp(\phi(\epsilon))+\lambda.$$

Now, various cases can be considered:

Case 1. If $\lambda^2-4\mu>0$ and $\mu\neq 0$, then

$$\phi_1(\epsilon)=\ln\left(\frac{-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\epsilon+C)\right)-\lambda}{2\mu}\right).$$

Case 2. If $\lambda^2-4\mu>0$, $\mu=0$, and $\lambda\neq 0$, then

$$\phi_2(\epsilon)=-\ln\left(\frac{\lambda}{\cosh(\lambda(\epsilon+C))+\sinh(\lambda(\epsilon+C))-1}\right).$$

Case 3. If $\lambda^2-4\mu<0$ and $\mu\neq 0$, then

$$\phi_3(\epsilon)=\ln\left(\frac{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\epsilon+C)\right)-\lambda}{2\mu}\right).$$

To procure the positive integer N in Eq. (5), we balance the terms in Eq. (4). Setting Eq. (5) in Eq. (4), results in

$$P(\exp(-\phi(\epsilon)))=0. \tag{6}$$

Through equating all the coefficients in (6) to zero, we will attain an algebraic set, which can be easily solved for determining the unknowns. Substituting them into (5), finally yields the exact solutions of original Eq. (3).

4 Applications

Now, we adopt the exp($-\phi(\epsilon)$) method to seek the exact solutions of the fractional forms of EW and modified EW equations in the conformable sense.

4.1 The conformable space-time fractional EW equation

By using the transformation

$$u(x,t)=f(\epsilon), \quad \epsilon=k\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha},$$

Eq. (1) can be changed into the following ODE

$$-lf'+\sigma k(f^2)'+\delta lk^2f'''=0. \tag{7}$$

Integrating (7) once with respect to ϵ, gives

$$-lf+\sigma kf^2+\delta lk^2f''=0, \tag{8}$$

where integrating constant is supposed to be zero.

4.1.1 Applying the exp($-\phi(\epsilon)$)-expansion method

By balancing f^2 and f'' in Eq. (8), we obtain $N=2$. Hence, Eq. (8) has the following formal solution

$$f(\epsilon)=a_0+a_1\exp(-\phi(\epsilon))+a_2\exp(-2\phi(\epsilon)). \tag{9}$$

Through setting Eq. (9) in Eq. (8) and equating all the coefficients to zero, we will achieve an algebraic set in the form

$$\begin{aligned}
&-la_0+\delta l\lambda\mu k^2a_1+2\delta l\mu^2k^2a_2+k\sigma a_0^2=0,\\
&(\delta l\lambda^2k^2+2\delta l\mu k^2-l)a_1+6\delta l\lambda\mu k^2a_2+2k\sigma a_0a_1=0,\\
&3\delta l\lambda k^2a_1+(4\delta l\lambda^2k^2+8\delta l\mu k^2-l)a_2+k\sigma a_1^2+2k\sigma a_0a_2=0,\\
&2\delta lk^2a_1+10\delta l\lambda k^2a_2+2k\sigma a_1a_2=0,\\
&6\delta lk^2a_2+\delta\sigma a_2^2=0.
\end{aligned}$$

Applying the symbolic computation package, results in

Case 1.

$$\begin{aligned}
a_0&=\mp\frac{6l\mu\sqrt{\delta(\lambda^2-4\mu)}}{\sigma(\lambda^2-4\mu)},\quad a_1=\mp\frac{6\delta l\lambda}{\sigma\sqrt{\delta(\lambda^2-4\mu)}},\\
a_2&=\mp\frac{6\delta l}{\sigma\sqrt{\delta(\lambda^2-4\mu)}},\quad k=\pm\frac{1}{\sqrt{\delta(\lambda^2-4\mu)}}.
\end{aligned}$$

Thus, the exact solutions to the fractional form of EW equation in the conformable sense can be constructed as follows

For $\lambda^2-4\mu>0$ and $\mu\neq 0$

$$\begin{aligned}
u_{1,2}(x,t)=&\mp\frac{6l\mu\sqrt{\delta(\lambda^2-4\mu)}}{\sigma(\lambda^2-4\mu)}\pm\frac{12\delta l\lambda\mu}{\sigma\sqrt{\delta(\lambda^2-4\mu)}\left(\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(\pm\frac{1}{\sqrt{\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C))+\lambda\right)}\\
&\mp\frac{24\delta l\mu^2}{\sigma\sqrt{\delta(\lambda^2-4\mu)}\left(\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(\pm\frac{1}{\sqrt{\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C))+\lambda\right)^2}.
\end{aligned}$$

For $\lambda^2-4\mu>0$, $\mu=0$, and $\lambda\neq 0$

$$\begin{aligned}
u_{3,4}(x,t)=&\mp\frac{6\delta l\lambda^2}{\sigma\sqrt{\delta\lambda^2}\left(\cosh\left(\lambda\left(\pm\frac{1}{\sqrt{\delta\lambda^2}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C\right)\right)+\sinh\left(\lambda\left(\pm\frac{1}{\sqrt{\delta\lambda^2}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C\right)\right)-1\right)}\\
&\mp\frac{6\delta l\lambda^2}{\sigma\sqrt{\delta\lambda^2}\left(\cosh\left(\lambda\left(\pm\frac{1}{\sqrt{\delta\lambda^2}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C\right)\right)+\sinh\left(\lambda\left(\pm\frac{1}{\sqrt{\delta\lambda^2}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C\right)\right)-1\right)^2}.
\end{aligned}$$

For $\lambda^2-4\mu<0$ and $\mu\neq 0$

$$\begin{aligned}
u_{5,6}(x,t)=&\mp\frac{6l\mu\sqrt{\delta(\lambda^2-4\mu)}}{\sigma(\lambda^2-4\mu)}\pm\frac{12\delta l\lambda\mu}{\sigma\sqrt{\delta(\lambda^2-4\mu)}\left(\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\pm\frac{1}{\sqrt{\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C\right)\right)+\lambda\right)}\\
&\mp\frac{24\delta l\mu^2}{\sigma\sqrt{\delta(\lambda^2-4\mu)}\left(\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\pm\frac{1}{\sqrt{\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha}-l\frac{t^\alpha}{\alpha}+C\right)\right)+\lambda\right)^2}.
\end{aligned}$$

Case 2.

$$a_0=\pm\frac{l(\lambda^2+2\mu)\sqrt{-\delta(\lambda^2-4\mu)}}{\sigma(\lambda^2-4\mu)},\quad a_1=\mp\frac{6\delta l\lambda}{\sigma\sqrt{-\delta(\lambda^2-4\mu)}},\quad a_2=\mp\frac{6\delta l}{\sigma\sqrt{-\delta(\lambda^2-4\mu)}},\quad k=\pm\frac{1}{\sqrt{-\delta(\lambda^2-4\mu)}}.$$

Consequently, the exact solutions to the fractional form of EW equation in the conformable sense can be established as follows

For $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$

$$u_{7,8}(x,t) = \pm\frac{l(\lambda^2+2\mu)\sqrt{-\delta(\lambda^2-4\mu)}}{\sigma(\lambda^2-4\mu)} \pm \frac{12\delta l\lambda\mu}{\sigma\sqrt{-\delta(\lambda^2-4\mu)}\left(\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\pm\frac{1}{\sqrt{-\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)+\lambda\right)}$$
$$\mp\frac{24\delta l\mu^2}{\sigma\sqrt{-\delta(\lambda^2-4\mu)}\left(\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\pm\frac{1}{\sqrt{-\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)+\lambda\right)^2}.$$

For $\lambda^2 - 4\mu > 0$, $\mu = 0$, and $\lambda \neq 0$

$$u_{9,10}(x,t) = \pm\frac{l\sqrt{-\delta\lambda^2}}{\sigma} \mp \frac{6\delta l\lambda^2}{\sigma\sqrt{-\delta\lambda^2}\left(\cosh\left(\lambda\left(\pm\frac{1}{\sqrt{-\delta\lambda^2}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)+\sinh\left(\lambda\left(\pm\frac{1}{\sqrt{-\delta\lambda^2}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)-1\right)}$$
$$\mp\frac{6\delta l\lambda^2}{\sigma\sqrt{-\delta\lambda^2}\left(\cosh\left(\lambda\left(\pm\frac{1}{\sqrt{-\delta\lambda^2}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)+\sinh\left(\lambda\left(\pm\frac{1}{\sqrt{-\delta\lambda^2}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)-1\right)^2}.$$

For $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$

$$u_{11,12}(x,t) = \pm\frac{l(\lambda^2+2\mu)\sqrt{-\delta(\lambda^2-4\mu)}}{\sigma(\lambda^2-4\mu)} \pm \frac{12\delta l\lambda\mu}{\sigma\sqrt{-\delta(\lambda^2-4\mu)}\left(\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\pm\frac{1}{\sqrt{-\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)+\lambda\right)}$$
$$\mp\frac{24\delta l\mu^2}{\sigma\sqrt{-\delta(\lambda^2-4\mu)}\left(\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\pm\frac{1}{\sqrt{-\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha} + C\right)\right)+\lambda\right)^2}.$$

4.2 The conformable space-time fractional modified EW equation

By using the transformation

$$u(x,t) = f(\epsilon), \quad \epsilon = k\frac{x^\alpha}{\alpha} - l\frac{t^\alpha}{\alpha},$$

Eq. (2) can be converted to an ODE as follows

$$-lf' + \sigma k(f^3)' + \delta l k^2 f''' = 0. \tag{10}$$

Integrating (10) once with respect to ϵ, yields

$$-lf + \sigma k f^3 + \delta l k^2 f'' = 0, \tag{11}$$

where integrating constant is assumed to be zero.

4.2.1 Applying the exp(−$\phi(\epsilon)$)-expansion method

By balancing f^3 and f'' in Eq. (11), we take $N = 1$. Consequently, Eq. (11) has the following formal solution

$$f(\epsilon) = a_0 + a_1\exp(-\phi(\epsilon)). \tag{12}$$

By inserting Eq. (12) along with its necessary derivative in Eq. (11) and equating all the coefficients to zero, we will derive an algebraic set as

$$\begin{aligned}
&-la_0 + \delta l\lambda\mu k^2 a_1 + k\sigma a_0^3 = 0,\\
&(\delta l\lambda^2 k^2 + 2\delta l\mu k^2 - l)a_1 + 3k\sigma a_0^2 a_1 = 0,\\
&3\delta l\lambda k^2 a_1 + 3k\sigma a_0 a_1^2 = 0,\\
&2\delta l k^2 a_1 + k\sigma a_1^3 = 0.
\end{aligned}$$

Using the symbolic computation package, we will find

$$a_0 = \frac{1}{2}\lambda a_1, \quad k = \pm\frac{2}{\sqrt{-2\delta(\lambda^2 - 4\mu)}}, \quad l = \mp\frac{\sigma a_1^2\sqrt{-2\delta(\lambda^2 - 4\mu)}}{4\delta}.$$

Thus, the exact solutions to the fractional form of modified EW equation in the conformable sense can be constructed as follows

For $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$

$$u_{1,2}(x,t) = \frac{1}{2}\lambda a_1 - \frac{2\mu a_1}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\pm\frac{2}{\sqrt{-2\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha} \pm \frac{\sigma a_1^2\sqrt{-2\delta(\lambda^2-4\mu)}}{4\delta}\frac{t^\alpha}{\alpha} + C\right)\right) + \lambda}.$$

For $\lambda^2 - 4\mu > 0$, $\mu = 0$, and $\lambda \neq 0$

$$u_{3,4}(x,t) = \frac{1}{2}\lambda a_1 + \frac{\lambda a_1}{\cosh\left(\lambda\left(\pm\frac{2}{\sqrt{-2\delta\lambda^2}}\frac{x^\alpha}{\alpha} \pm \frac{\sigma a_1^2\sqrt{-2\delta\lambda^2}}{4\delta}\frac{t^\alpha}{\alpha} + C\right)\right) + \sinh\left(\lambda\left(\pm\frac{2}{\sqrt{-2\delta\lambda^2}}\frac{x^\alpha}{\alpha} \pm \frac{\sigma a_1^2\sqrt{-2\delta\lambda^2}}{4\delta}\frac{t^\alpha}{\alpha} + C\right)\right) - 1}.$$

For $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$

$$u_{5,6}(x,t) = \frac{1}{2}\lambda a_1 - \frac{2\mu a_1}{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\pm\frac{2}{\sqrt{-2\delta(\lambda^2-4\mu)}}\frac{x^\alpha}{\alpha} \pm \frac{\sigma a_1^2\sqrt{-2\delta(\lambda^2-4\mu)}}{4\delta}\frac{t^\alpha}{\alpha} + C\right)\right) + \lambda}.$$

5 Conclusion

In this investigation, the fractional forms of EW and modified EW equations in the conformable sense were studied, successfully. First, by adopting a traveling wave transformation, the governing space-time fractional models were converted to the nonlinear ordinary differential equations; and then, the resulting NLODEs were solved using an effective method called the $\exp(-\phi(\epsilon))$-expansion method. As a consequence, a variety of exact solutions to the fractional forms of EW and modified EW equations were formally extracted; confirming the competence of the scheme.

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