
Stochastic traffic assignment of mixed electric vehicle and gasoline vehicle flow with path distance constraints

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Abstract

This paper addresses a general stochastic user equilibrium (SUE) traffic assignment problem (TAP) for transport networks with electric vehicles (EV), where EV paths are restricted by the EV driving range limits. A minimization model for path-constrained SUE is first proposed as an extension of path-constrained deterministic user equilibrium (DUE) TAP, which also extends the existing general SUE models with link-based constraints to path-based constraints. The resulting SUE model and solution algorithm can be used for other conditions with similar path-based constraints. The equilibrium conditions reveal that any path cost in the network is the sum of corresponding link costs and a path specific out-of-range penalty term, while path out-of-range term should equal to zero to ensure feasible flows. We develop a modified method of successive averages (MSA) with a predetermined step size sequence where both multinomial logit and multinomial probit based loading procedure are applied to solve the TAP. The suggested methods incorporate K-shortest paths algorithm to generate the path set on a need basis. Finally, two numerical examples are presented to verify the proposed model and solution algorithms.

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Keywords: Traffic assignment; stochastic user equilibrium; path distance constraints; K-shortest path algorithm; Multinomial Logit; Multinomial Probit

1. Introduction

Carbon-based emissions and greenhouse gases (GHG) are critical global issues as addressed by the Kyoto Protocol in 1998 (U.S. Environnemental Protection Agency, 2006). The transport sector is a significant contributor to

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GHG emissions in most countries, comprising 23% (worldwide) of CO2 emissions from fossil fuel combustion in 2005 while automobile transport is the principal CO2 production source. From the energy safety point of view, the transport sector as a whole is 98% dependent on fossil oil which is also exceedingly affected by changes in energy resources (OECD-ITF Joint Transport Research Centre, 2008). So changes to the current energy structure in transport sector are in urgent need.

Alternative fuels are addressed as a new fuel choice to reduce GHG emissions and electric vehicles (EV) are believed to be a sustainable solution (OECD-ITF Joint Transport Research Centre, 2008). Governments and automotive companies have recognized the value of these vehicles in helping the environment and are encouraging the ownership of EV through economic incentives (Hacker et al., 2009). It is mentioned that one million plug-in hybrid and electric vehicles will be on the road by 2015 in United States to reduce greenhouse gas emission and dependence on oil (Saber and Venayagamoorthy, 2009). According to Electric Drive Transportation Association, the plug-in electric vehicles (PEV) in US has exceeded 190,000 between January of 2011 and Mach of 2014 (Ghamami et al., 2014).

Although many cities are planning construction and expansion of charging infrastructures for EV, it is likely that in the foreseeable future EV commuters will need to charge their vehicles at home most of the time (Morrow et al., 2008). It is obvious that the driving range limit inevitably adds a certain level of restrictions to EV drivers’ travel behaviors, at least in a long future period prior to the coverage of recharging infrastructures reaching a sufficient level (Jiang et al., 2013). EV companies are trying to overcome this limited range requirement with fast charging stations, where a vehicle can be charged in only a few minutes to near full capacity. Besides being much more costly to operate rapid recharge stations, the vehicles still take more time to recharge than a standard gasoline vehicle would take to refuel (Botsford and Szczepanek, 2009).

However, the widespread adoption of PEV calls for fundamental changes to the existing network flow modelling tools for properly capturing changed behaviors and induced constraints in forecasting travel demands and evaluating transportation development plans (Jiang et al., 2013).

In order to take into consideration of driving range limit and insufficient charging facility status in traffic assignment, Jiang et al. (2012) proposed an approach to restrict flow of a path to zero if the path distance is greater than the driving range limit of EV. They employed a path travel time function that is the sum of the corresponding link cost such as the Bureau Public Road (BPR) function and showed the Lagrangian multiplier of its optimal solution stands for the unit out-of-range travel distance cost. Classic Frank-Wolfe algorithm with a constrained shortest path algorithm as its subroutine can be applied to solve this problem.

The deterministic user equilibrium (DUE) condition characterizes route choice behavior where users have perfect traffic network information and always choose the shortest path accurately. A convex minimization model for DUE conditions can be built by adding path distance constraints into the Beckmann’s conventional DUE model. A more realistic and general situation is that travel times are random variables or travel times are perceived by travellers in imperfect, stochastic manner. Although the stochastic user equilibrium (SUE) principle plays a more realistic role than DUE principle in addressing road user’s route choice behavior, the SUE traffic assignment problem with path-distance constraints has received little attention. To be consistent with the generalized DUE with path distance constraints, the SUE traffic assignment model with generalized path travel times are referred to as generalized SUE traffic assignment with path distance constraints. A milestone in formulating SUE conditions is Daganzo’s unconstrained minimization model (Daganzo, 1982) of conventional SUE conditions, which can lead to a convergent algorithm for solving the general SUE traffic assignment problem. However, adding side constraints (e.g. link capacity constraints) into Daganzo’s model cannot yield solution fulfilling generalized SUE conditions.

1.1. Literature Review

It is well known that the standard TAP under DUE can be solved efficiently with a Frank-Wolfe type algorithm
whose linearized sub-problem finds shortest paths for each OD pair at each iteration. The problem of finding the shortest path for an EV was originally discussed by Ichimori et al. (1981), where a vehicle has a limited battery and is allowed to stop and recharge at certain locations. Lawler (2001) developed a polynomial algorithm for its solution. Adler et al. (2014) proposed an EV shortest-walk problem to determine the shortest travel distance route which may include cycles for detouring to recharging batteries from origins to destinations with minimum detouring. Kobayashi et al. (2011) and Siddiqi et al. (2011) included battery recharging stations in their shortest weight-constrained path problem models, which is known to be NP-Complete (Desrosiers et al., 1984; Desrochers and Soumis, 1989), and proposed heuristic techniques as solution methodologies. There has been some recent consideration of the effect of EV on traffic assignment and DUE. (Jiang et al., 2012) studied the effect of restricted the EV path distances and assumes charging events only occur at OD nodes, which corresponds to the real circumstance of insufficient charging facilities.

As a rational extension to the DUE, the stochastic user equilibrium (SUE) principle can be adopted to formulate the TAP. Meng et al. (2007) found that adding link capacity constraints into Daganzo’s model would lead to a linearly constrained minimization problem. Nevertheless, any optimal solution of the induced minimization model did not fulfill the generalized SUE conditions. This indicates that the typical technique used in modelling the generalized DUE conditions was not available for the generalized SUE conditions except the logit-based generalized SUE conditions formulated by Bell (1995b). Meng et al. (2007) proposed a general stochastic user equilibrium (SUE) traffic assignment problem with link capacity constraints, inspired by Maher et al. (2005), who proposed a formulation for stochastic social optimum (SSO) with the objective of minimizing the total perceived travel time and found that the solution to SSO can be achieved by solving a SUE problem using the marginal cost function. Meng et al. (2007) found that SUE flow pattern can be generated by solving a SSO problem applying a modified link travel time function.

Previous studies on general SUE traffic assignment problem mainly tackled link-based constraints [see, e.g., (Meng et al., 2007; Meng and Liu, 2011; Meng et al., 2014)]. However, for EV users, the route choice is restricted by their driving range limit, which imposes path distance constraints to the general SUE model.

Early algorithms developed to solve the unconstrained logit-based SUE problem were link-based [e.g. Maher (1998)]. These link-based algorithms do not require path storage and often use Dial’s STOCH algorithm or Bell’s alternative as the stochastic loading step (Dial, 1971; Bell, 1995a). Path-based algorithms require explicit path storage to directly compute the logit route choice probabilities. Olof et al. (1996) developed a path-based algorithm based on the disaggregated simplicial decomposition algorithm to solve the multinomial logit (MNL) SUE problem. Bekhor and Toledo (2005) compared path-based algorithms for the MNL SUE problem, and showed that the disaggregated simplicial decomposition algorithm is superior to the path-based method of successive averages (MSA) algorithm. Among the path-based algorithms for the traffic equilibrium problem with additive path costs, much of the recent attention has been focused on the disaggregate simplicial decomposition (DSD) algorithm, which was proposed by Larsson and Patriksson (1992), and the gradient projection (GP) algorithm. A comparison work between these two path-based algorithm could be found in Chen and Lee (1999).

Xu et al. (2012) investigated different strategies for determination of step size of the path-based algorithms developed to solve the C-logit SUE models based on an adaptation of the GP method. Three strategies were investigated: (a) predetermined step size (Nagurney and Zhang, 1996), (b) Armijo line search (Larry, 1966; Bertsekas, 1976), and (c) self-adaptive line search (He et al., 2002; Chen et al., 2012). To have a fair comparison of different step size strategies, Bekhor et al. (2006). Used a working route set, obtained from a route choice set generation algorithm, such as labelling, link penalty, link elimination and simulation, in path-based problem. Behaviorally, it had the advantage of explicitly identifying those routes that were most likely to be used and also allowed greater flexibility to include route-specific attributes that might not be obtainable directly from the link attributes (Cascetta et al., 1997; Bekhor et al., 2006). A column generation procedure could also be readily embedded in the GP algorithm (Chen and Jayakrishnan, 1998).
To the best of our knowledge, it is still an open question to find an exact solution method for solving the general SUE problem with path distance constraints on a transport network with EV.

1.2. Objectives and Contributions

Since SUE relaxes the perfect information assumption of the DUE model by incorporating a random error term in the path cost function to model travelers’ imperfect perceptions of travel times, it would be more realistic to apply general SUE model to assign EV flows to the transport network. It is interesting to compare the assignment results with path distance constraints and DUE TAP with path distance constraints. In the end, Section 6 provides a few concluding remarks.

Meng et al. (2007) proposed a solution framework for general SUE problem with link-based constraints. For the general SUE traffic assignment problem addressed in this study, we investigate the properties of the path distance constraints and the solution method framework. Furthermore, K-shortest paths algorithm is applied to avoid path enumeration.

To sum up, the contributions of this study are twofold. First, a holistic methodology is proposed for general SUE traffic assignment model with path distance constraints on EV scheme, in which the classic unconstrained SUE model can be used to incorporate path distance constraints by modifying MSA algorithm and finding the distance-constrained K-shortest paths in stochastic network loading process. It is assumed that the EV route choices are restricted by the distance EV can travel with a single charge. Second, comparison results between SUE and DUE are provided. The major part of this paper is a discussion of the modelling and solution methods for the SUE traffic assignment problem with path distance constraints.

The remainder of this paper is organized in the following order. In Sections 2 & 3, we elaborate the problem formulation, and analyse its solution properties. Section 4 presents the solution algorithms for both Logit-based and Probit-based stochastic network loading models, while Section 5 presents the numerical results from applying the algorithm procedure for a small network and Sioux Falls network as well as the comparison work between SUE TAP with path distance constraints and DUE TAP with path distance constraints. In the end, Section 6 provides a few concluding remarks.

2. Notation, assumptions and problem description

Consider a strongly connected network, denoted by \( G = (N, A) \), where \( N \) and \( A \) are sets of nodes and links, respectively. \((r, s)\) stands for certain ordered pairs of nodes, \( r \in R \) and \( s \in S \), where node \( r \) is an origin and node \( s \) is a destination. \( R \subseteq N \) and \( S \subseteq N \) are sets of origins and destinations, respectively. There are non-negative travel demand \( q_n^r \) of n-th vehicle type between \((r, s)\). \( \mathbf{q} = \left( q_n^r \right)_{n} \), \( \forall (r, s)\) is a column vector for all the travel demands. Let \( K_{rs} \) be the set of paths connecting O-D pair \((r, s)\), \( f_{kn}^{rs} \) be traffic flow of n-th vehicle type on path \( k \in K_{rs} \), \( \mathbf{f} = \left( f_{kn}^r \right)_{rn} \), \( \forall (r, s) \) be a column vector of all these path flows between OD pair \((r, s)\), and \( \mathbf{f} = \left( f_{kn}^r \right)_{rn} \), \( \forall (r, s) \) be a column vector of all the path flows over the entire network. Let \( v_a \) denote traffic flow on link \( a \in A \) and \( \mathbf{v} = \left( v_a \right)_{a} \), \( a \in A \) is a column vector of all the link flows. The path flows and link flows should comply with fundamental flow conservation equations:
\[ v_a = \sum_{n} \sum_{(r,s,k)} f_{kn}^{rs} \delta_{a,k}^{rs}, \forall a \in A \]  
\[ \sum_{k} f_{kn}^{rs} = q_n^{rs}, \forall (r,s), n \]  
\[ f_{kn}^{rs} \geq 0, \forall (r,s), n, k \in K_{rs} \]

where \( \delta_{a,k}^{rs} = 1 \) if path \( k \in K \) between O-D pair \((r,s)\) traverses link \( a \in A \), and 0 otherwise.

Let \( t_a(v_a) \) denote the separable travel time function of link \( a \in A \), which is assumed to be a positive, strictly increasing, convex and continuously differentiable function of the traffic flow on the link. All the link travel time functions are grouped into a column vector \( t(v) = (t_a(v_a))^T, a \in A \). Travel time on path \( k \in K \) between O-D pair \((r,s)\) can be considered as a function of all the path flows, denoted by \( c_{rs}^k(f) \) with the expression

\[ c_{rs}^k(f) = \sum_{a} t_a(v_a) \delta_{a,k}^{rs} \] (4)

Given any positive feasible path flow pattern \( f \), \( f \) satisfies the conventional SUE conditions associated with the path travel time functions, \( c_{rs}^k(f) = (c_{rs}^k)^T, \forall (r,s), n, k \in K_{rs} \) is a column vector of all these path travel time functions between OD pair \((r,s)\), namely

\[ f_{kn}^{rs} = q_n^{rs} \cdot P_{kn}^{rs}(c_{rs}^k(f)) \] (5)

where \( P_{kn}^{rs} \) is the probability that vehicle type \( n \) choose path \( k \) between O-D pair \((r,s)\).

2.1. Path distance constraints and insufficient charging facility

Based on EV’s market potential, it is expected that in the future gasoline vehicles (GV) and EV will coexist in the automobile market. For this reason, the proposed model includes multiple classes of vehicles, namely GV and EV, which distinguish from each other in terms of driving distance range and travel cost composition. To derive the theoretical properties of the problem, we consider a set of assumptions regarding demand heterogeneity and travel behavior.

First without loss of generality, it is assumed that the demand population is only comprised of GV and EV. Plug-in hybrid electric vehicle (PHEV) are not explicitly considered since they can be simply treated as an in-between class of GV and EV in terms of the technological and economic features (i.e., driving range limit and travel cost composition), or a special type of GV with lower operating costs. Readily multiple types of EV with different driving range limits and operating costs can be incorporated into the model.

Second, we assume the total travel demand between each O-D pair for every vehicle type is deterministically known a-priori. SUE concept is devised for route choice procedure, in which each traveler chooses a route that minimizes his/her perceived travel cost while no one can reduce his/her perceived travel cost by unilaterally switching to an alternative route. For an individual GV traveler, stochastic user equilibrium simply implies a conventional stochastic traffic assignment problem of searching for perceived minimum cost (travel time); whereas for an EV traveler, it poses a path distance-constrained perceived minimum cost problem. In this paper, we scrutinize the integrated effect of different vehicle types with various path constraints.
Third, without loss of generality, we assume that both GV and EV travelers use a common form of systematic travel cost function for determining their travel choices. The link travel time functions are assumed to be separable between different network links and identical for different vehicle classes, implying the travel time on a particular link only depends on its own traffic flow. These functions are assumed to be positive, monotonically increasing, and strictly convex.

In our network equilibrium analysis, it is implicitly assumed that all EV are fully charged at their origins. The possible availability of commercial battery-charging or battery-swapping stations emerging in urban areas can be considered in the future when charging infrastructures achieve a certain level of coverage. EV users would choose a path whose distance $l^k$ is less than or equal to the driving range limit of the vehicle type, denoted by $D$.

any feasible path flow pattern should satisfy the path distance constraints:

\[ f^{rs}_k (D - l^k) \geq 0, \forall (r,s), n,k \in K \]  

(6)

which means that if the flow of that class of EV users going through this path is positive, the path distance is smaller than or equal to the driving range of a given class of EV; otherwise, the trip flow should equal to zero.

3. Mathematical model

Due to the complexity of probit-based SUE problem, directly adding side constraints into that general SUE model does not give us an equivalent minimization model to the probit-based SUE traffic assignment with side constraints (Meng and Liu, 2011). However, we can still add the path distance constraint into this minimization model developed by Sheffi (1985) as follows, only if predetermining path set to ensure distances of all the used paths are less than the range limit for each O-D pair.

\[
\min Z(v) = -\sum_{nrs} q_n^{rs} S^{rs}_n [c^{rs}_n(v)] + \sum_{\alpha} v_{a} t_{\alpha} (v_{\alpha}) - \sum_{t_{\alpha}(\omega)d\omega} a
\]

s.t.: (1)(2)(3)(6)

Compared to Sheffi’s model which can be solved as an unconstrained minimization problem and still yield a solution that satisfies the flow conservation constraints (1)(2)(3), the extra path distance constraints are the constraints that needs a careful consideration. To prove the equivalence between the solution of the problem given in Eq.(7) and the SUE equations, the first-order derivative of this problem have to coincide with the SUE conditions.

\[
L(x, \mu) = -\sum_{nrs} q_n^{rs} S^{rs}_n [c^{rs}_n(x)] + \sum_{\alpha} x_{\alpha} t_{\alpha} (x_{\alpha}) - \sum_{t_{\alpha}(\omega)d\omega} a - \sum_{nrs} \sum_{kn} \sum_{k} \mu_{kn}^{rs} \cdot (D_n - f^{rs}_k)
\]

The first-order derivative require that

\[
\nabla L(x, \mu) = 0
\]

The gradient is taken with respect to link flow vector $x$, and the derivatives of first three summation terms of Eq. (7) can be calculated as

\[
\frac{\partial L(x, \mu)}{\partial x_{b}} = (-\sum_{nrs} \sum_{k} q_n^{rs} P_{kn}^{rs} S_{b,k}^{rs} + x_{b}) \frac{dh_{b}}{d\gamma_{b}} - \sum_{nrs} \sum_{k} \mu_{kn}^{rs} \cdot (D_n - f^{rs}_k) \delta_{b,k}^{rs}
\]

Note that the extra path distance constraints could be infeasible. For some OD pairs, if none of those paths connecting them satisfies the range limit of a certain class of vehicles, the travel demand between them of this class of vehicles cannot be assigned to the network and the problem has no feasible solution.
However, those infeasible OD pairs can be found easily by checking distance of the shortest path from the start, and if the distance of shortest path is physically longer than range limit of a certain class of vehicles, then there would be no feasible path for the SUE TAP between this OD pair for this class of vehicles.

If it is feasible, which means there exists at least one path between each OD pair that is within the range limit of a certain class of vehicles. Then the term $\mu_{kn} \cdot D_n - l^D_k$, which is path out-of-range cost incurred when the path length exceeds the distance limit of that class of vehicles should equal to zero. $\mu_{rs}^{\ast kn}$ is a proxy of equivalent travel time value of the out-of-range cost per unit distance. Therefore, by ensuring the distance of each chosen path less than range limit, the derivative of the SUE objective function with respect to a link-flow variable becomes

$$\frac{\partial \tilde{f}(x, \mu)}{\partial x_b} = \left( - \sum_n \sum_{rs} \sum_{k \in \mathcal{K}_{rs}} q_{rs}^n P_{rs}^{kn} \hat{S}_{rs}^{kn} + x_b \right) \frac{df_b}{dx_b}, \forall b \tag{11}$$

Assume that link performance functions are strictly increasing, the gradient becomes zero if and only if

$$x_b = \sum_n \sum_{rs} \sum_{k \in \mathcal{K}_{rs}} q_{rs}^n P_{rs}^{kn} \hat{S}_{rs}^{kn}, \forall b \tag{12}$$

The above equation expresses the SUE link flows when it has any feasible solution, namely whenever we assign travel demand to paths between each OD pair, path distance should be less than the range limit of that class of vehicles.

Following the same procedure of demonstrating uniqueness in page 319, Chap. 12, Sheffi (1985), it is obvious that the Hessian matrix of the SUE objective function is positive definite, because the second derivative of

$$\sum_n \sum_{rs} \mu_{rs}^{\ast kn} f_{rs}^{\ast kn} (D_n - l^D_k)$$

with respect to path flow equals to zero. Therefor the model possesses two vital propositions as follows.

Proposition 1: Any local feasible minimum $x^{\ast}$ of this model satisfies the generalized SUE conditions, and the Lagrangian multipliers associated with path distance constraint (6) are path out-of-range costs.

Proposition 2: The SUE link flow pattern induced by any local minimum solution of the linear constrained minimization model is unique.

4. Solution method

4.1. Modified MSA algorithm

It was shown that the MSA algorithm can still be applied to the path distance constrained stochastic traffic assignment problem (STAP) with a direction-finding step different from that of classic STAP and feasibility check step.

For multinomial logit model (MNL), the steps are as follows:

Step 0: Feasibility check. For each OD pair, find the shortest path according to physical distance. If the distance of this path is longer than the range limit of a certain type of vehicle and the corresponding travel demand is positive, then there is no feasible path for this type of vehicle between this OD pair. Record this OD pair and infeasible vehicle type to Set A. If Set A is empty, go to the next step; if not, stop and display Set A.

Step 1: Initialization. Set $x_a(0) = 0$, $t_a = t_a[x_a(0)]$. For each OD pair, find K shortest path for each class of vehicles in terms of free flow travel time.
If the path distance is greater than the range limit of this class of vehicles, set the path travel time to infinite. Calculate the probability of choose each path, record them as initial path set and perform stochastic network loading to assign all the demand of each class of vehicles between this OD pair to the corresponding K shortest paths. This yields \( x_a^{(1)} \). Set iteration counter \( n = 1 \).

Step 2: Update. Calculate a new link cost in terms of \( t_a = t_a[x_a^{(1)}], \forall a \).

Step 3: Direction finding. Follow the same procedure described in step 1 to find K shortest path for each class of vehicles based on the current set of link travel times, \( \{ t^* \} \). If all the K paths between an OD pair exceed range limit of this type of vehicle, use initial path set in step 1 and perform stochastic network loading. This yields an auxiliary link flow pattern \( \{ y^n \} \).

Step 4: Step size. \( \{ \alpha_n \} \) is a predetermined step size sequence satisfying the three conditions:

\[
0 < \alpha_n < 1 \quad \text{and} \quad \lim_{n \to \infty} \alpha_n = 0
\]

\[
\sum_{n=1}^{\infty} \alpha_n = +\infty
\]

\[
\sum_{n=1}^{\infty} \alpha_n^2 < \infty
\]

There are a few step size sequences fulfilling the above conditions; for example

\[
\alpha_n = \frac{\rho}{n}, n = 1, 2, \ldots, \infty
\]

where parameter \( 0 < \rho \leq 1 \).

Step 5: Move. Find the new flow pattern by setting \( x_a^{n+1} = x_a^n + (1/n)(y^n - x^n) \).

Step 6: Convergence test. Let

\[
x_a = \frac{1}{m}(x_a^n + x_a^{n-1} + \cdots + x_a^{n-m+1})
\]

If the convergence criterion

\[
\sqrt{\sum_a (x_a - x_a^n)^2} / \sum_a x_a \leq \kappa
\]

is met, stop and \( \{ x_a^{n+1} \} \) is the set of equilibrium link flows; otherwise, set \( n = n + 1 \) and go to step 2.

4.2. Modified probit-based loading algorithm

For multinomial probit model (MNP), the steps are

Step 0: Feasibility check. This step is the same as that of logit model.

Step 1: Sampling. Set iteration counter \( n = 1 \). Sample \( T^n \) from \( T^n \sim N(t, \beta \sigma) \) for each link \( a \).

Step 2: All-or-nothing assignment. For each OD pair, find the distance-constrained shortest path based on
perceived link travel time $T_a^n$. Calculate the path distance of shortest path for this class of vehicles, if the path distance exceeds range limit, calculate second shortest one, etc. If all the K path distances are greater than range limit, use the initial shortest path generated in Step 0. Assign the travel demand of each class of vehicles between this OD pair to the distance-constrained shortest path based on perceived link travel time $T_a^n$. This yields the set of link flow $X_a^n$.

Step 3: Flow averaging. Let $X_a^n = \frac{(n-1)X_a^{n-1} + X_a^n}{n}$.

Step 4: Stopping test. Let $\sigma_a^n = \sqrt{\frac{1}{n(n-1)} \sum_{m=1}^{n} [X_a^m - X_a^n]^2}$, if $\max_a \sigma_a^n / X_a^n \leq \kappa$, stop; otherwise, set $n = n + 1$ and go to step 1.

5. Numerical example

Two numerical examples are adopted in this section to assess the proposed methodology.

5.1. Small network example

![Small network diagram](image)

Fig. 1. Small network

The first example consists of 9 nodes, 18 links, and 4 OD pairs: (1,3), (1,4), (2,3), and (2,4), as shown in Figure 1. The free-flow travel time is used as a proxy for the link length for each link. Travel time on each link is defined by the following BPR (Bureau of Public Road) type function

$$t_a(v_a) = t_a^0 \left(1 + 0.15 \times \left(\frac{v_a}{H_a}\right)^\rho\right), a \in A$$

(13)

where $t_a^0$ is the free flow travel time, $H_a$ is the link $a$ capacity and $\rho$ is a prescribed parameter. OD demands are assumed to be the same for both GV and EV (Given in Table 1). Free-flow travel time and link capacity are indicated in Table 2.

<table>
<thead>
<tr>
<th>Table 1. OD demand of small network example.</th>
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<tbody>
<tr>
<td>Origin</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
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</table>
We use this example to evaluate performance of proposed algorithms for solving both logit-based and probit-based SUE TAP with path distance constraints. The link flow patterns under two different cases, MNL and MNP with two classes of users (EV and GV), are estimated and compared in Table 2. EV and GV range limits are set as 20 and 100, respectively. K is set to be 8.

<table>
<thead>
<tr>
<th>Link #</th>
<th>Link length</th>
<th>Link Capacity</th>
<th>MNL flow</th>
<th>MNP flow</th>
</tr>
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<td></td>
<td></td>
<td>EV, D=20</td>
<td>GV, D=100</td>
<td>EV+GV</td>
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<tr>
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It is observed that the SUE assignment with different path distance limit resulted in different equilibrium link flows. For some links, e.g. link 1 & 2, both GV and EV obtained similar assignment results, while for the other links, it can be seen that the more EV use a certain link, the less GV choose that link, because the EV’s path choices are restricted by its driving range and EV user prefer paths of short distance. When EV users crowded into those links with short distance, they became over-saturated, thus increasing corresponding link travel time, and GV user would rather use those unsaturated links to reduce their travel time to obtain equilibrium.

Comparing MNL with MNP, it is clear that there are 7 links close to zero flow in MNL while all the links have been assigned some flow in MNP. This result might come from Independence of Irrelevant Alternatives (IIA) property of MNL, which makes MNP more realistic even if it suffers from its low efficiency.
5.2. Sioux falls network example

As shown in Figure 2, Sioux Falls network has a total of 24 nodes and 76 links. The travel demand table used in the application are from Suwansirikul et al. (1987). The link flow patterns under the same scenarios are compared. Without loss of generality, for the first three cases, only one class of vehicles are considered. Based on free-flow travel time, the range limit for EV is set to \( \beta \geq 1 \) where in this example maximum of all shortest path between each O-D pair and \( \beta \geq 1 \) is a parameter to ensure there is at least one feasible path connecting each OD pair. The K is set to 3 in K-shortest path algorithm.

A similar OD travel demand is applied to all the four experiments, including UE, MNL, MNP & MNL with MCU (Multiclass users). Two classes of users, i.e. EV and GV users, are involved in MNL with MCU, where the travel demand of each class is half of original travel demand, which means, the EV market share is 50%. For GV users, the range limit is set to \( \beta = 10 \).

The results illustrate that the equilibrium flows change significantly on a number of links. A few example links were selected randomly and their flow variations were observed in terms of different range limits (Figure 3).

As can be seen from Figure 2-6 that when \( \beta \) begins to increase, the network flows on these links behave in a different manner; UE flow patterns change little as \( \beta \) increases because the base range limit (\( \beta = 1 \)) is the maximum distance of shortest paths among all OD pairs while UE usually requires shortest path (All-or-nothing Assignment, for example). Comparing with MNP, flow patterns resulting from MNL model change dramatically.
As $\beta$ continuously increases, the flow rates change mildly and finally converge to values without range limit constraints. However, these changes may not be necessarily monotone.

![Equilibrium flow pattern](image)

Fig. 3. Equilibrium flow pattern (a) link 1 (b) link 6 (c) link 21 (d) link 51.

5.3. Sensitivity analysis

We conduct a sensitivity test with respect to the total demand in Sioux Falls network, multiplied its value by a constant factor and performed SUE assignment to observe the effect of the congestion level on algorithm performance. Since the Sioux Falls matrix is quite congested (Bekhor and Toledo, 2005), the factor ranges from 0.1 to 1.5, in intervals of 0.1. The number of iterations required by MNL and MNP to reach given convergence rate is presented in Figure 5. For all levels of demand, the modified MSA algorithm of MNL model requires less than 50 iterations to reach 0.01% precision level especially when demand level is low. However, comparing with MNL, the convergence rate of MNP model is quite slow, which requires more than 200 iterations to achieve 0.1 precision.

The performance of MNL and MNP for the Sioux Falls network is quite different in terms of convergence rate. For efficiency, the MNL model outperforms MNP. However, as is known to all, MNL model suffers from IIA property, and MNP might be better when distributed computing approaches are applied.
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Fig. 5. For all levels of demand, the modified MSA algorithm of MNL model requires less than 50 iterations to reach given convergence rate and MNP model needs more than 200 iterations to achieve the same rate of precision.

### 5.3. Sensitivity analysis

The performance of MNL and MNP for the Sioux Falls network is quite different in terms of convergence rate. For constant factor and performed SUE assignment to observe the effect of the congestion level on algorithm performance.

We conduct a sensitivity test with respect to the total demand in Sioux Falls network, multiplied its value by a factor range from 0.1 to 1.5, in total demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment network loading procedure. The direction finding step for MNL, involves finding K feasible paths to load the travel demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment step. The proposed algorithm is easy to understand and implement. The application of the algorithms in Sioux Falls network justifies the applicability of the solution procedures to general network with path-based constraints. The well-known and widely used MSA procedure and probit-based network loading method are adopted and modified to solve this problem, following the idea of putting the path distance constraints into the path selection rules of stochastic network loading procedure. The direction finding step for MNL, involves finding K feasible paths to load the travel demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment step. The proposed algorithm is easy to understand and implement. The application of the algorithms in Sioux Falls network justifies the applicability of the solution procedures to general network with path-based constraints. The numerical analysis results show the impact of range limit on network equilibrium flows.

### 6. Conclusions

This paper worked on the traffic assignment models with path distance constraints, where new SUE TAP is formulated, solved and numerically analysed. SUE models, which include perception error of travel time, are considered more rational than UE model. Multiclass users in SUE model represents a simplified case of current traffic networks that carry both EV and GV. More classes of users with various range limit can also be taken into consideration. The vehicles’ range limit is determined based on its travel distance only, while rationally the range limit should be related to both travel distance and travel time.

This paper shows that at the equilibrium point the selected paths to assign the travel demand are different from that of basic SUE TAP. The distance of each path must be less than the range limit of that class of vehicles. The well-known and widely used MSA procedure and probit-based network loading method are adopted and modified to solve this problem, following the idea of putting the path distance constraints into the path selection rules of stochastic network loading procedure. The direction finding step for MNL, involves finding K feasible paths to load the travel demand between each OD pair, while it requires finding feasible shortest path for MNP in all-or-nothing assignment step. The proposed algorithm is easy to understand and implement. The application of the algorithms in Sioux Falls network justifies the applicability of the solution procedures to general network with path-based constraints. The numerical analysis results show the impact of range limit on network equilibrium flows.

### References


