Results in Physics 7 (2017) 379–380

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Microarticle

A proposed test of special-relativistic mechanics at low speed

Boon Leong Lan

Electrical and Computer Systems Engineering, School of Engineering, Monash University, 47500 Bandar Sunway, Malaysia

ARTICLE INFO

Article history:
Received 8 November 2016
Accepted 24 December 2016
Available online 29 December 2016

Keywords:
Charged particle
Uniform magnetic field
Special-relativistic mechanics
Newtonian approximation

ABSTRACT

We show that the difference between the Newtonian and special-relativistic predictions for the angular position increases linearly with time for a charged particle moving at low speed in a circular path in a constant uniform magnetic field. Numerical results suggest that it is possible to test the two different predictions experimentally.

Introduction

Recently, it was shown numerically for a dissipative bouncing ball system that, although the speed of the ball is low and the gravitational field is weak, the Newtonian approximation to the chaotic general-relativistic trajectory breaks down rapidly [1]. The different Newtonian and general-relativistic chaotic trajectories could be tested in the laboratory but the parameters and initial conditions of the system must be known to very high accuracies so that sufficiently accurate trajectories can be calculated for comparison with experiment [2]. Similarly, for low-speed non-dissipative systems where gravity does not play a dynamical role, it has been shown that the special-relativistic trajectory is not always well-approximated by the Newtonian trajectory, regardless of whether the trajectories are chaotic or non-chaotic [3,4]. However, these systems are model systems [3,4], which are not realizable in the laboratory. In this paper, we present a non-chaotic system which, we show, could be used to test the different Newtonian and special-relativistic low-speed trajectories.

Consider the motion of a particle, with rest mass m_0 and charge q, in a constant uniform magnetic field $\mathbf{B}$, where the initial velocity $\boldsymbol{v}$ of the particle is perpendicular to $\mathbf{B}$. According to Newtonian mechanics, the particle moves with a constant linear speed v in a circular path of radius

$$r_{NR} = \frac{m_0 v}{qB}. \tag{1}$$

The angular speed of the particle is also constant given by

$$\omega_{NR} = \frac{qB}{m_0} \tag{2}$$

and thus the angular position of the particle varies linearly with time t

$$\theta_{NR}(t) = \theta_0 + \frac{qB}{m_0}t. \tag{3}$$

According to special-relativistic mechanics, the particle also moves in a circular path with constant linear speed v. However, the radius of the circular path is given by [5]

$$r_R = \frac{m_0 v}{qB\sqrt{1-(v/c)^2}}, \tag{4}$$

the constant angular speed is given by [5]

$$\omega_R = \frac{qB\sqrt{1-(v/c)^2}}{m_0} \tag{5}$$

and thus the angular position of the particle varies linearly with time t as

$$\theta_R(t) = \theta_0 + \frac{qB\sqrt{1-(v/c)^2}}{m_0}t \tag{6}$$

The only difference between the non-relativistic and relativistic expressions (which are all exact) for the radius, angular speed and angular position is in the mass term – rest mass m_0 and relativistic mass $\frac{m_0}{\sqrt{1-(v/c)^2}}$ in, respectively, the former and latter expressions.

E-mail address: lan.boon.leong@monash.edu

http://dx.doi.org/10.1016/j.rinp.2016.12.035
2211-3797/

Table 1
Time it takes for the difference between the non-relativistic and relativistic angular position of a proton, which moves in a circular path in a constant uniform magnetic field of 0.01 T, to grow to 0.1 rad for different ratio v/c. The relativistic radius of the circular path is also given in the last column.

v/c	t (sec)	r_R (meter)
1.00E−02	2.09E−03	3.1314066E+00
1.00E−03	2.09E−01	3.1312516E−01
1.00E−04	2.09E+01	3.1312500E−02
1.00E−05	2.09E+03	3.1312500E−03
1.00E−06	2.09E+05	3.1312500E−04
1.00E−07	2.09E+07	3.1312500E−05
1.00E−08	2.09E+09	3.1312500E−06

At low speed, where $v \ll c$, $\sqrt{1-(v/c)^2} \approx 1 - \frac{(v/c)^2}{2}$, which is close to one. The non-relativistic and relativistic radius and angular speed are therefore always close to one another

$$r_R \approx r_{NR} \tag{7}$$

$$\omega_R \approx \omega_{NR}. \tag{8}$$

However, the difference between the non-relativistic and relativistic angular position grows linearly with time t

$$\theta_{NR}(t) - \theta_R(t) \approx \frac{1}{2}\frac{v^2}{c^2}\frac{qB}{m_0}t. \tag{9}$$

The time it takes for the difference to grow to Δ is given by

$$t \approx \frac{2\Delta}{(v/c)^2(q/m_0)B}. \tag{10}$$

This time, which increases as v/c decreases, has a power-law dependence on v/c, with exponent -2. As an example, Table 1 shows the time it takes for the difference to grow to 0.1 rad (5.7 degree) for different v/c in the case of a proton in a 0.01 T magnetic field. For instance, for $v = 10^{-4}c$, the time is 0.348 min, whereas for $v = 10^{-5}c$, the time is 34.8 min. The relativistic radius of the proton's circular path, which decreases as v/c decreases [see Eq. (4)], is 3.13 cm and 3.13 mm, respectively. For comparison, for an electron in a 10^{-5} T magnetic field with $v = 10^{-5}c$, the time is 19.0 min and the relativistic radius is 1.71 mm. These results suggest that it is possible to test the different predictions of special-relativistic and Newtonian mechanics for the angular position of a charged particle moving at low speed in a circular path in a constant uniform magnetic field. Such a test of special-relativistic mechanics is essentially a test of the relativistic mass formula at low speed ($v \ll c$). In contrast, previous tests (see references in [6]) of the relativistic mass formula based on the motion of charged particles in electric and magnetic fields were for high speeds ranging from 0.26c to 0.99c (see Table 11.2 in [6]).

Acknowledgement

This work was funded by a Fundamental Research Grant FRGS/1/2013/ST02/MUSM/02/1 and the Advanced Engineering Platform at Monash Malaysia.

References

[1] Liang SN, Lan BL. PLoS ONE 2012;7(4):e34720.
[2] Liang SN, Lan BL. Res Phys 2014;4:187–8.
[3] Lan BL, Borondo F. Phys Rev E 2011;83:036201.
[4] Lan BL. Chaos 2006;16:033107.
[5] Barton G. Introduction to the relativity principle. West Sussex: John Wiley & Sons; 1999.
[6] Zhang YZ. Special relativity and its experimental foundations. Singapore: World Scientific; 1997.