Deep Learning Super-Resolution Reconstruction of Wall Quantities

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Abstract

Accurate prediction of the fluctuations of wall quantities – wall pressure and wall shear stress – is important for many applications including acoustics, and is strongly influenced by the spatial resolution of the grid in the near-wall region. For large industrial flows at very high Reynolds number, computing a solution on a grid with enough resolution to adequately resolve the small-scale fluctuations that contribute to high-frequency sound is infeasible. As such, a solution with lower spatial resolution is computed and other methods such as reconstruction may be used to estimate the small-scale structures that were not resolved by the grid. Recently proposed reconstruction methods such as super-resolution use deep learning to estimate missing detail and have shown promising results. In this work, super-resolution is applied to reconstruct wall pressure and wall shear stress fluctuations learnt from snapshots of a direct numerical simulation (DNS) of turbulent channel flow at friction Reynolds number $Re_\tau = 395$ that were filtered and down-sampled by a factor of four. Mechanisms are identified that allow more generalised application of the method, including dataset preparation and training parameters. With these mechanisms, the super-resolution method is shown to reconstruct instantaneous fields and time-averaged statistics such as energy spectra that are in very good agreement with the DNS.

1 Introduction

Accurately predicting the fluctuations of wall pressure and wall shear stress is important for many applications from small physiological flows to flows over large vehicles and can be used to predict the sound generated by the flow. For wall shear stresses, accurately capturing the gradient at the wall is dependent on the spatial resolution of the grid in the wall-normal direction. The spatial resolution in the wall-parallel directions affects the accuracy of the fluctuating component, and for acoustic applications also determines the cut-off or high-frequency limit of predicted sound. Small turbulent structures with characteristic size below the cell size are not resolved. For flows at high Reynolds number, the spatial resolution required to adequately resolve all the small-scale structures requires a number of grid points approximately proportional to Reynolds number cubed, 90% of which are in the boundary layer (Choi & Moin 2012), meaning a solution resolving all the turbulent motions at the wall is computationally infeasible. Large eddy simulation (LES) can be used where structures below the cell size are modelled, not resolved, which reduces the resolution requirement and hence computational cost. Further reductions can be achieved when the motions at the wall are modelled, as done in wall-modelled LES. For these solutions however, a frequency cut-off of around 2 kHz is typical (Croaker \textit{et al.} 2017), and in applications where high-frequency sound is important, this information is not resolved and must be estimated.

One approach is to estimate the information that was unresolved in LES – a process known as reconstruction. An explicit reconstruction is the process where unresolved information is estimated and interpolated directly onto a finer grid. Several reconstruction methods have been proposed, focused mainly on improving subgrid models for LES, but the main limitation is that constructing an

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expression for the model form is very difficult. More recent studies perform super-resolution using the deep-learning method proposed by Dong et al. (2015). Super-resolution (SR) is able to increase the spatial or temporal resolution of an input using a convolutional neural network that is trained with matching pairs of high- and low-resolution fields. Importantly, the model form is not required a priori. The majority of SR studies to-date focus on reconstruction of velocity fields in the wake and boundary layer regions, and show promising results including the reconstruction of velocity and vorticity fields in a two-dimensional laminar wake flow over a cylinder (Fukami et al. 2019, 2021), reconstruction of the wall-parallel velocity field in the boundary layer of a turbulent channel flow (Liu et al. 2020) and reconstructing wall-parallel velocity from wall shear stress over a flat plate (Guemes et al. 2019, Guastoni et al. 2021).

This paper outlines progress towards using super-resolution to recover wall pressure and wall shear stress fields from filtered DNS. Specifically, methods are presented that allow trained models to generalise in time and space which is crucial for their application to other flow configurations.

2 Method

DNS of turbulent channel flow is a common basis for training deep learning models owing to its well-studied characteristics. Here, data used for training and testing SR models were generated from DNS of turbulent channel flow at friction velocity-based Reynolds number \(Re_{\tau} = u_{\tau}\delta/u\), following the method of Moser et al. (1999). The case was constructed around a channel half-height \(\delta = 1\ m\) and a mean centreline velocity \(U = 1\ m/s\) on the domain \(2\pi \times 2\delta \times \pi\). The domain was discretised with a uniform \(N_x = 256\) in the wall-parallel streamwise and spanwise directions respectively, and \(Ny = 193\) following a geometric expansion in the wall-normal direction. The grid resolution in the wall-parallel directions, scaled by wall units \(\nu/u_{\tau}\) was \(\Delta x^+ = 9.69, \Delta z^+ = 4.85\). This is slightly higher resolution than is reported in Moser et al. (1999), and enough to capture greater than 99% of wall stress events (Yang et al. 2021). After initialisation from a lower Reynolds number solution, a spectral element solver (Blackburn et al. 2019) was used with elements of polynomial order 9 and Fourier expansion in the spanwise direction.

Data was sampled from snapshots of the DNS and for each grid point on both walls, three variables are stored – pressure and the computed tangential components of wall shear stress. To test the generalisation in time, models were trained and tested with data collected over three different periods of time: one flow-through time, defined as \(t_f = 2\pi/U\); one eddy turn-over time, defined as \(t_L = \delta/u_{\tau}\) and equivalent to approximately 3 flow-through times; or ten eddy turn-over times. The sampling frequency (write step) was varied to collect a fixed 2,500 examples per set over the specified time period. Several sets were generated over a total of 23 turn-over times, each serving as either training, validation or test data, and were chosen such that they are always separated in time by at least one turn-over time.

Training the SR models requires matching high- and low-resolution example fields (HR and LR respectively), and the LR fields were generated by filtering the DNS in physical space with a box filter of size \(r\), then sub-sampling every \(r\) grid points where \(r = \Delta/\Delta_{DNS}\) is the resolution ratio. Only \(r = 4\) is presented here, which has a similar resolution to a comparable channel flow LES at the same Reynolds number (Gullbrand 2003).

Data underwent further preparation for the training process. All variables were converted to their fluctuating component by subtracting the mean (e.g. \(p' = p - p_{\text{mean}}\)). The sample (set) mean was used rather than the global mean to ensure that all data sets seen by the model have zero mean. The optimisation used in the training process works best when all point values and hence error values are of similar magnitudes, which is not the case for the computed wall pressure and wall shear stress fluctuations – for example the fluctuations of the streamwise component of wall shear stress are an order of magnitude larger than the spanwise component. To address this, variables were scaled by
their respective ranges \((\text{range}_\theta = \text{max}(\theta) - \text{min}(\theta))\) computed from the DNS over the whole data collection period. This scaling was applied consistently to all data.

Generalisation in space focuses on the reduction of reconstruction artefacts that appear around the field boundaries. These artefacts are a consequence of using a convolutional network on data with periodic boundaries. Here, the domain is extended on each boundary by wrapping a number of data points around from the opposite boundary. After reconstruction, the fields are then trimmed to the correct dimensions. Reconstructions from fields with a wrapping treatment of two extra grid points are compared to reconstructions from fields without wrapping.

The static convolutional neural network (SCNN) presented by Liu et al. (2020) was chosen, implemented in TensorFlow 2. As shown in table 1, this is a simple sequential network consisting of three convolutional layers followed by an up-sampling operation. Models were trained using the AMSGrad optimiser and run on a single NVIDIA GeForce RTX 3060. The trained models were then applied to the LR filtered DNS fields of a test set, and several statistics were computed from the reconstructions including the probability distribution function, one-dimensional energy spectra and two-point correlations. Statistics are compared to those computed from the corresponding DNS fields.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Filters (kernel size)</th>
<th>Activation</th>
<th>Output dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
<td>–</td>
<td>–</td>
<td>((m, H, W, C))</td>
</tr>
<tr>
<td>conv1</td>
<td>64 (5 × 5)</td>
<td>tanh</td>
<td>((m, H, W, 64))</td>
</tr>
<tr>
<td>conv2</td>
<td>64 (5 × 5)</td>
<td>tanh</td>
<td>((m, H, W, 64))</td>
</tr>
<tr>
<td>conv3</td>
<td>(r^2C) (5 × 5)</td>
<td>tanh</td>
<td>((m, H, W, r^2C))</td>
</tr>
<tr>
<td>upsample</td>
<td>–</td>
<td>–</td>
<td>((m, rH, rW, C))</td>
</tr>
</tbody>
</table>

Table 1. Summary of the SCNN network architecture. \(m\) is the number of examples and \(C = 3\) is the number of variables.

### 3 Results and Discussion

To ensure that the model can be applied to any other data set, and to avoid any ambiguity or data leakage, it is important to apply data transforms consistently, particularly scaling and de-scaling. All image data for example can be scaled by the range of pixel values, 255, but this is difficult for flow data because there are no strict bounds on the range of values the velocity can take and due to the chaotic nature of turbulence, range and mean values computed over short sampling periods can vary significantly. The range is thus computed over a very long period of time such that the probability of any point value exceeding the training range becomes vanishingly low \(O(10^{-9})\). Conversely, the sample mean was used for converting raw data to fluctuations. With these transforms, all input and output data is bound to approximately \(±0.5\) and each set is centred about zero. As in figure 1, the probability distribution functions (PDF) for the LR inputs approximately collapse, and all HR outputs can be directly compared. Differences in the PDFs are attributed to the sampling period. Variables were also non-dimensionalised by the viscous pressure scale \(u_\tau^2\) which should allow the models to generalise to solutions calculated using different numerical methods, such as LES.

The trained model is then applied to the test set data. Each variable is simultaneously predicted, as shown in the contour plots in figure 2, generated at a single snapshot in time. The reconstructions are de-scaled by the ranges used for training to recover the original magnitudes of the non-dimensional fluctuations. Qualitatively, the fields are reconstructed very well and accuracy of the model on the test sets is comparable to the accuracy reached on the validation set during training. Large-scale flow structures are accurately reconstructed because they are adequately resolved in the LR input. Within these large structures, the peak magnitudes of extreme events are reconstructed as well as most of the fine detail around these structures that was filtered out. The model also predicts intricate small-scale flow structures that are not associated with large-scale motions. The reconstructions do show some
inaccuracies that are common to all variables. Specifically, some fine small-scale flow structures and appendages are merged together, particularly for the fine streamwise shear stress filaments, and some of the magnitudes of the peak stress events are slightly under-predicted. This effect is somewhat amplified by the fixed contour thresholds used to highlight these small prediction errors. Despite this, peak stress events are reconstructed very well which is evident in the tails of the PDFs, as shown for wall pressure fluctuations in figure 3, and the model is able to nearly match the PDF of the DNS when computed over the whole test set.

The performance of the model over time is assessed by the one-dimensional energy spectra computed from the time series of instantaneous reconstructions from the test set data. Two important characteristics are observed in the energy spectra of the wall pressure fluctuations, shown in figure 4. Firstly the total energy is recovered well and is a near-exact match to the DNS up to the cut-off
wavenumber determined from the LR grid. Secondly, energy is recovered above the cut-off wavenumber that was not present in the LR fields, though energy becomes increasingly under-predicted from about the cut-off wavenumber. This behaviour is not unexpected, having been observed in Fukami et al. (2019) and others, and can be attributed to the loss of correlation between the predicted large- and small-scale structures particularly as structures approach the size of prediction errors. Models with higher complexity may be able to improve prediction of the small-scale structures that contribute to high-wavenumber energy, though preference here was given to lower training cost.

To test the generalisability of models in time, three models were trained on data of a fixed number of samples generated over each of the three time periods: $1t_f$, $1t_L$, and $10t_L$. Two test set periods were also considered over 1 or ten turn-over times. Each model was applied to both sets and assessed as above. In order to generalise in time, a trained model needs to maintain its predictive performance when applied to data sampled from the DNS at any arbitrary time. That is, test accuracy and loss (mean square error) should be approximately the same as for the validation set during training, and when assessed with the statistics above.

The turn-over time $t_L$ was chosen because of its physical interpretation that within the time scale for large motions (of characteristic size $\delta$ for channel flow) to repeat, all motions of smaller characteristic size should have occurred. It was found that one turn-over time was the minimum period over which
a trained model would generalise to snapshots at a different time, however there was a significant improvement in reconstruction performance when training on the $10t_L$ set. There was no further improvement for time periods beyond $10t_L$. Models trained on the single flow-through data exhibited signs of over-fitting with degraded performance and much higher loss on any test data. Loss curves for training each model are shown in figure 5. For super-resolution, the training examples should have some correlation to create redundancy for the model to distinguish important features, however sampling over a short period of time results in examples that are very highly correlated and the model does not get the broad experience of wall stresses it does in the longer sets. This is particularly true for the streamwise wall shear stress where the long meandering streaks remain highly correlated over long periods of time. Similarly, assessing models on the longer test set provided more stable statistics which is expected because 10 turn-over times should be sufficient for collecting most flow statistics. Some statistical ‘wobbles’ were evident when using the $1t_L$ test set.

This result is important because generating training and test data, while a good investment, is a costly and time-consuming process which should be minimised where possible. Though there is a clear advantage for using data over longer periods of time that allow samples to further decorrelate, training and testing over shorter periods of time is desirable and is shown to provide acceptable results.

**Figure 5.** Loss curves for model training using data over a period of (left-right): $1t_f$, $1t_L$, $10t_L$, and $20t_L$.

Generalisation in space is the ability of a model to be applied to any domain of arbitrary dimensions and boundary conditions without degradation in performance. Any model with convolution in at least the first layer will accept inputs with arbitrary dimensions due to the translational invariance of the convolution operation. The focus here however is on removing reconstruction artefacts around the field boundaries, examples of which are shown in figure 6. These artefacts are a consequence of training on data with periodic boundaries and influence the statistics used to assess the models because the reconstructions are not strictly periodic. If these reconstructions were used in an acoustics application, such artefacts would result in spurious noise. Though this is an issue that is specific to flows with periodic boundaries, removing boundary artefacts can be considered generalisation in space because the models learn boundaries as part of their feature map and any arbitrary domain will have boundaries when input to these models.

The cause of the artefacts was thought to be the addition of zero-padding around the fields that is required to maintain the field dimensions after convolution (under ‘dimensions’ in table 1). To address the issue, a wrapping method was tested that extended the domain on each boundary by duplicating a number of data points around from the matching periodic boundary, and then trimming the reconstructed fields to the original dimensions. It was found that wrapping just two grid points around the domain was able to significantly reduce the amount and severity of artefacts around the boundaries. This was evident from the two-point correlations, such as in the spanwise correlations of wall pressure fluctuations shown in figure 7, that the periodicity of the reconstructed fields is recovered as the correlation coefficient approaches 1 at nodes approaching the periodic boundary. Periodicity is not completely recovered and some artefacts are not completely removed because of other small
prediction inaccuracies present in the model.

Figure 7. Spanwise two-point correlations for wall pressure fluctuations from fields reconstructed with and without the wrapping method.

Extending the boundaries by two grid points was sufficient because the filters in the model are of size $5 \times 5$ grid points which requires a padding of two grid points. Of course, to maintain the extended dimensions after convolution, zero-padding of two grid points is still required, and this padding is still learnt by the model. The method works however because the boundary artefacts occur at the extended boundaries which are then removed when fields are trimmed to the original dimensions. Periodicity is recovered because there is continuity of flow features over the original domain boundaries. Continuity of flow features is also possible in the tiling or sub-region approaches used by others, provided the tiles have some amount of overlap, however in those approaches each tile or sub-region still has boundaries and boundary artefacts. Wrapping more grid points increased the training cost without improving statistics or removing more artefacts. While the intent is to apply SR to other flow configurations, this result is important because turbulent channel flow data remains a common choice for training data and here, statistics that depend on periodicity such as two-point correlations and energy spectra are significantly improved.
4 Conclusions

Super-resolution of wall pressure and wall shear stress is reported for the first time. Models are able to reconstruct fields from low-resolution inputs that recover most of the detail of the original DNS fields and with statistics that are in very good agreement with the DNS. Methods of data preparation are presented that demonstrate the ability of the models to be applied to any data sampled from the DNS and should allow the models to generalise to data generated from solutions with different numerical methods. A significant improvement in reconstruction performance was observed when training and testing with data over a long period of time with lower correlation between examples. In cases where generating training and test data over long period of time is also infeasible, shorter periods down to a minimum of one turn-over time may be sufficient. A wrapping method was also presented that reduced the effects of boundary artefacts in reconstructed fields and improved statistics that rely on periodicity. While this is specific to training data with periodic boundary conditions, channel flow data remains a common choice for training deep-learning models and any arbitrary flow domain these models are applied to will have boundaries.

Methods are shown that allow the trained super-resolution models to generalise in time, space, and numerical method which is important for their intended application to other flow configurations such as LES. The potential for this method to be applied to large eddy simulation will be investigated in future work.

References


