Received: 12 May 2022 | Accepted: 11 June 2023

DOI: 10.1111/ecin.13165

ORIGINAL ARTICLE

Economic Inquiry

Optimal parental leave subsidization with endogenous fertility and growth

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Abstract

We examine the growth and welfare effects of parental leave subsidization in a life-cycle dynastic model with human capital externality. Such externality causes higher fertility, less parental time and expenditure on child human capital, and lower growth and welfare than the efficient levels. Efficient policies require subsidizing work leave, which includes parental leave, and labor income financed by a lump-sum tax. Calibration based on the U.S. data implies that a fully-covered leave duration of 7.8 weeks per parent financed by an increase in labor income tax would increase the annual growth rate by 0.06 percentage points and welfare by 0.027%.

KEYWORDS
fertility, growth, human capital, parental leave, welfare

JEL CLASSIFICATION
H2, J1, J22, O4

1 | INTRODUCTION

Human capital is not only a central factor of output production but also has been well-recognized as an engine of economic growth. Since children are the future builders of all nations, their human capital development crucially determines a nation's future economic performance. Among many factors that can affect human capital formation, parental time is an extremely crucial factor in child human capital development (e.g., Del Boca et al., 2014; Fiorini & Keane, 2014; Heckman & Cunha, 2007; Ruhm, 2004).

However, how much time should parents invest in their children? Furthermore, should, or how much should, the government subsidize parental time investment? In reality, the availability and generosity of paid parental leave such as

Abbreviations: ATUS, American Time Use Survey; BCE, baseline competitive equilibrium; FRED, Federal Reserve Economic Data; GDP, gross domestic product; IRS, Internal Revenue Service; LF, laissez faire; OECD, Organisation for Economic Co-operation and Development; OLD, optimal leave duration; PLS, parental leave subsidization; PLS_CI, parental leave subsidization financed by raising the capital income tax rate; PLS_ES, parental leave subsidization financed by reducing the education subsidy rate; PLS_LI, parental leave subsidization financed by raising the labor income tax rate; PLS_LS, parental leave subsidization financed by lump-sum taxation; PSID, Panel Study of Income Dynamics; SO, social optimum; UK, United Kingdom; US, United States.

Managing Editor: Dietrich Vollrath

the duration of leave and the rate of income support varies considerably across countries. For example, leave with full-payment rates available for mothers varies from 0 weeks in the U.S. to 84.4 weeks in Estonia (OECD Family Database).[1]

To gain insight into these questions, this study develops and calibrates a life-cycle dynastic family model with endogenous fertility and human capital accumulation to examine the growth and welfare effects of parental leave subsidization (PLS) and the optimal PLS, both analytically and quantitatively. Our basic model features a laissez-faire (LF) economy with overlapping generations of three-period-lived agents. In their first period of life, they develop human capital and do not make any decisions. In their second period of life, they make decisions regarding work and leisure time, consumption, savings, the number of children to have, and expenditure and time spent on each child's human capital development. In their third period of life, they retire and decide upon consumption and transfers to each child. The human capital of a child is specified as a function of parental time and expenditure inputs, parental human capital, and the economy-wide average human capital, that is, there are externalities in human capital accumulation.[2] The preference of an infinitely-lived dynastic family is defined as a discounted sum of utilities derived from the consumption levels of the old and young parents, leisure of young parent and the number of children in the family. This feature allows for a direct comparison of the social optimum (SO) and the competitive equilibrium based on the same welfare function which takes into account utilities of all generations, and hence facilitates the analysis of optimal PLS. The production side is characterized by a constant-return-to-scale production function that transforms physical capital, labor and human capital into a single final good. We show that the model globally converges to a balanced growth path which features a constant physical capital to effective labor ratio.

In the presence of human capital externalities, parents under-invest their time and expenditure inputs in children's human capital.[3] As a result, the growth rates of human capital and output per worker are lower than the socially optimal levels. Furthermore, when parents under-invest their time and expenditure inputs in children's human capital, the private marginal costs of raising a child are lower than the social marginal costs. Hence, fertility in the LF economy is higher than the socially optimal level through the trade-off between the quantity and quality of children (see Becker & Lewis, 1973; Fernihough, 2017; Hazan & Berdugo, 2002; Li et al., 2008; Moav, 2005; Willis, 1973, among others). A higher fertility rate also means that parents have less time available for their own leisure and labor as having more children requires parents to spend more time raising them. Also, compared with the SO, both young-age and old-age consumption are higher. As a result, the LF equilibrium is not socially optimal or efficient.

We then analyze if PLS can mitigate the efficiency loss that occur in the LF system. We interpret such subsidization as paid parental leave provided by governments to parents to care for newborn and young children, a benefit that is available in most advanced economies. To better understand the efficiency gain emerging from PLS, we first consider PLS financed by lump-sum taxation (PLS_LS), and then explore efficient policies that eliminate the efficiency loss of human capital externalities and achieve the SO.

We show analytically that PLS_LS increases parental time investment per child by lowering the marginal cost of investing time in child human capital and reduces fertility by increasing the marginal time cost of having a child. Through the quantity-quality trade-off of children, PLS_LS also increases parent's expenditure on child human capital. Since parental time investment in child human capital is higher now, PLS_LS reduces parental leisure and labor time. PLS_LS also lowers old-age consumption. Overall, PLS_LS can mitigate the efficiency loss arising from human capital externalities and raise the balanced steady-state growth rate of output per worker. We also show that an optimal leave subsidy rate exists which maximizes the welfare in the competitive equilibrium with PLS_LS but this optimal level of PLS_LS cannot achieve the SO.[4]

By considering general government policies including capital and income taxes and expenditure subsidy on child human capital, we find that parental leave subsidy is essential to achieve the SO. Specifically, efficient policies that achieve the SO require zero capital income tax and expenditure subsidy on child human capital but require subsidizing labor income and general work leave, including parental leave, by a lump-sum tax. The main idea is as follows. Leave subsidy reduces the time cost of having a child, while labor income subsidy increases the time cost of having a child as well as the marginal benefit of expenditure on child human capital. However, a higher marginal benefit of expenditure on child human capital increases the expenditure cost of raising a child via the quantity-quality trade-off of children. Thus, subsidizing general work leave, including parental leave, and labor income at the right levels can increase expenditure on child human capital and reduce fertility to their socially optimal levels. Lower fertility causes time investment per child, labor and leisure to increase to their socially optimal levels.

To evaluate the quantitative effects of PLS, we extend the basic model by including labor income tax, capital income tax, human capital expenditure or education subsidy, and a lump-sum tax (transfer if negative) and calibrate this extended model to the U.S. economy. The U.S. makes for an interesting case study as it is currently the only developed country that does not subsidize parental leave at the national level.[5] The quantitative exercise also aims to provide a policy recommendation for the U.S. to improve its current parental leave policy to achieve better outcomes.

In our calibration, we first choose an empirically plausible degree of externality in the human capital production function based on the estimated range in Borjas (1995). We then calibrate other parameters to match observed data moments in the U.S. data (American Time Use Survey, the Panel Study of Income Dynamics, and others) that capture the time use of households in childcare, leisure and work, parental expenditure on children's human capital development, consumption in young and old age, among others. We compute the optimal parental leave subsidy rate under three counterfactual cases: financed by raising the labor income tax rate (PLS_LI), financed by raising the capital income tax rate (PLS_CI) and financed by reducing the education subsidy rate (PLS_ES). We also convert the optimal parental subsidy rate into a corresponding parental leave duration that is fully covered. In all three cases, the introduction of the optimal parental leave subsidy improves welfare and raises the growth rate of output per worker but cannot achieve the SO. Not surprisingly, the improvement in welfare and growth is least significant in the PLS_CI case. In the PLS_LI case, the implied optimal leave duration is 7.8 weeks per child for a parent which is financed by a 0.58% points increase in the labor income tax rate. This raises the annual growth rate of output per worker by 0.06% points and welfare by 0.027% from the baseline economy. The results for the PLS_ES case imply a much longer leave duration and a corresponding large drop in the education subsidy rate which leads to more significant gains in welfare and growth compared with the PLS_LI case. However, we view such a policy combination to be more controversial since a substantial reduction in education subsidy could have other socioeconomic consequences that are beyond the scope of the model that we consider here. Finally, we present a summary of parental leave policies for a group of OECD countries. The optimal leave entitlement implied in the PLS_LI case is comparable to the practice in some European countries like Belgium and Netherlands.

To the best of our knowledge, this study is the first that examines the growth and welfare effects of PLS as well as the optimal PLS in the presence of human capital externalities both analytically and quantitatively. Our study contributes to the literature in several ways. First, our study contributes to a limited theoretical and quantitative literature that analyzes optimal parental leave policies (e.g., Barigozzi et al., 2018; Bastani et al., 2019) and the effects of parental leave policies (e.g., Bernal & Fruttero, 2008; Del Rey et al., 2021; Erosa et al., 2010). Among this literature, most studies focus on the effects of parental leave policies on labor market outcomes such as wages, employment and career choice (Barigozzi et al., 2018; Bastani et al., 2019; Bernal & Fruttero, 2008; Del Rey et al., 2021; Erosa et al., 2010). For example, Bastani et al. (2019) provide a theoretical justification for parental leave mandates to alleviate the under-provision of workplace flexibility and the gender wage gap in the labor market. Barigozzi et al. (2018) theoretically show that parental leave reduces welfare by exacerbating the negative externality generated by social norms concerning childcare activities and career choices. Erosa et al. (2010) quantitatively find that paid parental leave leads to welfare losses in a search and matching model since parental leave leads to inefficient matches in the labor market and encourages too much leave taking by fertile females. Our paper differs from the above literature in two ways: first, we focus on the effects of PLS on child human capital development and the subsequent effects on growth and welfare; and second, we consider human capital externalities as the source of inefficiency and discuss the optimal level of PLS both analytically and quantitatively.

Second, our study complements studies that analyze the effects of parental time investments on child human capital. For example, Bernal and Fruttero (2008) find a positive effect of parental leave on human capital quantitatively using a model of marriage and divorce with labor market frictions. Gahramanov et al. (2020) conduct a quantitative analysis of parental time investment on children's skill formation and find that parental time subsidization reduces fertility and has an uncertain effect on the labor supply of the primary caretaker. Focusing on the evaluation of education policies with a parental leave policy, Youderian (2019) quantitatively finds that education subsidization is more effective than PLS in promoting human capital and welfare. In contrast to our study, the above literature uses a partial equilibrium setup (Gahramanov et al., 2020) and abstract from endogenous fertility (Bernal & Fruttero, 2008; Youderian, 2019) and endogenous growth (Bernal & Fruttero, 2008; Gahramanov et al., 2020; Youderian, 2019).

Third, our study also contributes to the literature on public investment in human capital and economic growth. There is considerable literature on the effects of education subsidization or public education on economic growth (see Glomm & Ravikumar, 1992, 1998; de la Croix and Doepke, 2004, among others). However, this literature does not pay much attention to the influence of parental time subsidization on human capital development and economic growth. Our finding of positive impacts of parental time subsidization on economic growth and welfare suggests that parental time subsidization deserves more attention in this literature. This is in line with empirical evidence which finds that parental time is an extremely important factor in child human capital development (Del Boca et al., 2014; Ruhm, 2004) and that monetary expenditure is less beneficial than parental time in the cognitive development of children particularly in regard to young children (Del Boca et al., 2014).

Additionally, our theoretical and quantitative results concerning the effects of parental leave on fertility and child human capital are consistent with the empirical evidence. Carneiro et al. (2015) study and compare the effect of a change in parental leave entitlements in Norway and find that when parental leave entitlements are very low, similar to the current situation in the U.S., longer entitlements with PLS can increase parental time spent with children, and thus child human capital development. Using Austrian data, Lalive and Zweimuller (2009) find that increasing the parental leave duration increases fertility, as found by our quantitative analysis. Furthermore, the welfare implication of parental leave in our study agrees with the empirical evidence which shows that short to moderate periods of parental leave can increase economic efficiency (Ruhm & Teague, 1997).

The rest of the paper proceeds as follows. Section 2 describes the basic model without PLS, that is, the LF economy. Section 3 characterizes the SO. Section 4 characterizes the LF equilibrium, the competitive equilibria with PLS_LS and efficient policies that achieve the SO. Section 5 calibrates an extended model to the U.S. economy and discusses the quantitative implications of PLS in several cases. Section 6 provides concluding remarks.

2 | THE BASIC MODEL

We first describe the basic model without government policies. The model economy is inhabited by overlapping generations of a large number of identical agents who live for three periods. In their first period of life, these agents develop human capital and do not make any decisions. In their second period of life, they make decisions regarding work and leisure time, consumption, savings, expenditure and time spent on their children's human capital development, and the number of children. In their third period of life, they retire and decide upon consumption and transfers to their children. The mass of the working generation in period t is denoted by L_t.

The preferences of the coexisting old parent and young working parent in a family are assumed to be identical and are defined over the consumption levels of the old and young parents, $C_{o,t}$ and $C_{y,t}$ respectively, the young parent's leisure time, Z_t, and the number of children of the young parent, N_t. That is, the flow utility of the family in period t is

$$u_t = \beta \ln C_{o,t} + \alpha\left[\ln C_{y,t} + \gamma \ln Z_t + \rho \ln N_t\right]$$

where $\alpha \in (0,1)$ and $\beta \in (0,1)$ are discount factors across generations and life stages, $\gamma > 0$ is the weight on utility from leisure, and $\rho > 0$ is the weight on utility from the number of children. Therefore, the preference of the infinitely-lived dynastic family at time t is given by $U_t = u_t + \alpha U_{t+1}$,[6] and hence the time 0 preference is given by

$$U_o = \sum_{t=0}^{\infty} \alpha^t u_t. \tag{1}$$

U_o can serve as the Millian social welfare function, as the assumption of identical preferences shared by the old and the young avoids generational conflicts.

Each young parent in period t devotes one unit of time endowment to four activities: (i) a fixed time spent on looking after each child $\nu \in (0,1)$, and hence a total amount of time of νN_t; (ii) a variable time spent on looking after each child $e_t \in (0,1)$ which is a choice of the young parent and enters a child's human capital formation (to be described later), and hence a total amount of time of $e_t N_t$; (iii) leisure Z_t; and (iv) working time $(1 - \nu N_t - e_t N_t - Z_t)$.[7] Each young parent earns wage income $(1 - \nu N_t - e_t N_t - Z_t)W_t H_t$, where W_t is the wage rate per unit of effective labor, and H_t is the young parent's human capital. A young parent in period t receives transfers from her parent if $B_t > 0$ or provides transfers to her parent if $B_t < 0$. The young parent spends her wage income and transfers from her parent on young-age consumption $C_{y,t}$, retirement savings S_t, and resources spent on each child's human capital development Q_t. An old parent in period t spends her savings plus interest income on old-age consumption $C_{o,t}$ and transfers to her children B_t.

In the model, children are consumption goods as the number of children enters the utility function. The model allows for altruism from old members toward young working members and vice versa, which induces private intergenerational transfers (transfers to the young working members or old members in the family). When the altruism toward parents is sufficiently strong, transfers from the young parent to the old parent can be operative (i.e., $B_t < 0$) so that children are investment goods as well.[8]

The budget constraints of old and young parents can be written as follows:

$$C_{o,t} = S_{t-1}R_t - B_tN_{t-1}, \tag{2}$$

$$C_{y,t} = B_t + (1 - vN_t - e_tN_t - Z_t)W_tH_t - S_t - Q_tN_t, \tag{3}$$

where R_t is the gross interest rate on savings.

The production function of the single final good is given by

$$Y_t = DK_t^{\theta}[L_tl_tH_t]^{1-\theta}, D > 0, \theta \in (0,1) \tag{4}$$

where K_t is the aggregate stock of physical capital, L_t is total number of workers, l_t is labor time per worker, and H_t is human capital of a worker. In per worker terms, $y_t = Dk_t^{\theta}(l_tH_t)^{1-\theta}$, where $y_t = Y_t/L_t$ and $k_t = K_t/L_t$ denote output per worker and capital per worker, respectively. As one period in this model corresponds to 30 years in our calibration, assuming that both physical capital and human capital depreciate fully within one period is reasonable and can help obtain closed form solutions. Factors earn their marginal products, and the price of the sole final good is normalized to unity. Denoting $\mu_t = K_t/(L_tl_tH_t)$ as the physical capital-effective labor ratio, the wage rate per unit of effective labor and the real interest rate are then respectively given by:

$$W_t = (1-\theta)D\mu_t^{\theta}, \tag{5}$$

$$R_t = \theta D\mu_t^{\theta-1}. \tag{6}$$

The market clearing conditions for the labor and capital markets are given by:

$$l_t = 1 - vN_t - e_tN_t - Z_t, \tag{7}$$

$$K_{t+1} = L_tS_t. \tag{8}$$

The working population evolves according to $L_{t+1} = L_tN_t$. Then the resource feasibility constraint is given by

$$C_{y,t} = Dk_t^{\theta}((1 - vN_t - e_tN_t - Z_t)H_t)^{1-\theta} - k_{t+1}N_t - Q_tN_t - C_{o,t}/N_{t-1}. \tag{9}$$

The human capital of a child, H_{t+1}, is given by

$$H_{t+1} = AQ_t^{\delta}e_t^{\varepsilon}\left(H_t^{\zeta}\overline{H}_t^{1-\zeta}\right)^{1-\delta}, A > 0, 0 < \varepsilon, \delta < 1, 0 < \zeta \leq 1, \tag{10}$$

where Q_t is parental expenditure on a child's human capital development, e_t is the parental time input in a child's human capital development, H_t is the human capital of the child's parent, and $\overline{H}_t$ is the average human capital in the economy.[9] Parameters δ, ε and ζ are the weights of parental expenditure input, parental time input and parental human capital in the human capital technology, respectively. Additionally, the parameter $1-\zeta$ is the weight on average human capital in the economy, and hence $1-\zeta$ indicates the degree of human capital externalities. A higher value of $1-\zeta$ or a lower value of ζ indicates a higher externality. When $1-\zeta = 0$, that is, $\zeta = 1$, there is absence of human capital externalities. In equilibrium, $H_t = \overline{H}_t$ by symmetry.

To set an efficiency criterion for policy analyses, we first characterize the social planner's allocation.

3 | THE SO

In reference to Mill (1848), the social planner chooses a sequence $\left\{C_{o,t}, C_{y,t}, Q_t, e_t, Z_t, N_t, k_{t+1}, H_{t+1}, \overline{H}_{t+1}\right\}_{t=0}^{\infty}$ to maximize the utility in (1) subject to the resource feasibility constraint (9) and the human capital production function (10), given an initial state (H_0, k_0, N_{-1}). The social planner internalizes the human capital externalities by setting $\overline{H}_{t+1} = H_{t+1}$ and solves the following problem:

$$\max_{\{C_{o,t},Q_t,e_t,Z_t,N_t,k_{t+1},H_{t+1},\overline{H}_{t+1}\}} U_o = \sum_{t=0}^{\infty} \Big\{ \alpha^t \big[\beta \ln C_{o,t} + \alpha \ln\big(Dk_t{}^{\theta}((1 - vN_t - e_tN_t - Z_t)H_t)^{1-\theta} - k_{t+1}N_t - Q_tN_t - C_{o,t}/N_{t-1}\big) + \alpha\gamma \ln Z_t + \alpha\rho \ln N_t\big] + \lambda_t \Big[AQ_t^{\delta}e_t^{\varepsilon}\Big(H_t^{\zeta}\overline{H}_t^{1-\zeta}\Big)^{1-\delta} - H_{t+1}\Big]\Big\},$$

where λ_t is the multiplier on the human capital technology. Note that since the social planner internalizes human capital externalities ($0 < \zeta < 1$) by setting $\overline{H}_t = H_t$ for all $t > 0$ such that the human capital technology reduces to $H_{t+1} = AQ_t^{\delta}e_t^{\varepsilon}H_t^{1-\delta}$, the parameter ζ no longer affects the first-order conditions that characterize the social planner's allocation. For $t \geq 0$, the first-order conditions with respect to $C_{o,t}, k_{t+1}, Q_t, e_t, N_t, Z_t$ and H_{t+1} are given in Equations (11)–(17), respectively.

The social planner equates the marginal rate of substitution between consumption of old agent and consumption of young agent to their marginal rate of transformation $1/N_{t-1}$:

$$\frac{\beta C_{y,t}}{\alpha C_{o,t}} = \frac{1}{N_{t-1}}. \tag{11}$$

Equating the marginal rate of substitution between consumption per young parent and bequests to children in the form of physical capital to their marginal rate of transformation gives:

$$\frac{k_{t+1}C_{y,t+1}}{\alpha C_{y,t}} = \frac{\theta y_{t+1}}{N_t}. \tag{12}$$

Equating the marginal rate of substitution between consumption per young parent and expenditure on child human capital to their marginal rate of transformation gives:

$$\frac{\alpha^{t+1}Q_t}{C_{y,t}} = \frac{\delta\lambda_t H_{t+1}}{N_t}. \tag{13}$$

Equating the marginal rate of substitution between consumption per young parent and parental time in child human capital to their marginal rate of transformation gives:

$$\frac{\alpha^{t+1}e_t}{C_{y,t}} = \frac{\varepsilon\lambda_t H_{t+1}(1 - vN_t - e_tN_t - Z_t)}{N_t(1-\theta)y_t}. \tag{14}$$

Equating the marginal rate of substitution between fertility and consumption per young parent to their marginal rate of transformation gives:

$$\frac{\rho C_{y,t}}{N_t} + \frac{\alpha C_{y,t}C_{o,t+1}}{C_{y,t+1}N_t^2} = \frac{(v+e_t)(1-\theta)y_t}{(1 - vN_t - e_tN_t - Z_t)} + Q_t + k_{t+1}. \tag{15}$$

In Equation (15), the marginal utility obtained from having a child equals the marginal utility forgone from having a child which includes the time cost, the expenditure cost and the bequest cost.

In Equation (16), the social planner equates the marginal rate of substitution between the young parent's leisure and consumption to their marginal rate of transformation:

$$\frac{\gamma C_{y,t}}{Z_t} = \frac{(1-\theta)y_t}{(1 - vN_t - e_tN_t - Z_t)}. \tag{16}$$

In Equation (17), the social planner equalizes the discounted marginal utility gain from higher child human capital with the marginal utility loss from human capital investment:

$$\frac{\alpha^{t+2}(1-\theta)y_{t+1}}{C_{y,t+1}} + (1-\delta)\lambda_{t+1}H_{t+2} = \lambda_t H_{t+1}. \tag{17}$$

We now define and characterize the socially optimal allocation.

Definition 1. Given any initial levels of physical capital K_0 and human capital H_0, and any predetermined fertility rate N_{-1}, the SO is a sequence of allocations $\left\{C_{y,t}, C_{o,t}, N_t, e_t, Q_t, Z_t, Y_t, K_{t+1}, H_{t+1}, \overline{H}_{t+1}\right\}_{t=0}^{\infty}$ with $\overline{H}_t = H_t$ satisfying resource feasibility constraint Equation (9), technologies (4) and (10), the optimal conditions (11)–(17), and the transversality conditions (given in Supporting Information S2: Appendix A).

With the log utility, the Cobb–Douglas functions for both human capital and production technologies and full depreciation of physical and human capital within one period, fertility, time allocations and the proportional allocations of output are all constant over time given any initial state. Therefore, by letting the fraction of output per young parent or per worker y_t allocated to X_t be time-invariant lower-case variables: $x = X_t/y_t$, where $X_t = C_{y,t}$, $C_{o,t}$, Q_t, S_t or B_t, total savings $S_t = k_{t+1}N_t$ and bequests per child $B_t = \theta y_t - C_{o,t}/N_{t-1}$, we can transform the variables in the optimality conditions and the resource feasibility constraint into their relative ratios to output per worker, thus obtaining the following socially optimal constant allocation rules:

$$b = \theta - \frac{\beta c_y}{\alpha}, \tag{18}$$

$$s = \alpha\theta, \tag{19}$$

$$c_o = \frac{c_y \beta N}{\alpha} \ (\text{for } t > 0), \tag{20}$$

$$c_{o,0} = \frac{c_y \beta N_{-1}}{\alpha} \ (\text{for } t = 0), \tag{21}$$

$$c_y = \frac{\alpha(1 - s - qN)}{\alpha + \beta}, \tag{22}$$

$$q = \frac{\alpha\delta(1-\theta)}{N(1-\alpha(1-\delta))}, \tag{23}$$

$$e = \frac{(1-\nu N)\varepsilon q}{\delta(1-\theta) + \varepsilon q N + \delta\gamma c_y}, \tag{24}$$

$$Z = \frac{(1-\nu N)\gamma c_y \delta}{\delta(1-\theta) + \varepsilon q N + \delta\gamma c_y}, \tag{25}$$

$$N = \frac{\delta\left[(\rho+\beta)c_y - s\right] - qN(\varepsilon+\delta)}{\nu\delta\left[(\rho+\beta+\gamma)c_y - s - qN + 1 - \theta\right]} = \frac{N_n^{SP}}{\nu N_d^{SP}} \tag{26}$$

where

$$N_n^{SP} \equiv \alpha\{(\rho+\beta)[(1-\alpha)[1-\alpha\theta(1-\delta)]] - (\alpha+\beta)[(1-\alpha(1-\delta))\theta + (1-\theta)(\delta+\varepsilon)]\},$$

$$N_d^{SP} \equiv [\alpha+\beta+\alpha(\rho+\beta+\gamma)][(1-\alpha)[1-\alpha\theta(1-\delta)]] - \theta(\alpha+\beta)(1-\alpha(1-\delta)).$$

The inverse relationship between the ratio of expenditure on child human capital to output per worker, q, and the number of children, N, given by Equation (23), and the inverse relationship between parental time in child human capital, e, and the number of children, N, given by Equation (24), reflect the quantity–quality trade-off of children.

To ensure the existence of a solution to the social planner's problem, it is assumed that the utility weight for the number of children ρ is sufficiently large relative to the utility weight for the welfare of children α.

Assumption 1. $\rho > \underline{\rho}^{SP} \equiv \frac{(\alpha+\beta)[(1-\alpha(1-\delta))\theta+(1-\theta)(\delta+\varepsilon)]-\beta(1-\alpha)(1-\alpha\theta(1-\delta))}{(1-\alpha)(1-\alpha\theta(1-\delta))}$.

When Assumption 1 holds, fertility N is positive and hence, c_o, c_y, q, e, Z are also positive in Equations (20)–(25), yielding a unique interior solution to the planner's problem.[10] We now show that the model globally converges to a unique balanced growth path. By substituting $y_t = D\mu_t{}^{\theta} l_t H_t$ into the solutions for k_{t+1} and H_{t+1} by using the definition of total savings $S_t = k_{t+1}N_t$ and (10), respectively, we obtain the evolution equation for the physical capital-effective labor ratio

$$\mu_{t+1} = \frac{sD^{1-\delta}\mu_t{}^{\Gamma}}{A(ql)^{\delta}Ne^{\varepsilon}}. \tag{27}$$

Since $0 < \Gamma \equiv \theta(1-\delta) < 1$, μ_t globally converges to a constant

$$\mu_{\infty} = \left\{\frac{sD^{1-\delta}}{AN(ql)^{\delta}e^{\varepsilon}}\right\}^{\frac{1}{1-\Gamma}}. \tag{28}$$

Using Equations (27) and (28), the following lemma establishes the global convergence of the model toward a balanced growth path.

Lemma 1. *Starting from any initial state (N_{-1}, K_0, H_0), the economy globally converges toward its balanced growth path.*

To obtain an analytical expression for the balanced steady-state growth rate of output per worker, we combine the production function (4) and human capital technology (10) to get

$$\frac{H_{t+1}}{H_t} = A(Dql)^{\delta}e^{\varepsilon}\mu_t{}^{\theta\delta}. \tag{29}$$

As μ_t converges to μ_∞, given by Equation (28), H_{t+1}/H_t converges to $1+g_\infty$, where the balanced steady-state growth rate of output per worker g_∞ is obtained as:

$$g_{\infty} = \left\{A^{1-\theta}D^{\delta}(ql)^{\delta(1-\theta)}e^{\varepsilon(1-\theta)}\left(\frac{s}{N}\right)^{\theta\delta}\right\}^{\frac{1}{1-\Gamma}} - 1. \tag{30}$$

The steady-state growth rate positively depends on the productivity parameters (A, D), the ratio of parental expenditure on child human capital to output per worker q, labor input l, the parental time input in child human capital e, and the savings rate s, while it negatively depends on fertility N.[11]

With the full characterization of the optimal path for the model, we now solve for the level of welfare in Equation (1). Using the solutions for (c_y, s, q, e, Z, N, c_o, $c_{o,0}$) and the sequence $\{\ln y_t\}_{t=0}^{\infty}$, given an initial state (N_{-1}, H_o, k_o), we obtain

$$\begin{aligned} U_o &= \beta\ln(c_{o,0}y_0) + \alpha\sum_{t=0}^{\infty}\alpha^t\left[\ln c_{y,t} + \ln y_t + \gamma\ln Z_t + \rho\ln N_t + \beta\ln c_{o,t+1} + \beta\ln y_{t+1}\right] \\ &= \Psi_0 + \Psi_1\ln c_y + \Psi_2\ln l + \Psi_3\ln N + \Psi_4\ln q + \Psi_5\ln e + \Psi_6\ln s + \Psi_7\ln Z \end{aligned} \tag{31}$$

where Ψ_0 is a constant that consists of the initial state (N_{-1}, H_o, k_o) and the productivity parameters (A, D), and Ψ_i, i = 1,2,...7, are positive constants consisting of the model parameters and their expressions are given in Supporting Information S2: Appendix A. Note that the utility level in Equation (31) is bounded above with $0 < \alpha < 1$.

4 | COMPETITIVE EQUILIBRIUM

Having understood the conditions for the socially optimal allocation, we now turn to the analysis of competitive equilibrium allocations. We first characterize the competitive equilibrium in a LF system and compare it with the SO. We then add PLS financed by lump-sum taxation into the model and characterize the competitive equilibrium. Finally, we explore socially optimal policies that eliminate the efficiency loss of human capital externalities and achieve the SO. All proofs are provided in Supporting Information S2: Appendix A.

4.1 | The LF

The problem of a dynastic family is to maximize utility in (1) subject to budget constraints (2) and (3), and the human capital technology (10), taking the average human capital and prices as given. The budget constraints (2) and (3) can be combined into a single budget constraint:

$$C_{y,t} = \frac{(S_{t-1}R_t - C_{o,t})}{N_{t-1}} + (1 - \nu N_t - e_t N_t - Z_t)W_t H_t - S_t - Q_t N_t. \tag{32}$$

By using the combined budget constraint and the human capital technology for substitution, this problem can be rewritten as follows:

$$\max_{\{C_{o,t},S_t,Q_t,H_{t+1},e_t,Z_t,N_t\}} U_o = \sum_{t=0}^{\infty}\Big\{\alpha^t\big[\beta \ln C_{o,t} + \alpha \ln\big((S_{t-1}R_t - C_{o,t})/N_{t-1}$$

$$+ (1 - \nu N_t - e_t N_t - Z_t)W_t H_t - S_t - Q_t N_t\big) + \alpha\gamma \ln Z_t$$

$$+ \alpha\rho \ln N_t\big] + \lambda_t\left[AQ_t^{\delta}e_t^{\varepsilon}\left(H_t^{\zeta}\overline{H}_t^{1-\zeta}\right)^{1-\delta} - H_{t+1}\right]\Big\},$$

where λ_t is the multiplier on the human capital technology. By using $R_{t+1}/N_t = \theta y_{t+1}/S_t$ and $l_{t+1}W_{t+1}H_{t+1} = (1-\theta)y_{t+1}$ for substitutions, the first-order conditions with respect to $C_{o,t}$, S_t, Q_t, e_t, N_t, Z_t and H_{t+1} for $t \geq 0$ are given below respectively.

The family equates the marginal rate of substitution between old parent's consumption and young parent's consumption to their relative prices:

$$\frac{\beta C_{y,t}}{\alpha C_{o,t}} = \frac{1}{N_{t-1}}. \tag{33}$$

Equating the marginal rate of substitution between young-age consumption across generations to their relative prices yields:

$$\frac{S_t C_{y,t+1}}{\alpha C_{y,t}} = \theta y_{t+1}. \tag{34}$$

The marginal rate of substitution between young parent's consumption and expenditure on child human capital equals their relative prices:

$$\frac{\alpha^{t+1}Q_t}{C_{y,t}} = \frac{\delta\lambda_t H_{t+1}}{N_t}. \tag{35}$$

The marginal rate of substitution between young parent's consumption and the time input in child human capital equals their relative prices:

$$\frac{\alpha^{t+1}e_t}{C_{y,t}} = \frac{\varepsilon\lambda_t H_{t+1}(1-\nu N_t - e_t N_t - Z_t)}{N_t(1-\theta)y_t}. \tag{36}$$

Equating the marginal rate of substitution between fertility and young parent consumption to their relative prices gives:

$$\frac{\rho C_{y,t}}{N_t} + \frac{\alpha C_{y,t} C_{o,t+1}}{C_{y,t+1} N_t^2} = \frac{(\nu + e_t)(1-\theta)y_t}{(1-\nu N_t - e_t N_t - Z_t)} + Q_t + \frac{\alpha C_{y,t}\theta y_{t+1}}{C_{y,t+1} N_t}. \tag{37}$$

The marginal rate of substitution between young parent's leisure and consumption equals their relative prices:

$$\frac{\gamma C_{y,t}}{Z_t} = \frac{(1-\theta)y_t}{(1-\nu N_t - e_t N_t - Z_t)}. \tag{38}$$

Notice that the first-order conditions with respect to $C_{o,t}$,S_t,Q_t,e_t,N_t and Z_t in Equations (33)–(38) coincide with their counterparts in Equations (11)–(16) for the socially optimal allocation.[12]

However, the first-order condition with respect to child human capital H_{t+1} is different from that in Equation (17) for the SO. That is, the family equates the marginal rate of substitution between child human capital across generations to their relative prices:

$$\frac{\alpha^{t+2}(1-\theta)y_{t+1}}{C_{y,t+1}} + \zeta(1-\delta)\lambda_{t+1}H_{t+2} = \lambda_t H_{t+1}. \tag{39}$$

In Equation (39), the private marginal utility from child human capital is lower than the social marginal utility from child human capital when human capital externalities exist ($0 < \zeta < 1$). Hence, child human capital in the LF equilibrium is lower than the socially optimal level when human capital externalities exist. This is because individuals, unlike the social planner, cannot internalize human capital externalities as they fail to recognize the positive effect of average human capital on child human capital. As a result, expenditure and time inputs in a child's human capital (Q_t, e_t) given in Equations (35) and (36) are also lower than their socially optimal levels when human capital externalities exist. Through the trade-off of quantity and quality of children, fertility N_t in Equation (37) is higher compared to the socially optimal level. Moreover, since raising a child is time-intensive, when fertility is higher than the socially optimal level, leisure Z_t in Equation (38) and labor are lower than their socially optimal levels.

However, in the absence of human capital externalities ($\zeta = 1$), the first-order condition with respect to child human capital H_{t+1} in Equation (39) is identical to that for the SO. Therefore, the LF solution without human capital externalities corresponds to the social planner's solution for all t and hence is socially optimal. This is a feature of dynastic models, as opposed to conventional life-cycle models whose competitive equilibrium solutions are typically not socially optimal even in the absence of externalities or other frictions.

Definition 2. Given an initial state (N_{-1},K_0,H_0), a competitive equilibrium in the LF economy is a sequence of allocations $\{B_t, C_{y,t}, C_{o,t}, N_t, S_t, e_t, Q_t, Z_t, Y_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}$ and prices $\{R_t, W_t\}_{t=0}^{\infty}$ such that: (i) taking the average human capital and prices as given, firms and households optimize and their solutions satisfy the budget constraints (2) and (3), technologies (4) and (10), optimal conditions (5)–(6) and (33)–(39), and the transversality conditions (given in Supporting Information S2: Appendix A); (ii) all markets clear with $K_{t+1} = L_t S_t$ and per worker labor time $l_t = 1 - \nu N_t - e_t N_t - Z_t$; and (iii) consistency holds with $H_t = \overline{H}_t$.

By letting the fraction of output per worker spent on item X_t be a time-invariant lower-case variable $x = X_t/y_t$, where $y_t = Y_t/L_t$ and $X_t = B_t$,$C_{y,t}$,$C_{o,t}$,S_t or Q_t, we can transform the variables in the first-order conditions and budget constraints into their ratios to output per worker. The constant allocation rules for b,s,c_o,$c_{o,0}$ and c_y are the same as those for the SO in Equations (18)–(22); while the constant allocation rules for q,e,Z and N are given as follows:

$$q = \frac{\alpha\delta(1-\theta)}{N(1-\alpha\zeta(1-\delta))}, \tag{40}$$

$$e = \frac{\alpha\varepsilon(\alpha+\beta)(1-\theta)}{NN_d^{LF}}, \tag{41}$$

$$Z = \frac{\gamma\alpha[(1-\alpha\theta)(1-\alpha\zeta(1-\delta)) - \alpha\delta(1-\theta)]}{N_d^{LF}}, \tag{42}$$

$$N = \frac{N_n^{LF}}{\nu N_d^{LF}}, \tag{43}$$

where

$$N_n^{LF} \equiv \alpha\{(\rho+\beta)[(1-\alpha\theta)(1-\alpha\zeta(1-\delta)) - \alpha\delta(1-\theta)] - (\alpha+\beta)[(1-\alpha\zeta(1-\delta))\theta + (1-\theta)(\delta+\varepsilon)]\},$$

$$N_d^{LF} \equiv [\alpha+\beta+\alpha(\rho+\beta+\gamma)][(1-\alpha\theta)(1-\alpha\zeta(1-\delta)) - \alpha\delta(1-\theta)] - \theta(\alpha+\beta)(1-\alpha\zeta(1-\delta)).$$

Note that when human capital externalities are absent ($\zeta = 1$), it can be easily verified that the constant allocation rules for q,e,Z and N are identical to those for the SO in Equations (23)–(26).

As in the social planner's problem, a sufficient condition under which the competitive equilibrium features an interior solution is a sufficiently large utility weight for the number of children (ρ) relative to the utility weight for the welfare of children (α) such that an interior solution for fertility N exists.[13] When fertility N is positive, choice variables c_o,c_y,q,e,Z in Equations (20), (22) and (40)–(42) are also positive, yielding a unique competitive solution in the LF. This sufficient condition is given in Lemma 2 below. Note that Assumption 1 ensures the condition in Lemma 2 holds.

Lemma 2. *There exists a unique equilibrium interior solution* (N,c_y,s,q,e,Z,c_o,$c_{o,0}$) *if the utility weight for the number of children is large enough such that*

$$\rho > \underline{\rho}^{LF} \equiv \frac{(\alpha+\beta)[(1-\alpha\zeta(1-\delta))\theta + (1-\theta)(\delta+\varepsilon)] - \beta[(1-\alpha\theta)(1-\alpha\zeta(1-\delta)) - \alpha\delta(1-\theta)]}{(1-\alpha\theta)(1-\alpha\zeta(1-\delta)) - \alpha\delta(1-\theta)}.$$

The LF equilibrium also features a balanced steady-state growth path which is characterized by Equation (30) for the growth rate of output per worker (g_∞) and by Equation (31) for the welfare level (U_o). Next, we examine the effects of human capital externalities ($0 < \zeta < 1$) on the endogenous variables. Proposition 1 establishes these effects.

Proposition 1. *Larger human capital externalities (i.e., lower ζ) (i) do not affect the savings rate (s), but lead to (ii) lower ratio of parental expenditure on child human capital to output per worker (q), (iii) lower parental time in child human capital (e), (iv) lower leisure (Z), (v) lower labor time (l), (vi) higher young-age consumption relative to output per worker (c_y), (vii) higher old-age consumption relative to output per worker (c_o), and (viii) higher fertility (N). Results (i), (ii), (iii), (v) and (viii) show that larger human capital externalities lead to a lower balanced steady-state growth rate of output per worker (g_∞).*

The results in Proposition 1 imply that in the LF economy with human capital externalities, the ratio of parental expenditure on a child's human capital to output per worker q, the parental time input in child human capital e, leisure Z, labor time l, and the growth rate of output per worker g_∞ are lower than their counterparts in the SO; whereas the ratio of young-age consumption to output per worker c_y, the ratio of old-age consumption to output per worker c_o, and fertility N are higher than their counterparts in the SO. In terms of the welfare effect, intuitively, the deviation of allocations from their socially optimal levels arising from human capital externalities leads to a lower welfare level in the LF economy relative to the SO. The larger the human capital externalities, the larger the efficiency loss in the LF economy.

We next investigate if PLS can mitigate the efficiency loss of human capital externalities and explore the optimal rate of PLS.

4.2 | Parental leave subsidization financed by lump-sum tax (PLS_LS)

This section considers a PLS policy financed by a lump-sum tax (PLS_LS). Let $T_t > 0$ denote the amount of lump-sum taxes and $\pi_t \in (0,1)$ denote the parental leave subsidy rate. Then, the budget constraint in young adulthood becomes

$$C_{y,t} = B_t + (1 - \nu N_t - e_t N_t - Z_t) W_t H_t - T_t - S_t - Q_t N_t + \pi_t e_t N_t W_t \overline{H}_t, \tag{44}$$

where $\pi_t e_t N_t W_t \overline{H}_t$ is the amount of parental leave subsidy which is determined by the parental leave subsidy rate π_t, the total parental time input in children human capital development $e_t N_t$, and the average effective wage in the economy $W_t \overline{H}_t$.[14]

The government balanced budget constraint is given by

$$T_t = \pi_t \overline{e}_t \overline{N}_t W_t \overline{H}_t,$$

where the bar above each variable indicates its average level within the economy.

The problem of a dynastic family is to maximize utility in Equation (1) subject to the budget constraints (2) and (44), and the human capital technology in Equation (10), given the average human capital, prices, lump-sum taxes T_t, and the leave subsidy rate π_t. For $t \geq 0$, the first-order conditions with respect to $C_{o,t}$, S_t, Q_t, H_{t+1} and Z_t are the same as those in the LF economy. By using $R_{t+1}/N_t = \theta y_{t+1}/S_t$ and $l_{t+1} W_{t+1} H_{t+1} = (1 - \theta) y_{t+1}$ for substitutions, the first-order conditions with respect to e_t and N_t are given by Equations (45) and (46) as follows.

The marginal rate of substitution between young parent's consumption and the time input in child human capital equals their relative prices:

$$\frac{\alpha^{t+1} e_t}{C_{y,t}} = \frac{\varepsilon \lambda_t H_{t+1} (1 - \nu N_t - e_t N_t - Z_t)}{(1 - \pi_t) N_t (1 - \theta) y_t}. \tag{45}$$

By lowering the opportunity cost of parental time input in child human capital, the parental leave subsidy π_t tends to increase the parental time input in child human capital. Hence, PLS_LS can mitigate the under-investment of parental time in child human capital that occurs in the LF system.

Equating the marginal rate of substitution between fertility and the young-parent's consumption to their relative prices gives:

$$\frac{\rho C_{y,t}}{N_t} + \frac{\alpha C_{y,t} C_{o,t+1}}{C_{y,t+1} N_t^2} = \frac{[\nu + e_t(1 - \pi_t)](1 - \theta) y_t}{(1 - \nu N_t - e_t N_t - Z_t)} + Q_t + \frac{\alpha C_{y,t} \theta y_{t+1}}{C_{y,t+1} N_t}. \tag{46}$$

The parental leave subsidy π_t tends to increase the time cost of having a child, $e_t(1 - \pi_t)$. Hence, π_t tends to reduce the fertility rate.

Given a time-invariant parental leave subsidy rate and the corresponding lump-sum tax, the constant allocation rules for b, s, c_o, $c_{o,0}$ and c_y are the same as those in the LF. The constant allocation rules for q, e, Z, N and l are given as follows.

$$q = \frac{\alpha \delta (1 - \theta)}{N(1 - \alpha \zeta (1 - \delta))}, \tag{47}$$

$$e = \frac{\nu \varepsilon q N}{(1 - \pi)\left\{\delta\left[(\rho + \beta) c_y - q N - \alpha \theta\right] - \varepsilon q N\right\}}, \tag{48}$$

$$N = \frac{(1 - \pi)\left\{\delta\left[(\rho + \beta) c_y - q N - \alpha \theta\right] - \varepsilon q N\right\}}{\nu\left\{(1 - \pi)\delta\left[(\rho + \beta + \gamma) c_y - q N - \alpha \theta + 1 - \theta\right] + \pi \varepsilon q N\right\}}, \tag{49}$$

$$Z = \frac{(1 - \pi)\delta \gamma c_y}{\left\{(1 - \pi)\delta\left[(\rho + \beta + \gamma) c_y - q N - \alpha \theta + 1 - \theta\right] + \pi \varepsilon q N\right\}}. \tag{50}$$

By substituting the solutions for e,N and Z given above into $l = 1 - (\nu + e)N - Z$, the solution for labor time is:

$$l = \frac{(1-\pi)\delta(1-\theta)}{\left\{(1-\pi)\delta\left[(\rho+\beta+\gamma)c_y - qN - \alpha\theta + 1 - \theta\right] + \pi\varepsilon qN\right\}}. \tag{51}$$

Since $c_y = \alpha(1 - \alpha\theta - qN)/(\alpha + \beta)$ and $qN = \alpha\delta(1 - \theta)/(1 - \alpha\zeta(1 - \delta))$ are both independent of π, the effects of PLS_LS are straightforward, as summarized in Proposition 2.

Proposition 2. *Parental leave subsidization financed by lump-sum taxation (π) has (i) positive effects on parental time in child human capital (e), the ratio of parental expenditure on child human capital to output per worker (q), and the balanced steady-state growth rate of output per worker (g_∞); (ii) negative effects on fertility (N), leisure (Z), labor time (l), and old-age consumption relative to output per worker (c_o); and (iii) no effect on young-age consumption relative to output per worker (c_y), the ratio of transfers per child to output per worker (b), and the savings rate (s).*

Proposition 2 shows that when PLS_LS reduces the opportunity cost of the parental time input in child human capital, a young parent tends to spend more time on a child's human capital but less time on leisure and labor. Additionally, higher parental time input in child human capital also leads to higher parental expenditure on child human capital relative to output per worker and a lower fertility rate through the quantity-quality trade-off of children. The balanced steady-state growth rate of output per worker in Equation (30) increases with PLS_LS, as PLS_LS increases parental time input in child human capital, reduces fertility, and does not affect other variables in Equation (30).[15] The next proposition establishes the existence and uniqueness of a positive parental leave subsidy rate that maximizes the welfare in the PLS_LS economy.

Proposition 3. *When parental leave subsidization is financed by lump-sum taxation, the optimal parental leave subsidy rate, π^*, exists and it is positive and unique:*

$$\pi^* = \frac{\alpha(1-\zeta)(1-\delta)\left[(\alpha+\beta)(1-\theta-\alpha\theta) + \alpha(1-\alpha\theta)(\rho+\beta+\gamma)\right]}{p_d} > 0,$$

where p_d is a positive constant when Assumption 1 holds (see Supporting Information S2: Appendix A). In addition, π^* is zero when human capital externalities are absent ($\zeta = 1$), and is higher when the degree of human capital externalities is higher (lower ζ).

Proposition 3 implies that PLS_LS leads to higher welfare and a higher growth rate of output per worker than in the LF economy. However, PLS_LS cannot achieve the SO because a single parental leave subsidy cannot correct the efficiency loss in various dimensions when fertility, human capital investments and time allocations are all endogenous. Specifically, PLS_LS cannot reduce young-age consumption relative to output per worker and bring leisure and labor time up to their socially optimal levels as indicated by Proposition 2. As a result, fertility is higher and parental time and expenditure inputs in child human capital are lower than their socially optimal levels, as stated below in Corollary 1.

Corollary 1. *When parental leave subsidization is financed by lump-sum taxation, fertility (N) is higher but parental time in child human capital (e) and the ratio of parental expenditure on child human capital to output per worker (q) are lower than their socially optimal levels.*

Since PLS_LS mitigates the efficiency loss of human capital externalities but cannot achieve the SO, we next explore other taxes and subsidies along with parental leave subsidy that may achieve the SO.

4.3 | Efficient policies

To derive efficient subsidies and taxes to attain socially optimal outcomes, we consider a set of government policies: $\tau_{l,t}$, $\tau_{s,t}$,π_t,$\pi_{q,t}$ and T_t, where $\tau_{l,t}$ denotes labor income tax rate, $\tau_{s,t}$ denotes capital income tax rate, π_t denotes a subsidy rate on general work leave including parental leave, $\pi_{q,t}$ denotes expenditure subsidy rate on child human capital, and T_t

denotes a lump-sum tax (transfer if negative) in period t. The budget constraints can be combined into a single budget constraint:

$$C_{y,t} = \frac{S_{t-1}R_t\left(1-\tau_{s,t}\right) - C_{o,t}}{N_{t-1}} + (1 - vN_t - e_tN_t - Z_t)W_tH_t\left(1-\tau_{l,t}\right) - S_t \\ - Q_t\left(1-\pi_{q,t}\right)N_t - T_t + \pi_t(vN_t + e_tN_t + Z_t)W_t\overline{H}_t, \tag{52}$$

where paid work leave is based on the average effective wage in the economy $W_t\overline{H}_t$.[16]

The government balanced budget constraint is given by

$$\tau_{l,t}\left(1 - v\overline{N}_t - \overline{e}_t\overline{N}_t - \overline{Z}_t\right)W_t\overline{H}_t + \frac{\tau_{s,t}\overline{S}_{t-1}R_t}{\overline{N}_{t-1}} + T_t \\ = \pi_{q,t}\overline{Q}_t\overline{N}_t + \pi_t\left(v\overline{N}_t + \overline{e}_t\overline{N}_t + \overline{Z}_t\right)W_t\overline{H}_t, \tag{53}$$

where the bar above each variable indicates its average level within the economy.

The problem of a dynastic family is to maximize utility in Equation (1) subject to the single budget constraint (52) and the human capital technology (10), taking the average human capital, prices and government policies as given. For $t \geq 0$, the first-order condition with respect to $C_{o,t}$ is the same as in the LF. The first-order conditions with respect to S_t, Q_t, e_t, H_{t+1}, N_t and Z_t are given as follows, respectively.

$$\frac{S_tC_{y,t+1}}{\alpha C_{y,t}} = \left(1-\tau_{s,t}\right)\theta y_{t+1}, \tag{54}$$

$$\frac{\alpha^{t+1}Q_t}{C_{y,t}} = \frac{\delta\lambda_tH_{t+1}}{\left(1-\pi_{q,t}\right)N_t}, \tag{55}$$

$$\frac{\alpha^{t+1}e_t}{C_{y,t}} = \frac{\varepsilon\lambda_tH_{t+1}(1 - vN_t - e_tN_t - Z_t)}{\left(1-\tau_{l,t}-\pi_t\right)N_t(1-\theta)y_t}, \tag{56}$$

$$\frac{\alpha^{t+2}\left(1-\tau_{l,t+1}\right)(1-\theta)y_{t+1}}{C_{y,t+1}} + \zeta(1-\delta)\lambda_{t+1}H_{t+2} = \lambda_tH_{t+1}, \tag{57}$$

$$\frac{\rho C_{y,t}}{N_t} + \frac{\alpha C_{y,t}C_{o,t+1}}{C_{y,t+1}N_t^2} = \frac{(v+e_t)(1-\theta)y_t\left(1-\tau_{l,t}-\pi_t\right)}{(1 - vN_t - e_tN_t - Z_t)} + \left(1-\pi_{q,t}\right)Q_t + \frac{\alpha C_{y,t}\left(1-\tau_{s,t+1}\right)\theta y_{t+1}}{C_{y,t+1}N_t}, \tag{58}$$

$$\frac{\gamma C_{y,t}}{Z_t} = \frac{\left(1-\tau_{l,t}-\pi_t\right)(1-\theta)y_t}{(1 - vN_t - e_tN_t - Z_t)}. \tag{59}$$

If the subsidy and tax rates are time-invariant, the constant allocation rules for c_y, c_o and $c_{o,0}$ are the same as those in the LF economy. The constant allocation rules for b,s,q,e, N and Z are given as follows:

$$b = (1-\tau_s)\theta - \frac{\beta c_y}{\alpha}, \tag{60}$$

$$s = (1-\tau_s)\alpha\theta, \tag{61}$$

$$q = \frac{(1-\tau_l)\alpha\delta(1-\theta)}{N\left(1-\pi_q\right)[1-\alpha\zeta(1-\delta)]}, \tag{62}$$

$$e = \frac{\left(1-\pi_q\right)(1-vN)\varepsilon q}{\Upsilon_2 + \delta\gamma c_y}, \tag{63}$$

$$N = \frac{\delta Y_1 - (1-\pi_q)\varepsilon q N}{(\nu\delta)\left[(1-\tau_l-\pi)(1-\theta)+c_y\gamma+Y_1\right]}, \tag{64}$$

$$Z = \frac{(1-\nu N)\gamma c_y \delta}{Y_2+\delta\gamma c_y}, \tag{65}$$

where $Y_1 \equiv [(\rho+\beta)c_y - s - (1-\pi_q)qN]$ and $Y_2 \equiv (1-\tau_l-\pi)\delta(1-\theta)+(1-\pi_q)\varepsilon qN$. Denote $\tau_{LS} \equiv T_t/y_t$ as the constant lump-sum tax rate, we establish the efficient policies below.

Proposition 4. *In the presence of human capital externalities, the socially optimal policies* $(\tau_l, \tau_s, \tau_{LS}, \pi_q, \pi)$ *are determined as:*

(i) $\tau_s = 0$,
(ii) $\pi_q = 0$,
(iii) $\pi = \frac{\alpha(1-\delta)(1-\zeta)}{1-\alpha(1-\delta)}$,
(iv) $\tau_l = -\pi$,
(v) $\tau_{LS} = \frac{-\tau_l(1-\theta)}{l}$, *where* $l = 1 - \nu N - eN - Z$ *is evaluated at the socially optimal level.*

In the presence of human capital externalities ($0 < \zeta < 1$), Proposition 4 reveals that the socially optimal capital income tax rate should be zero to attain the socially optimal savings rate because capital income tax reduces savings from its socially optimal level. The socially optimal rate of expenditure subsidy on child human capital should also be zero. This is because expenditure subsidy tends to raise parental expenditure on child human capital and hence partially crowd out parental time input in child human capital as implied by the Cobb-Douglas form of child human capital technology. Thus, expenditure subsidy reduces the time input in child human capital from its socially optimal level. More importantly, the efficient policies require the socially optimal rate of general work leave subsidy, which includes parental leave, to be equal to that of labor income subsidy when these subsidies are funded by lump-sum taxes.

The intuition is as follows. First, labor income subsidy raises the time costs of having a child and leisure in Equations (64) and (65), respectively. Second, general work leave subsidy reduces the time costs of having a child and leisure in Equations (64) and (65). Hence, when the leave subsidy rate equals the labor income subsidy rate, the first and the second effects cancel out each other due to their opposing effects on fertility and leisure. However, there is a third effect, that is, labor income subsidy increases children's labor income after subsidies when they become workers in the future. As a result, labor income subsidy increases the marginal benefit of expenditure on child human capital, and hence induces parent to increase her expenditure on child human capital in Equation (62). Hence, by setting the right level of labor income subsidy, expenditure on child human capital q in Equation (62) can be raised to its socially optimal level. Higher q at the socially optimal level causes fertility to fall to its socially optimal rate via the quantity-quality trade-off of children. When fertility reduces to its optimal rate, labor, leisure and time input in child human capital also increase to their socially optimal levels. Accordingly, young-age and old-age consumption relative to output per worker also fall toward their socially optimal levels.

Hence, setting the right subsidy rates for labor income and work leave financed by a lump-sum tax can internalize human capital externalities and achieve the SO without requiring expenditure subsidization on child human capital. This illustrates the importance of introducing PLS as part of the efficient policies.

5 | A QUANTITATIVE ANALYSIS

5.1 | The U.S. baseline model

In this section we calibrate the model to the U.S. economy to examine the quantitative implications of introducing PLS. The U.S. is currently the only developed country that does not subsidize parental leave at the national level. The Family and Medical Leave Act in the U.S. only provides certain workers (not all workers) with 12 weeks of unpaid leave to care for a new child.

The basic model laid out in Section 2 allows us to obtain the analytical results discussed earlier, but it abstracts from many realistic features of the U.S. data that can be potentially important for the quantitative implications of PLS. We therefore extend the basic model by including labor income tax at rate τ_l, gross capital income tax at rate τ_s, human capital expenditure or education subsidy at rate π_q which captures public education expenditure on child human capital, and a lump-sum tax T_t (transfer if negative). The budget constraints (2) and (3) can be combined into a single budget constraint:

$$C_{y,t} = \frac{S_{t-1}R_t(1-\tau_s) - C_{o,t}}{N_{t-1}} + (1 - \nu N_t - e_t N_t - Z_t) W_t H_t (1-\tau_l) - S_t - Q_t(1-\pi_q)N_t - T_t. \tag{66}$$

The government balanced budget constraint is given by

$$\tau_l(1 - \nu\overline{N}_t - \overline{e}_t\overline{N}_t - \overline{Z}_t) W_t \overline{H}_t + \frac{\tau_s \overline{S}_{t-1} R_t}{\overline{N}_{t-1}} + T_t = \pi_q \overline{Q}_t \overline{N}_t \tag{67}$$

where the bar above each variable indicates its average level within the economy.

Following a similar approach as before, we obtain the following constant allocation rules:

$$b = (1-\tau_s)\theta - \frac{\beta c_y}{\alpha}, \tag{68}$$

$$s = (1-\tau_s)\alpha\theta, \tag{69}$$

$$c_o = \frac{c_y \beta N}{\alpha} \quad (\text{for } t > 0), \tag{70}$$

$$c_{o,0} = \frac{c_y \beta N_{-1}}{\alpha}, \tag{71}$$

$$c_y = \frac{\alpha(1 - s - qN)}{\alpha + \beta}, \tag{72}$$

$$q = \frac{(1-\tau_l)\alpha\delta(1-\theta)}{N(1-\pi_q)[1 - \alpha\zeta(1-\delta)]}, \tag{73}$$

$$e = \frac{(1-\pi_q)(1-\nu N)\varepsilon q}{\Upsilon_3 + \delta\gamma c_y}, \tag{74}$$

$$Z = \frac{(1-\nu N)\gamma c_y \delta}{\Upsilon_3 + \delta\gamma c_y}, \tag{75}$$

$$N = \frac{[\delta\Upsilon_1 - (1-\pi_q)\varepsilon q N]}{\nu\{[\delta\Upsilon_1 - (1-\pi_q)\varepsilon q N + c_y\gamma\delta] + \Upsilon_3\}}, \tag{76}$$

where $\Upsilon_1 \equiv [(\rho + \beta)c_y - s - (1-\pi_q)qN]$, as defined in Section 4.3, $\Upsilon_3 \equiv (1-\tau_l)\delta(1-\theta) + (1-\pi_q)\varepsilon q N$.

There is no PLS in the baseline model, given that parental leave entitlements in the U.S. at the national level is zero (Carneiro et al., 2015). In our counterfactual experiments, we introduce PLS into the baseline model to examine the quantitative implications of PLS. With PLS, the combined budget constraint is given by:

$$C_{y,t} = \frac{S_{t-1}R_t(1-\tau_s^c) - C_{o,t}}{N_{t-1}} + (1 - \nu N_t - e_t N_t - Z_t) W_t H_t (1-\tau_l^c) - S_t - Q_t(1-\pi_q^c)N_t - T_t + \pi e_t N_t W_t \overline{H}_t, \tag{77}$$

where π denotes the parental leave subsidy rate, and we put superscript "c" to other taxes and subsidies as they could be altered to balance government budget in the counterfactual economies with PLS. The government budget constraint is then given by

$$\tau_l^c\left(1-v\overline{N}_t-\overline{e}_t\overline{N}_t-\overline{Z}_t\right)W_t\overline{H}_t+\frac{\tau_s^c\overline{S}_{t-1}R_t}{\overline{N}_{t-1}}+T_t=\pi_q^c\overline{Q}_t\overline{N}_t+\pi\overline{e}_t\overline{N}_tW_t\overline{H}_t. \tag{78}$$

The constant allocation rules for $b,s,c_o,c_{o,0},c_y$ and q are the same as those in the baseline model. The constant allocation rules for e,N and Z are given as follows:

$$e=\frac{\left(1-\tau_l^c\right)\left(1-\pi_q^c\right)(1-vN)\varepsilon q}{\left(1-\tau_l^c\right)\mathrm{Y}_2+\left(1-\tau_l^c-\pi\right)\delta\gamma c_y}, \tag{79}$$

$$N=\frac{\left(1-\tau_l^c-\pi\right)\left[\delta\mathrm{Y}_1-\left(1-\pi_q^c\right)\varepsilon qN\right]}{v\left\{\left(1-\tau_l^c-\pi\right)\left[\delta\mathrm{Y}_1-\left(1-\pi_q^c\right)\varepsilon qN+c_y\gamma\delta\right]+\left(1-\tau_l^c\right)\mathrm{Y}_2\right\}}, \tag{80}$$

$$Z=\frac{\left(1-\tau_l^c-\pi\right)(1-vN)\gamma c_y\delta}{\left(1-\tau_l^c\right)\mathrm{Y}_2+\left(1-\tau_l^c-\pi\right)\delta\gamma c_y}, \tag{81}$$

where $\mathrm{Y}_1\equiv\left[(\rho+\beta)c_y-s-\left(1-\pi_q^c\right)qN\right]$, $\mathrm{Y}_2\equiv\left(1-\tau_l^c-\pi\right)\delta(1-\theta)+\left(1-\pi_q^c\right)\varepsilon qN$.

Comparing the government budget constraints in the baseline and counterfactual economies, Equations (67) and (78) respectively, we can see that the introduction of PLS must be financed by raising taxes and/or reducing other subsidies to balance government budget. We consider three cases: (i) the PLS is financed by an increase in labor income tax rate only; (ii) the PLS is financed by an increase in capital income tax rate only; (iii) the PLS is financed by a reduction in education subsidy rate only. Next, we describe how we calibrate the model to the U.S. economy and present the quantitative results.

5.2 | Calibration

We calibrate the baseline model to match a selected set of moments in the U.S. data. We first briefly describe the calibration procedure and the targeted data moments used in the calibration, then discuss the parameter values obtained. The targeted moments and parameter values obtained are reported in Tables 1 and 2, respectively.

5.2.1 | Calibration procedure

The parameters that need to be calibrated include: the preference parameters, α, β, γ, and ρ in the utility function; parameters in the production function θ and D; parameters in the human capital technology, δ, ε, ζ, and A; the fixed parental time spent on a child, v; and the various tax and subsidy rates, τ_l, τ_s, and π_q.

The income share of capital θ, the income tax and education subsidy rates τ_l and π_q, and fixed parental time v can be directly calibrated to match their empirical counterparts. Given these externally calibrated parameters and a value of ζ, the values of α, β, γ, ρ, δ, and ε can be uniquely and jointly determined to match the empirical counterparts of c_o, c_y, q, e, Z, and N, according to Equations (70) and (72)–(76). The value of $1-\zeta$ indicates human capital externality in the human capital technology. To calibrate ζ, we note that Borjas (1995) estimates a range of 0.18–0.3 for the elasticity of a person's human capital with respect to the average human capital of the father's generation, which corresponds to $(1-\zeta)(1-\delta)$ in our model. Hence, we consider the middle point of this range and choose the baseline value of ζ such that $(1-\zeta)(1-\delta)=0.24$. Estimates of Choi (2011) imply a much higher value of 0.44 for $(1-\zeta)(1-\delta)$. In Supporting Information S2: Appendix C, a robustness analysis is performed in which we consider alternative values of $(1-\zeta)(1-\delta)$

TABLE 1 Targeted data moments.

Description and notation	Value
Number of children per young parent (N)	0.9594
Consumption expenditure of a young household relative to income per young household (c_y)	0.5246
Consumption expenditure of an old household relative to income per young household (c_o)	0.3047
Total expenditure on child education relative to income per young household (q)	0.0675
Fraction of time allocated to a child when the child is young (0–6 years old) for a young parent (e)	0.0471
Fraction of time allocated to a child when the child is elder (7–17 years old) for a young parent (v)	0.0327
Fraction of time allocated to leisure for a young parent (Z)	0.3313
Externality in the human capital technology $(1-\zeta)(1-\delta)$	0.24
Annual growth rate of real GDP per capita (g_∞)	1.46%
Income share of labor $(1-\theta)$	0.5941
Labor income tax rate (τ_l)	22.4%
Net capital gain tax rate ($\tau_s R/(R-1)$)	15%
Parental expenditure on childcare and education relative to income per young household ($(1-\pi_q)\, q$)	0.0104

Note: The data source for N, c_y and c_o is PSID, for constructing q is OECD, and for v, e and Z is ATUS. Human capital externality is from Borjas (1995). Annual growth rate of real GDP per capita and income share of labor are from World Bank and FRED, respectively. Labor income tax rate and net capital gain tax rate are obtained from Guvenen et al. (2023) and IRS, respectively. Finally, parental expenditure on childcare and education is from Lino et al. (2017).

TABLE 2 Baseline calibration.

Utility	
Discounting factor	$\alpha = 0.2313$
Utility weight for old-age consumption	$\beta = 0.1401$
Utility weight for number of children	$\rho = 0.1700$
Utility weight for leisure	$\gamma = 0.4917$
Production of final output	
Income share of capital	$\theta = 0.4059$
Production of human capital	
Weight of expenditure input	$\delta = 0.0788$
Weight of parental time input	$\varepsilon = 0.2779$
Degree of externality $(1-\zeta)$	$\zeta = 0.7395$
Productivity parameters	$A^{1-\theta}D^{\delta} = 2.724$
Tax and subsidy rates	
Tax rate on labor income	$\tau_l = 0.224$
Tax rate on gross capital income	$\tau_s = 0.0091$
Subsidy rate on child education	$\pi_q = 0.8459$

and re-calibrate the parameters. Based on Equation (30), the value of the constant $A^{1-\theta}D^{\delta}$ can be obtained by matching the empirical counterpart of g_∞.[17] Finally, the value of capital income tax rate τ_s is obtained by matching the capital gains tax in the data.

5.2.2 | Data moments

The values of the data moments for the U.S. economy and their data sources are presented in Table 1. Supporting Information S2: Appendix B describes in detail the construction of all targeted data moments. Unless otherwise stated, the data moments are obtained by taking the average of the annual figures for years between 2011 and 2015. The main reason we choose these years is that data for education expenditure on children (Lino et al., 2017) are only available for these years. One period in the model is set to 30 years, so young age and old age correspond to ages 30–59 and 60–89. In both micro-data sets that we use, American PSID and American Time Use Survey (ATUS), we consider two age groups. The young parent age group includes households with at least one child under 18 years of age. The reference person for each household in this age group is required to be between age 30 and 59 and work full time. The old age group includes households who do not have any child below 18 years of age and whose reference persons are retired and between age 60 and 89.

First, we use PSID to construct the average number of children per young parent (N) and the average consumption expenditure of a household relative to output per worker in the young parent age group (c_y) and in the old parent age group (c_o), where output per worker is calculated as income per young household (total household income divided by the number of young households). Second, the empirical counterpart of the expenditure on child human capital to output per worker (q) is constructed using the ratio of total public and private expenditure on child education to GDP in the U.S. data. The education subsidy rate (π_q) is calibrated by matching the ratio of parental expenditure on child human capital relative to output per worker (given by $(1 - \pi_q)\ q$), where the former is obtained as the average expenditure on childcare and education in Lino et al. (2017).

Third, the fraction of time allocated to a child (v and e) and leisure (Z) are obtained from ATUS. We interpret e as the fraction of a young parent's time spent on a child when the child is young (0–6 years old), while the fixed parameter v as the fraction of time allocated when the child is elder (7–17 years old). This interpretation is motivated by a few considerations. First, as documented in Ruhm (2004) and Del Boca et al. (2014), parental time input in children in their early childhood is particularly important for children's human capital development. Our formulation that e enters the child human capital technology is in line with this empirical evidence. Second, the amount of time spent with younger children before they reach the compulsory school age is more of a choice to parents.[18] Some parents choose to look after their young children by themselves while others choose to use childcare services. For children of school age, the amount of time parents spent with them tends to be much shorter and less variable across families.[19] Third, this interpretation allows us to connect parental time subsidization considered here with parental leave policies in practice, the majority of which concern maternity or paternity leave for looking after newborns and younger children.

Fourth, data for annual growth rate of GDP per capita (g_∞) and share of labor compensation in GDP $(1 - \theta)$ are collected from World Bank and Federal Reserve Economic Data, respectively. Finally, labor income tax rate (τ_l) is from Guvenen et al. (2023). And capital income tax rate (τ_s), which is imposed on gross capital income, is obtained by matching the net capital gains tax rate in Internal Revenue Service.

5.2.3 | Parameter values

Given the targeted data moments reported in Table 1 and following the calibration procedure described in Section 5.2.1, we obtain the values of the internally calibrated parameters. For parameters in the utility function, the discount factor across generations is $\alpha = 0.2313$, implying an annual discount factor of 0.9524. The utility weight for old age consumption is $\beta = 0.1401$, suggesting that the weight households put on old age consumption relative to young age consumption is $\beta/\alpha = 0.6054$. Moreover, the utility weight for the number of children is $\rho = 0.17$ and the utility weight for leisure in the utility function is $\gamma = 0.4917$.

For parameters in the human capital technology, the weight of parental expenditure on child human capital (which can be interpreted as physical capital input in the production of human capital) is $\delta = 0.0788$, which is smaller than the weight of physical capital in the production of final goods ($\theta = 0.4059$). This reflects the fact that the production of human capital is less physical capital (or more human capital) intensive than the production of final goods. The calibrated value of ζ is 0.7395, not too far below a value of 1 which indicates no externality. This suggests that the degree of human capital externality is not very high in our baseline calibration. We consider alternative calibrations with higher or lower degree of human capital externality in Supporting Information S2: Appendix C. The weight of parental time input in the production of child human capital, ε, is 0.2779. As ε is larger than δ, this result suggests that parent's time input is more important than expenditure input in child human capital development. This finding is consistent with the empirical finding of Del Boca et al. (2014), where they examine children's human capital development and find that the parent time input is more important than expenditure or physical input for younger children. The constant that consists of productivity parameters A and D, $A^{1-\theta}D^{\delta}$, is 2.7240. Finally, the tax rate on gross capital income $\tau_s = 0.0091$.

The values of both externally and internally calibrated parameters are reported in Table 2, referred to as the baseline calibration hereafter.

5.3 | Quantitative results

In this section, we compare the equilibrium outcomes based on the U.S. baseline calibration for five different economies: (1) the U.S. baseline competitive equilibrium, BCE; (2) the SO; (3) the counterfactual economy with optimal PLS financed by raising labor income tax rate, PLS_LI; (4) the counterfactual economy with optimal PLS financed by raising capital income tax rate, PLS_CI; (5) the counterfactual economy with optimal PLS financed by reducing education subsidy rate, PLS_ES. We focus on these five economies because we believe these scenarios are empirically relevant and the differences in their welfare and growth outcomes have important policy implications. We do not consider a counterfactual economy where PLS is financed by a lump-sum tax since we view this case as less applicable in real practice.

We define the welfare gain in economy i, where i refers to each of the economies described above, over the baseline competitive equilibrium (BCE) using the notion of consumption equivalent variation, which is defined as the percentage change in both young-age and old-age consumption in every period (Δ^i) such that the welfare in the BCE economy (U_0^{CE}) reaches the welfare in economy i (U_0^i). Using Equation (1), it is straightforward to show

$$U_0^i = U_0^{CE} + \sum_{t=0}^{\infty} \alpha^t (\beta + \alpha) \ln \left(1 + \Delta^i\right),$$

and thus

$$\Delta^i = exp\left[\frac{\left(U_0^i - U_0^{CE}\right)(1-\alpha)}{\alpha + \beta}\right] - 1.$$

The equilibrium outcomes for the five different economies and the welfare gains over the BCE are reported in Table 3. Before discussing these quantitative results, it's worth to verify our theoretical results in Section 4 concerning the efficiency of the LF equilibrium, the PLS financed by lump-sum taxation (PLS_LS), and the efficient policies that attain the SO. The results for LF and PLS_LS are reported in Table A1 in Appendix, which are obtained by shutting down all or relevant taxes and subsidies in the U.S. baseline model but keeping other parameters at their baseline values.

The quantitative results for LF are consistent with the theoretical results established in Proposition 1. In particular, expenditure and time inputs (q and e) in child human capital are significantly lower than their counterparts in the SO while fertility is significantly higher. The welfare in LF is lower, as indicated by a 0.07% welfare gain of the SO over the LF outcome. Consistent with Proposition 3, there exists an optimal PLS rate of 7.07% that maximizes the welfare in the PLS_LS economy. Introducing this optimal PLS improves upon the LF outcome, achieving a welfare gain of 0.014% which is much smaller than the welfare gain in the SO case. The efficient policies that attain the SO involves introducing a subsidy rate of 7.6% for both labor income and general work leave. Again, this is consistent with the theoretical characterization in Proposition 4.

5.3.1 | Comparison between BCE and SO

First, we compare the BCE economy with the SO. As shown in Table 3, compared with the SO, fertility (N), expenditure on child human capital relative to output per worker (q), leisure (Z), old-age consumption relative to output per worker (c_o) and bequests per child relative to output per worker (b) are all substantially higher, while the parental time input in

TABLE 3 A comparison of equilibria.

Variables	(1) Baseline competitive equilibrium	(2) Social optimum	(3) Optimal PLS_LI	(4) Optimal PLS_CI	(5) Optimal PLS_ES	(6) Alternative PLS_ES
N	0.9594	0.5288	0.9754	0.9628	1.1424	0.9813
q	0.0675	0.0260	0.0659	0.0673	0.0269	0.0621
e	0.0471	0.0985	0.0510	0.0476	0.0730	0.0507
Z	0.3313	0.2932	0.3312	0.3311	0.3235	0.3304
l	0.5921	0.6374	0.5872	0.5917	0.5558	0.5878
c_y	0.5246	0.5558	0.5249	0.5247	0.5458	0.5270
c_o	0.3047	0.1779	0.3100	0.3058	0.3775	0.3131
s	0.0930	0.0939	0.0930	0.0930	0.0930	0.0930
b	0.0846	0.0694	0.0844	0.0842	0.0718	0.0832
g_a(%)	1.46	2.00	1.52	1.47	1.57	1.50
Δ(%)	-	4.45	0.027	0.004	1.52	0.35
π(%)	-	-	7.66	1.11	38.13	7.66
OLD (weeks)	-	-	7.8	1.0	58.4	7.7

Note: Column (6) refers to an experiment in which the parental leave subsidy rate is the same as the optimal subsidy rate in PLS_LI, but it is financed by reducing the education subsidy rate. For variables, g_a is the annual growth rate of output per worker, and OLD denotes the optimal leave duration in weeks. The corresponding changes in labor income tax rate (column (3)) and capital income tax rate (column (4)) are $\Delta\tau_l = 0.58$ and $\Delta\tau_s = 0.09$ percentage points. The changes in the education subsidy rate in column (5) and (6) are $\Delta\pi_q = -17.02$ and $\Delta\pi_q = -0.97$ percentage points.

a child's early childhood (e), labor time (l), young-age consumption relative to output per worker (c_y) and savings (s) are substantially lower in the BCE. These results suggest that the presence of human capital externalities as well as distortionary taxes and subsidies causes the endogenous variables to deviate from their socially optimal levels substantially. As a result, the annual growth rate of output per worker (g_a) is also much lower in the BCE, 1.46% versus 2% in the SO. And there is also a large welfare gain of 4.45% of the SO over the BCE.

The results for alternative calibration in Supporting Information S2: Appendix C reveal that the welfare gain and the increase in the growth rate of output per worker in the SO compared with the BCE are larger (smaller) than under the baseline calibration when the degree of human capital externalities is higher (lower).

5.3.2 | Optimal parental leave subsidy financed by raising labor income tax

Next, we compare the competitive equilibrium outcomes of the economy with optimal parental leave subsidy financed by raising labor income tax (PLS_LI) and the BCE economy. We first solve for the optimal parental leave subsidy rate π^* financed by raising labor income tax, which is chosen to maximize the welfare U_o. The optimal subsidy rate is obtained as $\pi^* = 7.66\%$, and the corresponding increase in the labor income tax rate, found by equating the government budget constraint, is $\Delta\tau_l = 0.58$ percentage points.

Under the optimal subsidy rate, the total subsidy a worker receives for her parental time on one child as a fraction of income per worker is given by $p_t\bar{e}_t W_t \overline{H}_t/\bar{y}_t = p_t\bar{e}_t(1-\theta)/\bar{l}_t$, which is 0.3953%. This optimal subsidy can be converted into an optimal leave period that is fully covered. Suppose a worker is entitled to take a 1-year parental leave for every child with full replacement rate, which corresponds to the worker receiving a subsidy of 1 year's labor income for each child she has. Such a subsidy as a fraction of income per worker over 30 years can be approximated by a young household's annual labor income divided by the income per young household over 30 years, which is about 2.65% in the data. So, in the model, the optimal subsidy rate corresponds to an optimal fully-covered parental leave of (0.3953/2.65) (52), that is, 7.8 weeks.

The annual growth rate of output per worker is 1.52%, 0.06% points higher than in the BCE. The welfare gain over the BCE is 0.027%. This welfare gain arises mainly due to an increase in the parental time input in a child's early childhood (e) and young parent's consumption (c_y) as well as a fall in expenditure on child human capital (q) and leisure (Z) from the baseline equilibrium levels toward the socially optimal levels. However, the welfare gain is much lower than the welfare gain in the SO, suggesting that the efficiency loss from human capital externality and distortionary taxes and subsidies can only be slightly mitigated by the PLS financed with higher labor income tax.

The left panel of Figure A1 in the appendix depicts the welfare gains in the PLS_LI for a range of parental leave duration. The welfare gains have an inverted U-shape with the peak at a leave duration of 7.8 weeks. The welfare gains become negative once the leave duration is beyond 15.5 weeks. The right panel of Figure A1 shows the percentage changes in the growth rate of output per worker from the corresponding baseline levels, which are monotonically increasing with the duration of parental leave.

The results for alternative calibration in Supporting Information S2: Appendix C show that the optimal parental leave duration in the PLS_LI with a higher (lower) degree of human capital externalities is longer (shorter) than discussed above. The welfare gain and the increase in the growth rate of output per worker are also larger or smaller, respectively.

5.3.3 | Optimal parental leave subsidy financed by raising capital income tax

When the parental leave subsidy is financed by raising capital income taxes (PLS_CI), the optimal parental leave subsidy rate, again chosen to maximize U_o, is solved as $\pi^* = 1.11\%$, which is substantially lower than the optimal subsidy rate in the PLS_LI economy. The corresponding increase in the capital income tax rate is given by $\Delta\tau_s = 0.09$ percentage points. As discussed above, this optimal parental leave subsidy can also be converted into a fully-covered parental leave period of 1 week, much shorter than the optimal parental leave duration implied in the PLS_LI case.

The optimal PLS_CI has similar effects as the optimal PLS_LI. However, compared with the optimal PLS_LI, the optimal PLS_CI is less effective in altering the parental time input (e), young parent's consumption (c_y) and expenditure on child human capital (q) from the baseline equilibrium levels toward their corresponding socially optimal levels. Hence, the overall effect is a welfare gain of 0.004% and an increase of 0.01% points in the annual growth rate of output per worker over the BCE. The much smaller increases in both the welfare level and growth rate compared with the PLS_LI case reflect a more distortionary effect of capital income taxation than labor income taxation as a way to finance parental leave subsidy.

5.3.4 | Optimal parental leave subsidy financed by cutting education subsidy

The comparison between the BCE and the SO earlier reveals that the parental time input (e) is smaller while expenditure on a child's human capital (q) is much higher than their socially optimal levels in the BCE. Hence, for the last experiment we consider optimal PLS financed by reducing education subsidy, PLS_ES. This also gives us the optimal combination of parental leave subsidy and education subsidy implied by the model.

In this case, the optimal parental leave subsidy rate is $\pi^* = 38.13\%$, which is substantially higher than the optimal subsidy rate in the PLS_LI and PLS_CI cases. The corresponding reduction in education subsidy is $\Delta\pi_q = 17.02$ percentage points, implying an education subsidy rate of 67.57%. The optimal fully-covered parental leave period in this case is 58.4 weeks, which is also substantially higher than that in PLS_LI and PLS_CI cases.

The optimal PLS_ES leads to 35.48% increase in the parental time input (e) and 60.15% decrease in the expenditure on a child's human capital (q) from their counterparts in the BCE. These changes make e and q much closer to their socially optimal levels. Compared with the BCE, there is a reduction in both leisure and hours of work while there is an increase in consumptions and the number of children. The overall effect is a welfare gain of 1.52% over the BCE, and an increase of 0.11% points in the annual growth rate of output per worker. These changes are much larger than in the PLS_LI and PLS_CI cases.

The results discussed above suggest that reducing the current education subsidy in the U.S. economy while introducing parental leave subsidy could significantly promote welfare and growth.[20] The optimal combination of education and parental leave subsidy we find requires a large cut in the education subsidy. However, implementing a substantial reduction in public education expenditure on children could have important socioeconomic consequences that are beyond the scope of the model that we consider here. For instance, children in lower socio-economic background would suffer the most, and as a result, the education gap between students with lower and higher socioeconomic statuses would be widened.

Given the consideration above, in our final experiment we consider a moderate change in parental leave and education subsidies. Specifically, we introduce the optimal parental leave subsidy rate of 7.66% in the PLS_LI case and finance it by reducing the education subsidy rate. The results of this experiment are reported in column (6) of Table 3. The reduction in the education subsidy rate is 1% point ($\Delta\pi_q = 0.0097$). The welfare gain of such a policy is 0.35% over the baseline economy, much larger than the welfare gain in the PLS_LI case. The annual growth rate of output per worker increases by 0.04% points, comparable to the growth effect in the PLS_LI case.

5.4 | Leave policies among OECD countries

This subsection summarizes the parental leave policies for a group of OECD countries. This allows us to compare our quantitative results for the U.S. economy with other countries. In this comparison, we focus on the results for the optimal leave duration when it is financed by raising labor income tax (PLS_LI).[21]

Parental leave policies for early childhood vary substantially across countries. In general, there are four types of leave policy for early childhood: maternity leave, paternity leave, parental leave and home care leave (for details, see OECD, 2019). Given that this paper does not distinguish between these types of leave, we look at the total leave available to mothers and fathers for the group of OECD countries. Additionally, parental leave policies across countries vary in leave duration and average payment rate. To make the comparison across the countries easier, we consider full-rate equivalent leave duration which is obtained by multiplying the duration of leave in weeks by the payment rate (i.e., payment relative to average earnings) received by the claimant over the duration of the leave. Table 4 reports the full-rate equivalent leave (in weeks) per child available to parents for a group of OECD countries in 2018, which shows great variations across these countries.

The U.S. is the only country in the sample that does not offer statutory entitlement for paid leave on a national basis. Ireland has the second lowest paid leave entitlements in this sample, 7.5 full-rate equivalent weeks.[22] In addition to the U.S. and Ireland, six other countries provide a leave entitlement below 14 weeks which is the minimum duration of maternity leave recommended by International Labor Organization. At the other extreme, nine countries provide

TABLE 4 Paid leave entitlements per child available to parents in 2018 (weeks).

Country	Full-rate equivalent leave	Country	Full-rate equivalent leave	Country	Full-rate equivalent leave
Australia	8.6	Hungary	69.2	New Zealand	8.4
Austria	55.9	Iceland	26.6	Norway	52.4
Belgium	18.0	Ireland	7.5	Poland	43.6
Canada	26.6	Israel	15.0	Portugal	32.9
Chile	31.0	Italy	26.0	Slovak Rep.	53.1
Czech Rep.	47.5	Japan	66.1	Slovenia	52.3
Denmark	27.6	Korea	40.6	Spain	20.3
Estonia	86.4	Latvia	52.8	Sweden	45.4
Finland	46.0	Lithuania	66.0	Switzerland	8.2
France	23.4	Luxembourg	45.3	Turkey	11.7
Germany	48.3	Mexico	13.0	UK	12.1
Greece	21.7	Netherlands	16.4	US	0.0

Note: Here we report the sum of full-rate equivalent (weeks) available to mothers and fathers (refer to Tables PF2.1.A and PF2.1.B in OECD (2019)). Paid leaves available to fathers are far shorter than that for mothers.

full-rate equivalent leave longer than a year. For example, Estonia and Hungary provide 86.4 and 69.2 weeks of full-rate equivalent leave per child, respectively.

Our quantitative analysis shows that for the U.S. economy, the optimal leave duration when it is financed by raising labor income tax is 7.8 weeks per child for a single parent, implying a full-rate equivalent leave entitlement of 15.5 weeks per child for two parents. Such a leave entitlement is comparable to the practice in some European countries like Belgium and Netherlands. Our alternative calibrations suggest a minimum full-rate equivalent leave duration of 13.7 weeks and a maximum of 21.4 weeks for two parents for the U.S. economy (see Table C2 in Appendix C). This range appears reasonable, considering that about a third of the OECD countries fall in this range.

Our results suggest that for the U.S. economy, introducing a parental leave entitlement of 15.5 weeks per child for two parents would raise output growth and improve welfare. However, as shown in Figure A1 in Appendix, a leave duration that is longer than 31 weeks for two parents would reduce welfare. This result suggests that the extremely long parental leave duration among some OECD countries could be welfare reducing for the entire society.

6 | CONCLUSION

This study examines the growth and welfare effects of PLS and explores optimal PLS and efficient policies in a life-cycle dynastic family model featuring endogenous fertility, labor and accumulation of physical and human capital in presence of human capital externalities. We provide analytical results in a basic model and then conduct a quantitative analysis for the U.S. economy based on an extended model.

Our analytical results show that human capital externality causes higher fertility, less parental time and expenditure inputs in child human capital, and lower growth and welfare in the LF compared with the SO. Introducing PLS financed by lump-sum taxation (PLS_LS) leads to lower fertility, more parental time and expenditure inputs in child human capital and higher growth and welfare over the LF outcome. We also show the existence and uniqueness of an optimal parental leave subsidy rate in the PLS_LS economy but the optimal parental leave subsidy cannot achieve the SO. We find that subsidizing general work leave, including parental leave, and labor income by a lump-sum tax can achieve the SO. These results highlight the importance of PLS, consistent with the importance of parental time on child human capital development that is documented in empirical studies.

Our quantitative results suggest that, for an empirically plausible degree of externalities, introducing parental leave subsidy financed with alternative distortionary tax devices improves welfare and raises the growth rate of output per worker from the baseline economy that is calibrated to the U.S. economy. In particular, the optimal parental leave subsidy financed by raising labor income tax rate implies an optimal fully-covered leave duration of 7.8 weeks per child for one parent, which increases the annual growth rate by 0.06% points and welfare by 0.027% compared with the baseline equilibrium outcome. Hence, our quantitative policy exercises suggest that the U.S. may benefit in the long-term by subsidizing parental leave given that paid parental leave entitlements are absent at the national level.

The findings of our study may also shed light on parental leave policy for other economies. As developing countries typically have lower standards for parental leave provisions, inadequate parental care for young children, lower levels of human capital and lower growth rates of output per capita in comparison to developed countries, our results support the implementation of subsidized parental leave policy as a way to improve the economic performance in developing countries. In addition, developed economies such as Estonia and Hungary that currently provide very generous PLS or allow a long leave duration may benefit in the long-term by reducing PLS and the leave duration, as implied by our result that shows the existence of an optimal level of PLS. Indeed, the empirical evidence in Ruhm and Teague (1997) shows that short to moderate periods of parental leave can increase economic efficiency (Ruhm & Teague, 1997).

The model can be extended to incorporate several other aspects that are potentially important to analyze the impact of parental leave provisions. For instance, the model can be extended to include more periods to capture different stages of child human capital development and the different roles of parental time at each stage. Another possible extension is to consider heterogeneous agents. Our quantitative results suggest that introducing a parental leave subsidy financed by reducing the current level of education subsidy in the U.S. can achieve more significant gains in welfare and growth compared with the labor income tax case. However, a substantial reduction in education subsidy could have other socioeconomic consequences that cannot be addressed in our current model that abstracts from household heterogeneity. We leave these aspects for future research.

ACKNOWLEDGMENTS

We would like to thank the co-editors, Dietrich Vollrath and Farasat Bokhari, and an anonymous reviewer for giving us valuable comments and suggestions to improve the paper. We also like to thank Maria Racionero and seminar participants at the 2nd Australian Workshop on Public Finance 2022, Australian Gender Economics Workshop 2021, Computing in Economics and Finance Conference 2021, Workshop of the Australasian Macroeconomics Society 2021 and Australian Government Department of Education, Skills and Employment for their helpful comments.

Open access publishing facilitated by Monash University, as part of the Wiley - Monash University agreement via the Council of Australian University Librarians.

CONFLICT OF INTEREST STATEMENT

None.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in OPENICPSR at https://doi.org/10.3886/E190482V3.

ENDNOTES

[1] Under the most basic form of parental leave, parents are entitled to take leave to take care of a newborn child. However, in many countries, parental leave extends beyond the basic entitlement, so mothers or fathers can take some time off work to take care of an infant or older child. In this study, we use the term "parental time subsidization" and "parental leave subsidization" interchangeably to refer to paid parental leave policies to care for and nurture children.

[2] The notion of human capital externalities is old in economics (Marshall, 1890), and has received a great deal of attention by economists especially when endogenous growth theories emerge (see Nelson & Phelps, 1966; Romer, 1990; among others). Many empirical studies find evidence of human capital externalities through channels such as ethnic groups, neighborhoods, workplaces, or schools, see, for example, Borjas (1992, 1994, 1995), Rauch (1993), Moretti (2004a,b), Mas and Moretti (2009), Choi (2011), and Wantchekon et al. (2015).

[3] We normalize all the quantity variables by output per worker to detrend the model. So, the expenditure input here and other quantity variables (such as consumption) to be discussed below are all relative to output per worker.

[4] Throughout the paper, optimal PLS refers to the parental leave subsidy rate that maximizes the welfare in the decentralized economy with PLS which in general cannot achieve the socially optimal welfare level.

[5] In the U.S., some workers have access to paid leave through their employers or because they live in states that have state paid leave policies. However, there is no access to paid leave at the national level.

[6] The model is an extension of Zhang and Zhang (2007) and Yew and Zhang (2009, 2013) to include leisure of the young parent in the preferences to explore the growth and welfare implications of parental time subsidization. The formulation of having average preference of children U_{t+1}, instead of total preference $N_t U_{t+1}$, in the parental preference here follows Mill (1848), Razin and Ben-Zion (1975), Zhang and Zhang (2007), and Yew and Zhang (2009, 2013) for its analytical convenience.

[7] In our calibration, we interpret e as parental time raising a child when the child is younger, while v as that when the child is elder. See Section 5 for a detailed discussion on this interpretation.

[8] Ioannides and Kan (2000) and Raut and Tran (2005) find supporting evidence for two-sided altruism using U.S. and Indonesian data, respectively.

[9] Any possible effect of the fixed parental time v on a child's human capital development is captured in the constant parameter A in Equation (10).

[10] To see that c_y is positive, substituting Equations (19) and (23) into Equation (22), and so $c_y = \alpha(1-\alpha)[1-\alpha\theta(1-\delta)]/[(\alpha+\beta)(1-\alpha(1-\delta))]$ which is positive.

[11] The productivity parameters A and D in Equation (30) are assumed to be sufficiently large. Thus, steady-state positive growth exists.

[12] By substituting the market clearing condition $k_{t+1}N_t = S_t$ into (34), the first-order condition with respect to S_t in the laissez faire is identical to that with respect to k_{t+1} in the social planner's problem. Then, by substituting (34) into (37), the first-order condition with respect to N_t in the laissez faire is identical to that in the social planner's problem.

[13] This result has been established in Zhang (1995), Zhang et al. (2001) and Yew and Zhang (2009) in dynastic family models.

[14] If the parental leave subsidy rate depends on a parent's effective wage, the positive growth and welfare effects would be stronger than that if the subsidy rate depends on the average effective wage in the economy. To be more conservative about the welfare gains of parental leave subsidization, we assume the parental leave subsidy rate depends on the average effective wage, which is in line with leave policies in some countries such as Australia.

[15] The product of the ratio of parental expenditure on child human capital to output per worker and labor input is given by $ql = \nu\alpha[\delta(1-\theta)]^2/\{[\delta((\rho+\beta)c_y - qN - \alpha\theta) - \varepsilon qN][1-\alpha\zeta(1-\delta)]\}$, which is independent of π.

[16] Here, work leave refers to any leave for family and personal reasons, given by the term $(\nu + e_t)N_t + Z_t$, which includes parental leave as introduced in Section 4.2. We will show that such a general work leave subsidy is essential for achieving the social optimum.

[17] We do not need separate values for the scale parameters A and D. The constant $A^{1-\theta}D^{\delta}$ is sufficient.

[18] For the majority of states in the U.S., age of required school attendance starts at 6 years old. See Table 5.1 (available at https://nces.ed.gov/programs/statereform/tab5_1.asp) which reports compulsory school attendance laws in different states in the U.S.

[19] According to American Time Use Survey (ATUS), the mean and standard deviation of the number of hours parents spent with children of age 7–17 are both lower than that of parents spent with children of age 0–6.

[20] According to OECD (2022), the U.S. public spending on primary and secondary education relative to GDP (4.6%) exceeds many other advanced economies and the OECD average (4.4%). Nevertheless, as reported in Schleicher (2019), students' performance in various academic tests (e.g., reading, math and science) is below the OECD average and even much worse than less advanced OECD countries (e.g., Hungary, Lithuania and Slovenia). This suggests that the substantial public spending on early education does not appear to be translated into a better education outcome (i.e., higher human capital) in the U.S.

[21] We focus on the results in the PLS_LI case because raising labor income tax is less distortionary than raising capital income tax, as shown in our results in Sections 5.3.1 and 5.3.2, and reducing education subsidy has other implications that cannot be addressed in our current framework.

[22] This is mainly due to a low average payment rate of 26.7% in Ireland, despite that it provides relatively long maternity leave entitlements, 26 weeks for mothers and 2 weeks for fathers.

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SUPPORTING INFORMATION

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How to cite this article: Yew, S.L., Li, S.M. & Moslehi, S. (2024) Optimal parental leave subsidization with endogenous fertility and growth. *Economic Inquiry*, 62(1), 97–125. Available from: https://doi.org/10.1111/ecin.13165

APPENDIX

Additional quantitative results

TABLE A1 A comparison of equilibria between laissez-faire, social optimum and optimal parental leave subsidization financed by lump-sum taxation (PLS_LS).

Variable	(1) Laissez faire	(2) Social optimum	(3) Optimal PLS_LS
N	0.6683	0.5288	0.6658
q	0.0192	0.0260	0.0193
e	0.0727	0.0985	0.0782
Z	0.2931	0.2932	0.2920
l	0.6365	0.6374	0.6341
c_y	0.5564	0.5558	0.5564
c_o	0.2251	0.1779	0.2243
s	0.0939	0.0939	0.0939
b	0.0691	0.0694	0.0691
g_a(%)	1.61	2.00	1.68
Δ(%)	-	0.07	0.014
π(%)	-	-	7.07
OLD (weeks)	-	-	10.2

Note: g_a is the annual growth rate of output per worker, Δ refer to welfare gains over the laissez-faire outcome, and OLD denotes the optimal leave duration in weeks. The efficient policies (not reported in the table) that attain the social optimum involves introducing a subsidy rate of 7.6% for both labor income and general work leave, financed with a lump-sum tax.

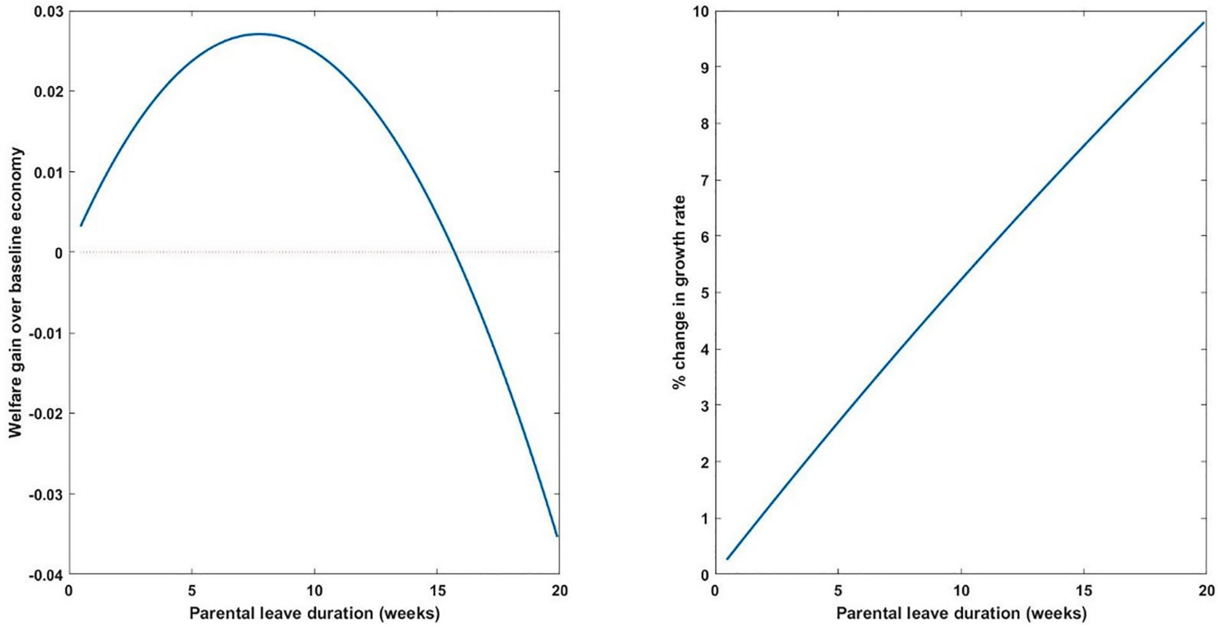

FIGURE A1 Welfare gains (left) and percentage changes in growth rate of output per worker (right) in PLS_LI from the baseline equilibrium for different parental leave durations. PLS_LI refers to the economy with parental leave subsidization financed by raising labor income tax. To construct this figure, we consider different parental leave subsidy rates in the PLS_LI, which imply different parental leave durations per child for one parent (as shown on the horizontal axis). For the PLS_LI with each parental leave subsidy rate, we calculate the welfare gain over the baseline equilibrium and the percentages changes in the growth rate of output per worker from its level in the baseline equilibrium.