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Optimal tax enforcement with productive public inputs☆

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ABSTRACT

We study optimal public expenditure and tax enforcement in a simple one-sector, dynamic endogenous growth model where agents optimize consumption and evasion; evasion is costly, while public expenditure increases private capital productivity. We show that tax evasion costs and the efficiency of endogenous audits play a crucial role in determining the relationship between tax evasion, tax rates, public expenditure, and growth. The key elements to improve tax enforcement are efficiency in the audit process and increased productivity in public expenditure. Increasing tax evasion costs could reduce tax evasion, but when tax enforcement is inefficient, this might trigger a perverse effect in which a tax rate increase reduces tax revenue. This finding implies that government spending depends on the efficiency of the audit process: expanding government expenditure optimally or increasing private productivity is impossible without improvements in tax compliance.

1. Introduction

Tax evasion – defined as an illegal misrepresentation of the tax base to the tax authorities – is a severe problem in many developed and developing countries. Chaudhuri et al. (2006), Schneider and Buehn (2018), and Schneider (2012) show that the shadow economy – a good proxy for tax evasion – has been increasing in both OECD and transition economies. In Europe, the level of tax evasion is about 20% of the gross domestic product, accounting for a potential loss of about 1 trillion euros (EUR) each year (Buehn and Schneider, 2012; Murphy, 2014). Recent estimates by Feige and Cebula (2011) suggest that intentional under-reporting of income is about 18%–19% of the total reported income in the US, leading to a tax gap of about 500 billion US dollars (USD), which may increase to about one trillion USD if tax avoidance is considered (Davison, 2021).

Despite significant efforts, most countries are still far from decreasing this undesirable behavior. From a policy perspective, it is important to find the optimal balance between the tax rate and audit parameters so that governments can raise the revenue necessary to implement optimal public expenditure policies. This paper contributes to this debate by investigating the government's optimal tax enforcement policies (in terms of tax rates, audit frequency, and fines) in a context where the quality of the audit and public spending process may influence tax evasion. We develop a dynamic general equilibrium model where government spending contributes to economic growth by increasing the productivity of private capital while prohibiting public debt; the government maximizes welfare (and determines the best tax rate to that aim) but delegates tax collection and enforcement to a tax authority that sets the optimal level of audits and fines. Furthermore, both the taxpayer and the tax authority incur tax evasion-related costs.

We show that the optimal levels of audit and fines depend on the efficiency and the cost of the audit process. When tax audit is efficient, even in the presence of low tax evasion costs, the level of public expenditure is close to optimal. In contrast, inefficiencies and corruption reduce the ability of the government to enforce tax rules and lessen the optimal audit rates and fines. In other words, when the quality of the audit process is poor, tax enforcement through audits and fines is rather ineffective, and the government may increase noncompliance costs to reduce tax evasion; however, our model shows that if these costs are sufficiently high and tax enforcement is inefficient, a Laffer curve effect in the relationship between tax rate and tax revenue arises. That is, increasing the tax rate may result in higher tax evasion and a drop in tax collection. In this context, the only approach to reduce tax evasion is to combine improvements in tax administration with increased efficiency in public expenditure and tax rate cuts. From a policy standpoint, this situation implies that inefficient tax audits and public goods provision constrain effective tax enforcement. Furthermore, the efficiency of tax administration and noncompliance costs also affect the optimal size of government expenditure. We show that in an environment where raising tax rates increases tax evasion, the optimal tax rate is lower than the value established by Barro's (1990) rule. This

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finding reinforces the conclusion that an optimal fiscal policy depends on tax enforcement and noncompliance costs taxpayers face. Unless tax compliance is improved, expanding government expenditure is not feasible because the link between tax evasion and tax rates via public spending may be self-perpetuating. Tax evasion caused by increased tax rates reduces output growth due to falling productive public input, reinforcing the factors that facilitate tax evasion; in the long run, this may lead to production stagnation and a decline in taxable capacity.

The rest of this paper is organized as follows. In the following section, we show this paper's main contributions by comparing our approach to the existing literature. Section 3 presents our model and the main results of our analysis, Section 4 presents government tax policy choices to maximize growth, and Section 5 concludes.

2. Related literature

Traditional tax evasion models rely on a portfolio allocation process (Allingham and Sandmo, 1972; Yitzhaki, 1974, 1979) and are cast in a static framework. In these models, tax evasion yields the so-called "Yitzhaki's puzzle" by predicting that higher marginal tax rates encourage tax compliance. This outcome is rejected by most empirical studies, which suggest a clear positive relationship between taxes and tax evasion, or hidden economy, in general (Clotfelter, 1983, Crane and Nourzad, 1990, Alm et al., 1993, Giles and Caragata, 2001). In this context, the role of tax authorities is often overlooked, with some remarkable exceptions, such as Keen and Slemrod (2017), Paramonova-Kuchumova (2017), who argue that tax authorities could be close to the point where expanding tax enforcement is no longer optimal. In other words, tax enforcement efforts may lead to decreasing returns to this activity. Static models can only capture the reduction in the tax revenue and the change in the distribution of the tax burden. For example, Keen and Slemrod (2017) shows that optimal audit depends on the enforcement elasticity (which should be equal to the adjusted marginal cost-revenue ratio), implying that optimal compliance is always positive, and tax evasion would always result in some revenue loss.

Notably, the revenue loss is only one facet of tax evasion (Slemrod, 2007; Alm, 2012; Dzhumashev and Gahramanov, 2011; Markellos et al., 2016; Dzhumashev and Gahramanov, 2010). Tax evasion negatively affects the production of public goods and, ultimately, economic growth. From this perspective, static models fail to consider long-run consequences, which may be even more pervasive in the economy. Tax evasion may lead to stagnation due to a self-reinforcing mechanism when lower tax revenue further reduces the tax base (or increases the relative burden of public debt if deficits are allowed) and thus can perpetuate the bad growth outcome (Cooray et al., 2017; Turnovsky and Basher, 2009; Bardhan, 1997; Bekoe, 2012).

To date, few contributions have examined tax evasion and its implications for growth when public expenditure contributes to increasing private capital productivity. In particular, Celimene et al. (2016) show that tax evasion accompanied by corruption increases the volatility of growth. Caballé and Panadés (2007) find that when fines on tax evasion are proportional to evaded taxes, the economic growth rate decreases with higher tax rates; however, they do not consider the effect of tax evasion on growth through productive public inputs. Chen (2003) shows that tax evasion may be effectively discouraged by increasing the cost of tax evasion, establishing more severe punishments, and increasing tax auditing; however, these strategies do not produce significant growth effects unless the productivity of public goods is very high. We differ from these studies by considering the role of public sector spending in explaining tax evasion behavior.

The existing studies primarily investigate the nexus between tax evasion and growth by assuming that the tax administration parameters are exogenous. For example, Chen (2003) assumes that the cost of tax administration is a fixed share of total income, whereas Kafkalas et al. (2014) defines tax-audit expenditure as a constant share of tax revenues; however, the institutional structure and the efficiency and costs of the audit process are themselves factors in determining tax compliance. Audit effectiveness (i.e., the tax administration's capacity to detect noncompliance) is still understudied, although it appears to have important implications for tax evasion outcomes (Kasper and Alm, 2022).

We depart from this literature by choosing tax and audit parameters endogenous in a context where the audit rate may not coincide with the detection rate due to the efficiency of public institutions and corruption. In this way, we make endogenous the probability of audit, following Petrohilos-Andrianos and Xepapadeas (2016), and the probability of detection. The quality of institutions (i.e., the ability of the government to enforce rules) and their associated level of corruption creates a dependency between fines, audit rates, and probability of detection.[1] This dependency implies that tax evasion may flourish because of corrupt bureaucrats, (Marjit et al., 2017) which we duly consider. Along these lines, Cerqueti and Coppier (2011) study the role of institutional settings and local conditions on the evolution of corruption and tax evasion while Ivanyna et al. (2016) find that actions to reduce tax evasion are ineffective unless corruption is mitigated. Dzhumashev (2014) highlights the endogeneity of the tax audit process concerning economic development, which goes hand-in-hand with institutional development.[2]

Another distinct aspect of our model is that we consider a stochastic growth environment, where a rise in evasion and tax rates increases the risk on the taxpayer's capital. In a static model, the interaction between taxes and evasion only partially captures the risk associated with the evolution of the individual's capital profile. Following Levaggi and Menoncin (2013), we model auditing as a Poisson process (with constant intensity). We choose this formulation because while evasion is a continuous decision, the audit is a discrete process that happens only at given dates (and a finite number of times in any finite period); thus, this assumption seems to be more consistent with the actual discrete audit process rather than the continuous Wiener process used, for instance, in Dzhumashev and Gahramanov (2011) and Lin and Yang (2001).

Our analysis contributes to the current literature in several ways. First, the optimal levels of fines and audits are determined endogenously instead of assuming that they are exogenous, as most of the current literature does (Chen, 2003; Kafkalas et al., 2014). Second, accounting for the feedback from the public inputs to private productivity *a la* Barro (1990) in a dynamic setting allows us to highlight the crucial role of institutional quality in the tax evasion-tax decision. Third, our analysis reveals a novel link between Barro's (1990) rule for setting the optimal size of government expenditures and the "Yitzhaki puzzle" (Yitzhaki, 1974, 1979), which states that tax evasion increases with the nominal tax rate. Specifically, Barro's (Barro, 1990) rule only holds if costs from tax-evasion activities are sufficiently low so that a higher tax rate results in greater tax revenue. In this case, the best policy option is to set the tax rate close to the productivity of public expenditure, as Barro (1990) established.; however, this outcome falls into the so-called "Yitzhaki puzzle", which is rejected by the empirical evidence above. This puzzle can only be solved if the cost incurred to conceal tax evasion is sufficiently high.

3. The model

We model tax evasion in a dynamic, general equilibrium framework with a deterministic production function, whose productivity also depends on the level of public expenditure. A representative agent owns

[1] See Alm, 2012 for review. Furthermore, Bernasconi et al. (2015) find that audits and fines, not fiscal uncertainty, should be used to control tax evasion.

[2] Specifically, Dzhumashev (2014) finds that with economic development, the wage rate rises and makes private rent-seeking costs higher, discouraging tax evasion.

a production technology; the government produces a public good that improves private capital productivity, finances its activity using a linear income tax, and hires an agency for tax collection and audit activities. The consumer may conceal part of their income from the government by incurring a cost proportional to the evaded tax; they must pay any taxes owed along with a fine if they are audited and found to be evading taxes. The audit process is delegated to the tax agency, which chooses the audit parameters to maximize the expected tax revenue; the agency observes the average level of tax evasion, not the decision-making process of the taxpayers. The audit's outcome depends both on the resources spent by the government and on the efficiency of the tax authority in making the audit. In this setting, the government sets the tax rate to maximize social welfare, defined as the sum of the lifetime utility of the representative consumer.

3.1. The environment

A representative agent is endowed with an initial amount of capital k_0, which is used in a production function, together with public expenditure g_t. The production function has the following Cobb–Douglas form

$$y_t = \bar{A} g_t^{\psi} k_t^{1-\psi}, \tag{1}$$

where $\bar{A}$ is the total factor productivity, $\psi \in (0,1)$ is the constant output elasticity of public expenditure, and $1-\psi$ is the output elasticity of capital. The government levies a statutory tax τ on income y_t to finance public expenditure and the cost of tax collection and audits. Audit and tax collection activities are devolved to the tax authority, which is assumed to be a perfect government agent.

The agent may decide to conceal a fraction e_t of the income; thus, they would evade taxes in the amount of $\tau e_t y_t$. However, as in Chen (2003) and Dzhumashev and Gahramanov (2010, 2011),[3] the taxpayer incurs a private, time-independent cost of s for each unit of income evaded $e_t y_t$ (we assume $s < \tau$). These costs include the choice of the best tax regime to pursue tax evasion and price discounts, sell products on the black market (Davidson et al., 2007), and the costs relating to concealing illegal income, as explained by Cowell (1990), and Hillman (2009). These costs generally increase with both the degree of tax evasion and the income level.[4] In light of these intuitions, parameter s captures the positive effect of income level and evasion rate on the cost. Finally, if a taxpayer is audited and the tax authority detects evasion, they must pay a fine at a rate $\theta > 1$ proportional to the tax evaded as in Yitzhaki (1974).

3.2. The audit and tax collection process

Taxes are collected, and the tax authority conducts tax audits. The tax authority is an agency hired by the government, and its role is to maximize the tax revenue for a given level of the tax rate (Rablen, 2013). This paper incorporates some of the real-world features of the fiscal systems, where audits do not necessarily mean that the taxpayer will ultimately pay taxes (see Kasper and Alm, 2022 for a review). As in Rablen (2013), we assume that the probability of paying what is due (plus a fine) depends on two different processes: (a) the frequency of audits and (b) the fraction of audits that successfully detect evasion. Following Levaggi and Menoncin (2013, 2011) and Bernasconi et al. (2015), the event of an audit is represented by a Poisson jump, allowing us to capture the binary nature of the audit process. A Poisson jump $d\Pi_t$ is characterized by having the same expected value and variance

$$\mathbb{E}\left[d\Pi_t\right] = \mathbb{V}\left[d\Pi_t\right] = \lambda \cdot dt,$$

where λ measures the frequency of tax audits.

Following an audit, we assume that only a fraction $p_d \in (0,1)$ of the evaded tax revenue is successfully detected because of inefficiencies and corruption. Examining the factors determining the value of p_d reveals that the probability of detection is higher if tax codes are simple and less ambiguous, the quality of the auditors is good, and the courts have higher effectiveness (Slemrod and Yitzhaki, 2002; Balios and Tantos, 2019). We model these characteristics through the probability p, which depends on the quality of public institutions and is a given parameter. That is, a better tax administration stemming from a good institutional system increases the efficiency of tax audits and allows for the detection of tax evasion more frequently; however, we depart from the existing literature by explicitly modeling the endogenous part of the probability of detection. We assume the following form for the probability of detection.

$$p_d = p + f,$$

where the second term f captures the relationship with fines and other audit parameters.

We next examine how fines may influence the effectiveness of audits. Given that the tax authority maximizes tax revenue, higher fines should increase the amount of taxes collected, i.e., they should create an incentive to improve tax evasion detection; however, assuming that fines θ would linearly increase the probability of detection would be simplistic. First, according to Allingham and Sandmo (1972), if detection is likely and penalties are severe, taxpayers reduce the evasion rate. Furthermore, the higher the fine, the more effort the tax evaders exert in tax-concealing activities (Polinsky and Shavell, 2001). Moreover, Paramonova-Kuchumova (2017) finds that the tax enforcement efforts may reduce returns on detection. We model these behavioral changes by assuming that the marginal increase in the fine's effect on the detection probability is decreasing.

Based on the above intuition, the contribution of the penalty rate to the detection probability can be written as

$$f = \pi \theta^{-\xi}, \tag{2}$$

where $\pi > 0$ captures the positive effect of a change in the fine on the detection probability. The intuition is that higher penalty rates motivate greater detection of tax evasion. Since $-\xi < 0$, this effect is diminishing in θ. The higher the value of ξ, the lower the contribution of the fine to the probability of detection for the given fine rate. Parameter ξ reflects the inefficiency of the tax administration. If ξ is low, the penalty for evasion becomes a stronger deterrent to tax evasion, even in the presence of imperfect audit instruments.

From the above discussion, we write the overall detection probability as follows:

$$p_d = p + \pi \theta^{-\xi}. \tag{3}$$

Finally, we assume the existence of collection and audit costs. The pre-audit collection costs are proportional to gross tax revenue; thus, they depend on tax evasion: $\mu(1-e_t)\tau y_t$, where $\mu \in [0,1]$ is constant through time. Audit costs are assumed to be quadratic in the frequency of the audits (λ) and proportional to the amount of tax evasion and fines ($\theta \tau e_t y_t$);[5] We justify this assumption by a reasonable possibility that an increase in the frequency of audits may require increased efforts

[3] We depart from Lamantia and Pezzino (2021), Bethencourt and Kunze (2019), and Luttmer and Singhal (2014) who use non-pecuniary costs stemming from social and cultural norms and tax morale that affect the decision to evade taxes because it requires substantial changes in the standard utility function.

[4] For example, Barreto (2000) explains this link intuitively; as the size of illegal income rises, it becomes increasingly difficult to conceal it.

[5] This is a standard way to model the costs increasing in the related effort; see, for example, Chen, 2003.

to detect marginal tax noncompliance.[6] Thus, the total cost of the collection and audit process is

$$C(\theta,\lambda) = \mu(1-e_t)\tau y_t + \frac{\omega}{2}\theta\tau e_t y_t \lambda_t^2 = \tau y_t\left(\mu(1-e_t) + \frac{\omega\theta e_t}{2}\lambda^2\right), \tag{4}$$

where $\frac{\omega}{2}$ is the parameter associated with the running costs.

3.3. Capital dynamics

The consumer uses the net income flow in Eq. (1) to increase their capital, k_t, and to buy consumption goods c_t. If they also decide to evade a fraction e_t of their income, the after-tax stochastic capital dynamics can be written as

$$dk_t = \left[\left(1-\tau+e_t\tau - se_t\right)y_t - c_t\right]dt - \theta\tau e_t y_t p_d\, d\Pi_t, \tag{5}$$

where c_t is consumption, and the expected value is

$$\mathbb{E}_t\left[dk_t\right] = \left[\left(1-\tau+e_t\tau - se_t - \lambda p_d e_t\theta\tau\right)y_t - c_t\right]dt.$$

The actual average tax rate ($\tilde{\tau}$) paid by the agent solves the equation

$$\left(1-\tau+e_t\tau - se_t - \lambda p_d e_t\theta\tau\right)y_t = (1-\tilde{\tau})\,y_t,$$

which can be written as

$$\tilde{\tau} = \tau - \left(\tau\left(1-\theta\lambda p_d\right) - s\right)e_t. \tag{6}$$

Note that $\tilde{\tau}$ differs from the effective tax rate. The average tax rate paid by the agent is the actual burden of the tax-related payments the agent makes, which includes the tax liability itself and related costs stemming from the tax evasion activity. The agent knows that the government sets public expenditures with the zero-balance rule but perceives collection and audit costs $C(\theta,\lambda)$ as a form of inefficiency. The true budget constraint for the government is

$$\tau\left(1-e_t\right)y_t + \lambda\theta\tau e_t y_t p_d = g_t + C(\theta,\lambda),$$

which can be written as

$$\tau\left(1-e_t\right)y_t + \lambda\theta\tau e_t y_t p_d = g_t + \tau y_t\left(\mu(1-e_t) + \frac{\omega\theta e_t}{2}\lambda^2\right). \tag{7}$$

The government's net revenue can be written as

$$\tau(1-E)\,y_t = g_t, \tag{8}$$

where $\tau(1-E)$ is the effective tax rate for the economy, and E is a measure of the revenue lost because of tax collection costs, tax evasion, and tax audits.

Thus, by definition, we have $E \in (0,1)$, which is a measure of the compliance gap, i.e., the difference between the amount of tax legally due and that collected $(1-E)$ (Keen and Slemrod, 2017). Even though taxpayers know that evasion reduces revenue, in line with the classical theory of public goods provision and free riding, they think their contribution to public expenditure is marginal enough not to change the tax revenue; they interpret E as a measure of inefficiency, which reduces the productivity (in terms of public expenditure) of their tax effort. In other words, the taxpayer would expect each unit, τy_t, of tax revenue to produce public expenditure; the actual revenue is only $\tau(1-E)\,y_t$, which is interpreted as a measure of the government's inability to run an efficient system.

If we substitute the relationship given by (8) into the production function (1), we obtain

$$y_t = \left(\bar{A}\left(\tau(1-E)\right)^{\Psi}\right)^{\frac{1}{1-\Psi}} k_t,$$

which can be written as

$$y_t = Ak_t, \tag{9}$$

where

$$A := \left(\bar{A}(1-E)^{\Psi}\,\tau^{\Psi}\right)^{\frac{1}{1-\Psi}}. \tag{10}$$

Given that the agent perceives E as independent of their behavior, they believe their capital evolves with the standard rules of an AK production function given as (9); thus, this equation is used in their maximization problem.

3.4. The solution to the taxpayer's problem

The representative consumer maximizes their lifetime log utility and chooses their optimal levels of consumption and tax evasion to solve the inter-temporal problem.

$$\max_{\{c_t,e_t\}_{t\in[0,\infty[}} \mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t}\ln c_t dt\right],$$

where ρ is the subjective constant discount rate. This problem is subject to the budget constraint

$$\begin{aligned} dk_t = &\left[\left(1-\tau+(\tau-s)\,e_t\right)\left(\bar{A}(1-E)^{\Psi}\,\tau^{\Psi}\right)^{\frac{1}{1-\Psi}} k_t - c_t\right]dt - \\ &-\theta\tau e_t\left(\bar{A}(1-E)^{\Psi}\,\tau^{\Psi}\right)^{\frac{1}{1-\Psi}} k_t p_d\, d\Pi_t. \end{aligned} \tag{11}$$

The solution to this problem yields the following:[7]

$$\begin{aligned} c_t^* &= \rho k_t, \\ e_t^* &= \frac{1}{\theta p_d\left(\bar{A}\tau(1-E)^{\Psi}\right)^{\frac{1}{1-\Psi}}}\left(1-\frac{\lambda p_d\theta\tau}{\tau-s}\right). \end{aligned} \tag{12}$$

The capital accumulation of the agent is given by

$$\begin{aligned} \frac{dk_t^*}{k_t^*} = &\left[(1-\tau)\left(\bar{A}(1-E)^{\Psi}\,\tau^{\Psi}\right)^{\frac{1}{1-\Psi}} + \frac{\tau-s}{\theta p_d\tau} - \lambda - \rho\right]dt \\ &-\left(1-\frac{\lambda p_d\theta\tau}{\tau-s}\right)d\Pi_t, \end{aligned}$$

whose expected value yields the steady-state growth rate for the economy,

$$\gamma := \frac{1}{dt}\mathbb{E}_t\left[\frac{dk_t^*}{k_t^*}\right] = (1-\tau)\left(\bar{A}(1-E)^{\Psi}\,\tau^{\Psi}\right)^{\frac{1}{1-\Psi}} - \rho + \frac{\tau-s}{\theta\tau p_d} - 2\lambda + \frac{\theta\tau p_d}{\tau-s}\lambda^2. \tag{13}$$

The optimal level of consumption is a constant fraction of capital, which coincides with the subjective discount rate ρ, as in any log-preference model. The optimal tax evasion rate is constant over time and falls with the intensity of the audit, the fine rates, and the productivity and costs of tax evasion. Interestingly, the perceived inefficiency of government activity (E) increases tax evasion, as one might expect; however, from Eq. (12), we know that tax evasion also contributes to the growth of E, which might lead to a self-fueling effect of tax evasion. An internal solution to the optimal tax evasion problem implies that e_t must lie in the interval $[0,1]$. For this condition to hold, the audit intensity should satisfy[8]

$$\frac{\tau-s}{p_d\theta\tau}\left(1-\theta\tau p_d\left(\bar{A}(1-E)^{\Psi}\,\tau^{\Psi}\right)^{\frac{1}{1-\Psi}}\right) < \lambda < \frac{\tau-s}{p_d\theta\tau}. \tag{14}$$

Condition (14) is interpreted as follows. If the frequency of audit λ is sufficiently high ($\lambda > \frac{\tau-s}{p_d\theta\tau}$), the expected loss from fines is higher than the gain from evasion; thus, there is no incentive

[6] Keen and Slemrod (2017) show that the additional revenue gained from stricter enforcement equals the associated additional compliance and administration costs; however, we differ from this paper by explicitly modeling the effect of enforcement on tax compliance.

[7] See Appendix A.

[8] The value of λ can be higher than 1 since λ is an intensity not a probability.

to evade. Conversely, if the frequency λ is sufficiently low ($\lambda < \frac{\tau-s}{p_d\theta\tau}\left(1-\theta\tau p_d\left(\bar{A}(1-E)^{\psi}\tau^{\psi}\right)^{\frac{1}{1-\psi}}\right)$), evading the full tax liability is beneficial. When the condition given by (14) is satisfied, tax evasion is expedient. Condition $\lambda > \frac{\tau-s}{p_d\theta\tau}$ can also be written in terms of the tax rate. That is, evasion occurs only if the tax rate τ is higher than τ_u:

$$\tau > \tau_u := \frac{s}{1-\lambda p_d\theta}, \tag{15}$$

where τ_u denotes the threshold value given on the right-hand side of (14). In other words, for $\tau > s$ and $1-\lambda p_d\theta > 0$, if the tax rate, τ, set by the government is higher than the threshold τ_u, evasion will occur; given the costs associated with tax evasion, the taxpayer could expect a positive gain from tax evasion. In other words, τ_u represents the value of the statutory tax rate for which tax evasion is non-expedient.

From (15), this threshold depends positively on tax evasion cost parameter s and the effective cost of audits $\lambda p_d\theta$. This mechanism is in line with Yitzhaki's (1974) outcome, implying that we observe greater tax evasion with higher tax rates. Moreover, the existence of such a threshold, driven by the costs stemming from the economy's institutional structure, offers an additional explanation for why tax evasion rates are widely different across countries. Conditions (14) and (15) show that the extent of tax evasion depends on the economy's features (including the efficiency of public institutions and the productivity of the private sector), combined with the fiscal policy stance in a given economy. This situation implies that no simple, one-size-fits-all policies exist to reduce tax evasion, as economies differ in terms of the efficiency of public institutions, the productivity of private capital, and fiscal-policy parameters.

We next consider some comparative statics based on the optimal values of tax evasion. The derivative of e_t^* concerning τ allows us to determine the reaction function of the private sector to a change in the tax rate; this problem has long been debated in the literature, beginning with the so-called "Yitzhaki (1974) puzzle". For our model, we can write the elasticity of e_t^* with respect to τ as (we recall that E is independent of τ)

$$\frac{\partial e_t^*}{\partial\tau}\frac{\tau}{e_t^*} = -\frac{1}{1-\psi} + \frac{\lambda p_d\theta\tau s}{(\tau-s)^2}\frac{1}{1-\frac{\lambda p_d\theta\tau}{\tau-s}}. \tag{16}$$

For $s = 0$, this derivative is negative; however, for s, which is sufficiently high, the sign of the derivative may change. In fact, from Eq. (16), if

$$s > \tau\left(1-\frac{\psi}{2(1-\psi)}\lambda p_d\theta\left(\sqrt{1+4\frac{1-\psi}{\psi\lambda p_d\theta}}\right)\right), \tag{17}$$

the effect of an increase in the tax rate on tax evasion is positive.[9] This result reconciles theory with empirical evidence by showing that for $s > 0$, the Yitzhaki puzzle may not be confirmed. From a policy perspective, the elasticity of tax evasion concerning s, which is the private cost that the taxpayer incurs while evading taxes, is worth considering:

$$\frac{\partial e_t^*}{\partial s}\frac{s}{e_t^*} = -\frac{\lambda p_d\theta\tau s}{(\tau-s)^2}\frac{1}{1-\frac{\lambda p_d\theta\tau}{\tau-s}}.$$

This elasticity is always negative when evasion is expedient. In other words, an increase in s reduces the level of tax evasion, but it may change the sign of the derivative concerning the tax rate (see (16)) from negative to positive. The level of tax evasion costs for which the derivative is positive is itself increasing in the tax rate and decreasing in p_d (i.e., the efficiency of audit). The relationship between e_t^* and ξ is complicated; whether this is a negative or positive relationship depends on the level of the other parameters of the model. We consider some possible properties of this relationship in the context of the tax administration problem in Section 3.5.

The condition for a stable equilibrium, obtained by merging (7) and (8), can be written as

$$E^* = \mu(1-\bar{e}_t) + \left(1-\lambda\theta p_d + \frac{\omega\theta}{2}\lambda^2\right)\bar{e}_t, \tag{18}$$

where the mean of e_t, denoted by $\bar{e}_t$, is represented by Eq. (12). In equilibrium, we can write:

$$E^* = \mu(1-\bar{e}_t) + \frac{1-\lambda\theta p_d + \frac{\omega\theta}{2}\lambda^2}{\theta p_d\left(\bar{A}\tau(1-E^*)^{\psi}\right)^{\frac{1}{1-\psi}}}\left(1-\frac{\lambda p_d\theta\tau}{\tau-s}\right),$$

which implies that E^* has an implicit solution.

In the equation above, all the variables are time-independent; hence, the value E^* is constant. In deciding their optimal evasion (12), the taxpayer interprets E as a measure of government inefficiency and reacts with tax evasion. This situation means that the long-run equilibrium $\bar{e}_t$ depends on how the government sets an optimal tax policy and determines $E = E^*$; in contrast, an individual taxpayer thinks that their tax evasion does not affect the value of E. This perception causes tax evasion to be self-reinforcing—a rise in tax evasion lowers A (see Eq. (12)), which reinforces the persistence of evasion in the long run by increasing the value of E over time (see Eq. (18)). Thus, a self-reinforcing channel through productive input of the public sector can push tax evasion further up. This result might lead to a vicious cycle of tax evasion and low public input for less developed countries with relatively low public productive inputs. The marginal effect of productive public input on total factor productivity may be stronger; therefore, a decrease in productive public input might have a greater impact on tax evasion.

3.5. The tax authority problem

The tax authority is an agency hired by the government whose objective is to maximize the amount of tax revenue for the given tax parameters. It observes an average evasion level $\bar{e}$, and chooses the audit parameters that maximize the expected revenue for the rate, τ, set by the government. If we denote by dT_t the government's net revenue in each instant, we can write the following:

$$dT_t = \tau\left(1-e_t\right)y_t dt + \theta\tau e_t y_t\left(p+\pi\theta^{-\xi}\right)d\Pi_t - \tau y_t\left(\mu(1-e_t) + \frac{\omega\theta e_t}{2}\lambda^2\right)dt,$$

the expected value of which is

$$\mathbb{E}_t\left[dT_t\right] = \tau y_t\left[1-e_t+\theta e_t\left(p+\pi\theta^{-\xi}\right)\lambda - \mu(1-e_t) - \frac{\omega\theta e_t}{2}\lambda^2\right]dt. \tag{19}$$

Therefore, the problem of the tax administration is given by

$$\max_{\lambda,\theta}\mathbb{E}_t\left[dT_t\right].$$

The solution to this tax administration problem yields the following:

$$\lambda^* = \frac{p}{\omega}\frac{2\xi}{2\xi-1}, \tag{20}$$

$$\theta^* = \left(\pi\frac{2\xi-1}{p}\right)^{\frac{1}{\xi}}.$$

The first equation shows the optimal frequency of audits, which is increasing in the quality of the audit system (p) and decreasing in its cost (ω), as expected; however, it is also decreasing in ξ. The optimal

[9] By solving for s the equation $\frac{\partial e_t^*}{\partial\tau}\frac{\tau}{e_t^*} = 0$, we can find the conditions for the existence of a positive derivative:

$$1+\frac{\psi}{2(1-\psi)}\lambda p_d\theta\left(1-\sqrt{1+4\frac{1-\psi}{\psi\lambda p_d\theta}}\right) < \frac{s}{\tau} < 1+\frac{\psi}{2(1-\psi)}\lambda p_d\theta\left(1+\sqrt{1+4\frac{1-\psi}{\psi\lambda p_d\theta}}\right).$$

However, the upper limit of the existing set is incompatible with the restriction, $s < \tau$; hence, the condition is derived in the text.

fine is decreasing in p because a more efficient tax system does not require such incentive to detect more tax evaders; it is increasing in π and decreasing in ξ.[10]

The optimal levels of audit λ^* and fine θ^* depend on the efficiency and the cost of the audit process; however, they are not related to the income level, which implies that (λ^*, θ^*) are time-invariant.[11] Finally, since $p_d = p + \pi\theta^{-\xi}$, from Eq. (20) we get

$$p_d = \frac{p\xi}{\xi - 1/2}. \tag{21}$$

Given that $0 < p_d < 1$, (21) implies that $\xi > \frac{1}{2}$. As ξ rises, p_d decreases, which is in line with the assumption of a decreasing marginal effect of the fine; hence, ξ is a proxy for the inefficiency in tax administration.

After substituting the optimal values (λ^*, θ^*) into the threshold of the tax rate τ_u defined in Eq. (15), we obtain the following:

$$\tau_u = \frac{s}{1 - \frac{4\xi^2\pi^{\frac{1}{\xi}}}{\omega}\left(\frac{p}{(2\xi-1)}\right)^{\frac{2\xi-1}{\xi}}}. \tag{22}$$

Eq. (22) shows that the level of the tax rate compatible with zero evasion ($\tau = \tau_u$) depends on the efficiency and the cost of audits and on the private costs that the taxpayer incurs while evading. Interestingly, for $s = 0$, there is no positive tax rate compatible with zero tax evasion. In (22), if ξ is low, the fines are an effective deterrent to tax evasion even in the presence of less efficient audit instruments; when ξ is low enough, it pays to increase detection through higher fines according to Eq. (20). Furthermore, when ξ is low, the tax rate threshold, τ_u, compatible with the no-tax-evasion outcome increases. In other words, the government can afford to impose a higher tax rate without pushing the taxpayers into tax evasion activities. Moreover, a sufficiently efficient audit process (p is sufficiently high) can be a suitable deterrent even if the tax administration is inefficient because the threshold τ_u is rather high. In summary, the quality of the audit process and the inefficiency of the tax inspectors are crucial factors to consider when seeking to reduce tax evasion; therefore, improving the institutions that determine the audit quality must be the focus of any policy to reduce tax evasion.

Given the stochastic nature of the tax-collection process, the expected tax revenue is given by

$$\mathbb{E}_t\left[dT_t\right] = \bar{\tau} y_t dt,$$

where the effective tax rate $\bar{\tau}$ can be written as

$$\bar{\tau} := \tau\left(1 - e_t + \theta e_t\left(p + \pi\theta^{-\xi}\right)\lambda - \mu(1 - e_t) - \frac{\omega\theta e_t}{2}\lambda^2\right). \tag{23}$$

Now, we examine the reaction of the effective tax rate, $\bar{\tau}$, to an increase in the nominal tax rate τ. The derivative is

$$\frac{\partial\bar{\tau}}{\partial\tau} = \frac{\bar{\tau}}{\tau} - \frac{\partial e_t}{\partial\tau}\frac{\tau}{e_t}\left(1 - \mu - \frac{\bar{\tau}}{\tau}\right). \tag{24}$$

If the elasticity $\frac{\partial e_t}{\partial\tau}\frac{\tau}{e_t}$ is negative and the cost parameter μ is sufficiently small, the derivative given by (24) is positive. It becomes negative if the elasticity, $\frac{\partial e_t}{\partial\tau}\frac{\tau}{e_t}$, is positive and higher than the following threshold

$$\frac{\partial e_t}{\partial\tau}\frac{\tau}{e_t} > \frac{\frac{\bar{\tau}}{\tau}}{1 - \mu - \frac{\bar{\tau}}{\tau}}.$$

As shown above, if s is "sufficiently high" (see Eq. (17)), $\frac{\partial e_t}{\partial\tau}\frac{\tau}{e_t}$ is positive, indicating that an increase in the tax rate may decrease tax revenue. This mechanism results in an effect similar to that described by the Laffer curve (Wanniski, 1978; Sanyal et al., 2000). If this is the case, increasing the tax rate reduces government spending and the economy's aggregate productivity.

4. Welfare maximization

In an ideal world, a benevolent government maximizes taxpayers' welfare. In a model of endogenous growth, the tax rate that maximizes welfare coincides with the one that maximizes growth (see Appendix B and Dzhumashev, 2014). In the absence of tax evasion, the growth(welfare)-maximizing tax rate is equal to the output elasticity of public expenditures (i.e., the first-best optimal tax rate, $\tau^{FB} = \psi$) as shown in Appendix B. This situation is the benchmark on which to evaluate government fiscal policies. The following subsections consider the effect of tax administration and audit on the optimal fiscal policy.

4.1. Efficient tax authority

Tax evasion reduces growth by decreasing the tax revenue for a given tax rate; therefore, a benevolent government should first consider if its tax authority is efficient enough to eradicate tax evasion. The key variable in this context is τ_u, which represents the tax rate value for which tax evasion is non-expedient. In fact, from Eq. (22), we know that any

$$\tau < \tau_u = \frac{s}{1 - \frac{4\xi^2\pi^{\frac{1}{\xi}}}{\omega}\left(\frac{p}{(2\xi-1)}\right)^{\frac{2\xi-1}{\xi}}}$$

is compatible with $e_t = 0$. This implies that if $\tau_u > \psi$, an audit is sufficiently efficient to stop tax evasion, and $\tau^* = \psi$ is the optimal level of the tax rate. In this case, public expenditure is suboptimal only due to the presence of the fixed audit and collection costs μy_t. Although the tax evasion rate is zero, an audit is still necessary to prevent such behavior, and from (23), we obtain $\bar{\tau} = \psi(1 - \mu)$. Thus,

$$\tilde{\tau} = \psi \neq \bar{\tau} = \psi(1 - \mu).$$

is compatible with $e_t = 0$.

Since public expenditure is suboptimal, we conclude that the growth rate is lower than in the first-best case; however, the optimal tax rate is still equal to the rate that maximizes private capital growth. In other words, extracting resources from the private sector is optimal, provided they are efficiently employed as publicly-supplied input to improve capital productivity. Thus, an economy with higher productivity of public expenditures can set a higher optimal tax rate; however, this optimal result may not be achieved if the efficiency of the tax administration does not allow $\tau^* = \psi$. We consider the implications of having such low-efficiency tax administration in the next subsection.

4.2. Inefficient tax authority

If $\tau_u < \psi$, the relationship between tax revenue and the tax rate must be examined to determine the optimal rate. As Section 3.4 shows, the level of the private evasion costs becomes important in this context.

We first consider the optimal fiscal policy when s is sufficiently high; an increase in the tax rate raises tax evasion, which in turn increases the cost of an audit and causes a reduction in government revenue. In this case, tax evasion becomes self-fueling; the reduced productivity due to lower government inputs (captured by a decrease in the technology coefficient A; e.g., see (10)) increases tax evasion (see (13)), which decreases tax revenue even more. Other things being equal, the higher the level of tax evasion, the lower the growth rate without any benefit to the private agent. Such a reduction in government revenue produced by tax evasion is similar to the effect captured by the Laffer curve (Wanniski, 1978; Sanyal et al., 2000). In this scenario, the government may

[10] The derivatives of λ and θ are respectively:

$$\frac{\partial\lambda^*}{\partial\xi} = -\frac{2p}{\omega(2\xi - 1)^2},$$

$$\frac{\partial\theta^*}{\partial\xi} = ln\left(\frac{\pi(2\xi - 1)}{p}\right)(2\xi - 1) < 0.$$

[11] From a more general perspective, the income level might be related to the quality of institutions; however, our arrangement does not consider that possible channel.

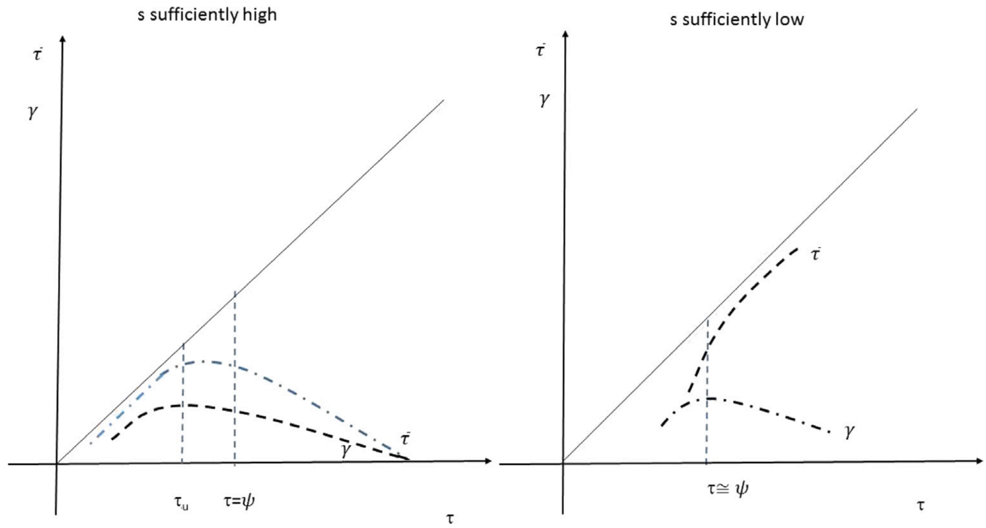

Fig. 1. Statutory tax rate, growth, and actual tax rate.

increase its tax revenue only by reducing the tax rate to the point where the tax administration parameters allow it to drive the evasion rate to zero. That is, the optimal government policy is to set $\tau^e = \tau_u < \psi$, which is the tax rate that maximizes growth in this second-best environment. Furthermore, this tax rate also allows us to achieve a long-run equilibrium because $E = 0$.

Fig. 1 provides some insights into the role of s in the optimal tax rate setting, where the actual tax rate is drawn for different levels of the nominal tax rate. On the horizontal axis, we depict the nominal tax rate τ, while the vertical axis depicts the effective tax rate $\bar{\tau}$ and the growth rate γ. The diagram on the left is drawn for a level of s compatible with $\frac{\partial \bar{\tau}}{\partial \tau} < 0$, while the diagram on the right is drawn for $s = 0$. When s is sufficiently high, tax evasion creates an effect similar to the Laffer curve. When the nominal tax rate is below τ_u, the effective tax rate $\bar{\tau}$ increases (i.e., the dash-dotted line in Fig. 1) because the tax evasion rate is zero. When the nominal tax rate is above τ_u, an increase in the tax rate raises tax evasion and decreases the effective tax rate $\bar{\tau}$. The highest growth rate (and associated optimal tax rate) are obtained for the highest level of the tax rate compatible with zero tax evasion (i.e., the dashed line in Fig. 1). The optimal tax rate will be lower than that without tax evasion.

When s is sufficiently low, an increase in the tax rate raises tax revenue because the latter reduces tax evasion. Let us consider the extreme case and assume $s = 0$; in this case, the government's problem can be written as

$$\max_{\tau} \gamma = (1-\tau)\left(\bar{A}\bar{\tau}^{\psi}\right)^{\frac{1}{1-\psi}} - \rho + \frac{1}{\theta p} - 2\lambda + p\theta\lambda^2, \tag{25}$$

which in the long-run equilibrium can be written as

$$\bar{\tau} = \tau(1-\mu-E)$$
$$= \tau(1-\mu) - \frac{1-\left(p_d - \frac{\omega\lambda}{2}\right)\theta\lambda}{\theta p_d A}\left(1-\lambda p_d \theta\right),$$

and the maximum of γ is reached when the following condition holds:[12]

$$\tau^* \simeq \psi.$$

In the real world, it is realistic to assume $s > 0$. Therefore, in practice, it is crucial to determine what the government must do when $s > 0$, but sufficiently low to get $\frac{\partial \bar{\tau}}{\partial \tau} > 0$. In this case, the government should set the tax rate to maximize Eq. (25), i.e., the best policy option is still to set the tax rate to a level similar to the output elasticity of public expenditure *a la* Barro (1990). As discussed above, a closed-form solution cannot be obtained for an optimization problem in this case; however, several simulations using a value for s compatible with an increase in the tax revenue show that the rule $\tau^* \simeq \psi$ is still valid.[13]

We next illustrate the case when s is sufficiently low using the right-hand side panel of Fig. 1. In this case, the actual tax rate – and hence, the tax revenue – increases with τ, but growth starts to decrease for values of τ higher than the output elasticity of public input ψ. This situation implies that if any tax rate increase allows the collection of more tax revenue and a fall in economic growth rate accompanies such policy change, the tax burden must be greater than the optimal level. As mentioned above, this condition drives the "Yitzhaki puzzle". Nonetheless, the empirical evidence indicates that the real-world tax systems appear to have sufficiently high noncompliance cost of s; thereby, a rise in tax rates is associated with an increase in tax evasion.

5. Discussion and conclusions

Tax evasion is a severe problem for many developing and developed countries. Our model proposes a framework wherein growth, government inputs, tax evasion, and tax enforcement can be studied together. We show that tax evasion may be self-reinforcing even in an environment where the government is benevolent. Inefficiencies in tax administration cause agents to engage in tax evasion activities that allow them to increase their present income at the cost of reducing growth in the long run. Our model highlights the role of private costs incurred to evade income taxes; in particular, if these costs are sufficiently high, increasing the tax rate as a response to tax evasion produces a self-reinforcing mechanism, the final result of which is a decrease in tax revenue.

The economy may find itself in a vicious circle where an increase in tax evasion decreases government inputs, leading to more tax evasion if the government tries to increase the tax rate. In this case, our model suggests that the government should try to reduce tax rates in the short run and improve the efficiency and productivity of public goods rather than only engaging in audit activities. This result aligns with the recent literature on the relationship between corruption, tax evasion, and economic growth. An increase in the frequency of audits and fines may not lead to the expected results unless corruption-induced inefficiencies are first addressed (Ivanyna et al., 2016; Rijckeghem and Weder, 2001; Kafkalas et al., 2014). By increasing the intrinsic motivation of the

12 See Appendix C.2.

13 The simulations are not given here but can be obtained from the authors.

staff, (Kadous and Zhou, 2019) corruption-induced inefficiencies can, in turn, be reduced, ensuring that public officials are less corruptible.

From a policy perspective, it is interesting to study another form of incentives that the government can use to reduce tax evasion. As Section 3.4 indicates, private costs related to evasion can reduce the latter since the derivative of e concerning s is decreasing. This indicates that one way to fight tax evasion might be to increase the associated cost by making it harder to spend income from tax evasion. In this respect, it may seem that policies aimed at reducing the use of cash for payments, as well as some form of audit based on conspicuous consumption (Levaggi and Menoncin, 2016), may produce the right incentives to curb tax evasion.

Our analysis adds an interesting result to this debate. An increase in the cost of tax evasion, captured by s, may reduce the level of tax evasion; however, it should also be associated with a study on the effects of such an increase on the marginal change in tax revenue caused by an increase in tax rates. If the increase in s changes the derivative's sign, a concurrent reduction in the tax rate will more effectively reduce incidents of tax evasion. A second interesting conclusion is that the usual tax administration instruments in the form of more audits and heavier penalties may also reduce tax evasion; however, this relationship depends on the efficiency parameters, and this may not be the most effective way to reduce tax evasion.

This paper's main policy recommendation is that one cannot reduce tax evasion by combating it as a stand-alone phenomenon. Our results show that the optimal tax enforcement measures (intensity of audit and fines) depend on the quality of institutions and the productivity impact of public sector input; therefore, significantly improving tax enforcement is not feasible without improving the overall institutional setting.

Another important point is that the efficiency of tax administration and noncompliance costs have implications for the optimal size of government expenditures. Our results show that in an environment where raising tax rates increases tax evasion, the optimal tax rate is lower than the value established by Barro's (1990) rule. This finding reinforces the conclusion that an optimal fiscal policy depends on tax enforcement and noncompliance costs taxpayers face. Expanding the optimal size of government expenditure is not feasible unless tax compliance is improved.

The model can be extended in several directions. Bureaucrats could have a more central role, and the relationship between penalties, corruption, and the outcome of audits could be more explicitly modeled. Another interesting avenue would be to have different agents in the model with different risk attitudes, which would allow a study of the welfare implications of changes to fiscal parameters. Finally, the corruption of tax auditors can be incorporated into the tax administration in the model to analyze its implications for tax evasion.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Optimal tax evasion

The optimal value of the problem

$$\max_{c_t,e_t}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\ln c_t dt\right],$$

is the value function $J(t,k_t)$ that solves the Hamilton–Jacobi–Bellman (HJB) differential equation

$$0=\frac{\partial J}{\partial t}-\rho J+\max_{c_t,e_t}\left\{\ln c_t+\frac{\partial J}{\partial k_t}\left[\left(1-\tau+e_t\tau-se_t\right)y_t-c_t\right]\right.$$
$$\left.+\left[J\left(t,k_t-p_d\theta\tau e_t y_t\right)-J\left(t,k_t\right)\right]\lambda_t\right\}.$$

The first-order condition (FOC) on c_t is

$$c_t^*=\frac{1}{\frac{\partial J}{\partial k_t}},$$

while the FOC on e_t is

$$\frac{\partial J}{\partial k_t}(\tau-s)-\lambda\frac{\partial J\left(t,k_t-p_d\theta\tau e_t^* y_t\right)}{\partial\left(t,k_t-p_d\theta\tau e_t^* y_t\right)}\theta=0.$$

The guess function is

$$J\left(t,k_t\right)=\frac{1}{\phi}\ln\left(\phi k_t\right)+B,$$

where B is a constant whose value must solve the HJB equation. The derivatives of this function are

$$\frac{\partial J}{\partial t}=0,$$

$$\frac{\partial J}{\partial k_t}=\frac{1}{\phi k_t},$$

that can be substituted into the FOCs to obtain

$$c_t^*=\phi k_t,$$

$$e_t^*=\frac{1}{\theta\tau p_d A}\left(1-\frac{\lambda p_d\theta\tau}{\tau-s}\right).$$

Accordingly, the HJB becomes

$$0=-\frac{\rho}{\phi}\ln\left(\phi k_t\right)+\ln\left(\phi k_t\right)-\rho B+\frac{1}{\phi}(1-\tau)A+\frac{1}{\phi}\frac{\tau-s}{\theta\tau p_d}-\frac{\lambda}{\phi}-1+\frac{\lambda}{\phi}\ln\frac{\lambda\theta\tau p_d}{\tau-s},$$

which can be split into

$$0=-\frac{\rho}{\phi}\ln\left(\phi k_t\right)+\ln\left(\phi k_t\right),$$

$$0=-\rho B+\frac{1}{\phi}(1-\tau)A+\frac{1}{\phi}\frac{\tau-s}{\theta\tau p_d}-\frac{\lambda}{\phi}-1+\frac{\lambda}{\phi}\ln\frac{\lambda\theta\tau p_d}{\tau-s}.$$

The first equation gives

$$\phi=\rho,$$

while the second one becomes

$$0=-\rho B+\frac{1}{\rho}(1-\tau)A\frac{1}{\phi}\frac{\tau-s}{\theta\tau p_d}-\frac{\lambda}{\phi}-1+\frac{\lambda}{\rho}\ln\frac{\lambda\theta\tau p_d}{\tau-s},$$

the solution of which is

$$B=\frac{1}{\rho^2}\left((1-\tau)A-\lambda-\rho+\frac{\tau-s}{\theta\tau p_d}+\lambda\ln\frac{\lambda\theta\tau p_d}{\tau-s}\right).$$

Appendix B. Proof that social welfare maximization equal to growth maximization

Assume that the government chooses a set of policy variables, Φ, so that social welfare is maximized. Since the economy is populated with identical individuals, this problem reduces to the maximization of the utility of the representative agent,

$$\max_{\Phi}U(\tau)=\int_0^\infty ln(c_t)\exp(-\rho t)dt \tag{26}$$

$$s.t.\ \gamma = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = [(1-\tau)\frac{\partial y_t}{\partial k_t} - \rho], k(0) = k_0.$$

In time t, capital per capita is given by

$$k_t = k_0 \exp(\gamma t). \tag{27}$$

Recall

$$y_t = \bar{A}k_.$$

Thus, one can write

$$\dot{k}_t = \frac{dk_t}{dt} = \bar{A}k_t - c_t - \delta k_t. \tag{28}$$

Dividing both sides of (28) by k_t yields:

$$\frac{\dot{k}_t}{k_t} = \bar{A} - \frac{c_t}{k_t} - \delta.$$

Substituting for $\rho = \bar{A} - \frac{\dot{k}_t}{k_t} - \delta$, and after some manipulation, we obtain:

$$c_t = \rho k_t.$$

Now, we can write the representative individual's utility function in the following form:

$$u(c_t) = \ln(\rho k_0 \exp(\gamma t)). \tag{29}$$

Then the optimization problem becomes

$$\max_{\Phi} \tilde{U}(\Phi) = \int_0^\infty [\ln(\rho k_0 \exp(\gamma t))] \exp(-\rho t) dt. \tag{30}$$

This is simplified further as

$$\tilde{U} = \gamma \int_0^\infty t \exp(-\rho t) dt + \ln(\rho k_0) \int_0^\infty \exp(-\rho t) dt. \tag{31}$$

We note that the second term of (31) is not a function of policy variables; therefore, this and other constant terms can be ignored. In other words, $\max_\Phi \tilde{U}(\Phi)$ is equivalent to $\max_\Phi \hat{U} = \gamma \int_0^\infty t \exp(-\rho t) dt$. This integration is solved as

$$\int_0^\infty t e^{-\rho t} dt = -\frac{t}{\rho} e^{-\rho t}\Big|_0^\infty + \frac{1}{\rho} \int_0^\infty e^{-\rho t} dt = -\frac{1}{\rho^2} e^{-\rho t}\Big|_0^\infty = \frac{1}{\rho^2}.$$

Then, the first term of (31) becomes

$$\hat{U} = \frac{\gamma(\Phi)}{\rho^2}. \tag{32}$$

The welfare maximization problem given by (30) is equivalent to maximizing the objective function given by (32). In other words, the welfare maximization in this model is equivalent to the maximization of the growth rate of the individual's consumption.

Appendix C. Taxation and growth

C.1. Optimal tax rate when agents do not evade

When tax evasion is equal to zero, growth can be written as

$$\gamma_0 = (1-\tau)\left(\bar{A}\tau^\psi\right)^{\frac{1}{1-\psi}} - \rho. \tag{33}$$

If the government wants to set τ such that γ_0 is maximized, the FOC would be

$$\frac{\partial \gamma}{\partial \tau} : \bar{A}^{\frac{1}{1-\psi}} (\tau(1-\mu))^{\frac{\psi}{1-\psi}} \frac{\tau - \psi}{(1-\psi)\tau} = 0,$$

the solution of which is

$$\tau^{FB} = \psi.$$

C.2. Optimal tax rate when agents evade and s=0

$$\begin{aligned} Max_\tau \gamma &= (1-\tau)\left(A(\tau(1-\mu-E))^\psi\right)^{\frac{1}{1-\psi}} - \rho + \frac{1}{\theta p} - 2\lambda + p\theta\lambda^2 \\ &= (1-\tau) A^{\frac{1}{1-\psi}} \left(\tau(1-\mu-e^*)(1-\lambda\theta(p_d - \frac{\omega\lambda}{2}))\right)^{\frac{\psi}{1-\psi}} \\ &- \rho + \frac{1}{\theta p} - 2\lambda + p\theta\lambda^2 \end{aligned}$$

where e_E is the equilibrium tax evasion rate that can be written as $e^* = \frac{1}{\theta\tau p_d A}\left(1 - \lambda p_d \theta\right)$. The FOC can be written as

$$\frac{\partial \gamma}{\partial \tau} : = -A^{\frac{1}{1-\psi}} \left(\frac{1}{2} \frac{\left(2\theta\tau p_d A(1-\mu) - 2\left(1-\lambda p_d\theta\right)^2 - \lambda^2\theta\omega\left(1-\lambda p_d\theta\right)\right)}{\theta p_d A} \right)^{\frac{\psi}{1-\psi}} \times$$
$$\left(\frac{\tau-\psi}{(1-\psi)\tau} + \psi \frac{\left(1-\lambda p_d\theta\right)\left(2\lambda p_d\theta - 2 - \lambda^2\theta\omega\right)(1-\tau)}{(1-\psi)\tau\left(2\theta\tau p_d A(1-\mu) - 2\left(1-\lambda p_d\theta\right)^2 - \lambda^2\theta\omega\left(1-\lambda p_d\theta\right)\right)}\right) = 0$$

and the solution is

$$\begin{aligned} \tau &= \psi + \frac{\left(1-\lambda p_d\theta\right)\left(1-\lambda p_d\theta + \frac{\lambda^2\theta\omega}{2}\right)(1-\psi)}{p_d\theta(1-\mu)A} \\ &= \psi + \frac{(1-\psi)(2\xi-1)}{(1-\mu)A} \frac{\left(1-2p^2\frac{\xi^2}{\omega(2\xi-1)^2}\left(\pi\frac{2\xi-1}{p}\right)^{\frac{1}{\xi}}\right)\left(1-4p^2\frac{\xi^2}{\omega(2\xi-1)^2}\left(\pi\frac{2\xi-1}{p}\right)^{\frac{1}{\xi}}\right)}{2p\xi\left(\pi\frac{2\xi-1}{p}\right)}. \end{aligned} \tag{34}$$

For a feasible set of parameters, we can verify that $\tau \simeq \psi$, as the magnitude of the second term on the right-hand side of (34) becomes relatively small compared to ψ.

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