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Nonlinear dynamics of Kimchi premium[☆]

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ABSTRACT

Kimchi premium, the persistent non-zero price difference exists between the US and Korean crypto-markets. Not only does the premium represent a violation of the law of one price but it may also reflect the bubble aspect of crypto-markets or crypto-market segmentation. Contrary to the literature relying on linear modelling, we employ threshold regression with multiple regimes to show the nonlinear dynamics of the premium and identify its determinants. We find that the premium is mean-reverting when it exceeds a certain level of thresholds but displays a random walk inside the range, which implies that only for relatively large-sized premiums, arbitrageurs exploit the premium. Kimchi premium has a non-zero long run steady-state level of 1.24% for Bitcoin aligned with the violation of the law of one price. We demonstrate that the non-zero premium exists due in part to market frictions given that the trading fee is positively correlated with the threshold.

1. Introduction

Cryptocurrency has been a buzzword for more than half a decade thanks to its remarkable growth in its price as a type of investment vehicle and its potential to replace cash. In the contemporary literature, cryptocurrencies have been treated as assets although, to some, it has the potential to become future currencies in digitalised worlds, e.g. Fousekis and Tzaferi (2021), Charfeddine et al. (2020) and Liu et al. (2020) for cryptocurrencies as investment assets. While quite a few papers employ methodologies developed for financial assets to investigate the dynamics of cryptocurrencies, it has been documented that cryptocurrencies have several unique features that are not observed in other financial assets. See Bariviera et al. (2017), Zhang et al. (2018), Hu et al. (2019), Scaillet et al. (2018), and Liu and Tsyvinski (2021) among many others for stylised facts of cryptocurreny markets.

Among many unique features of cryptocurrency, considerable, persistent and recurrent price differences for the identical cryptocurrency across countries have been observed, i.e. violation of the law of one price, see, for example, Makarov and Schoar (2019). In line with this, "Kimchi premium" is a phenomenon that is not easily observed in the other financial markets: Persistent non-zero premium exists for a cryptocurrency traded in Korean-won denoted exchanges compared to the same cryptocurrency traded in the US-dollar denoted exchanges. Unlike other financial markets, these arbitrage opportunities are not instantly exploited, which provides us with research questions as to whether this premium is transient or permanent, and what factors drive the premium and the dynamics of the premium. In particular, the premium does seem to exhibit a non-zero stable level of equilibrium over the long run while it displays considerable short-run variations.

Against this backdrop, we analyse Kimchi premium and various cryptocurrency-related time series, study the premium's dynamics and investigate its nonlinearity. We also examine what determinants drive the premium. In doing so, we obtain a long-run equilibrium level for the premium and the half-life of its dynamics and discuss why the steady state level of the premium is non-zero, which implies the violation of the law of one price. To this end, unlike the extant literature, we consider threshold regression in Hansen (2000) in order to capture the nonlinear dynamics of the premium. The threshold regression model we consider can accommodate a potential presence of different dynamics depending on the magnitude of Kimchi premium in the previous period.

The rationale behind this threshold regression model is as follows. For relatively small-sized Kimchi premiums, there is not much incentive for arbitrageurs to exploit price differences across exchanges. Consequently, the movement of Kimchi premium follows a random walk without any strong arbitrage trading. Meanwhile, for relatively

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large-sized Kimchi premiums, arbitrage trading for exploiting the premium could kick in and the price deviation would be cleared. As a result, the premium displays a mean reverting nature. This regime-switching feature arises due in part to inadequate financial services, restrictions on cross-border capital flow and market frictions such as miner fees. For instance, arbitrage in cryptocurrencies between Korean and foreign markets encounters regulatory obstacles. Generally, transferring funds from cryptocurrency trading in Korea to overseas locations is prohibited. There are reporting obligations for significant transactions. Moreover, foreign residents face limitations in opening Korean bank accounts for trading in the local market. See Yim et al. (2018) for a discussion from legal perspectives. All of these contribute to the regime-switching feature in the dynamics of cryptocurrency price differentials.

Contemporary literature on Kimchi premium merits explanation. The majority of the literature focuses on cryptocurrency and its price dynamics, not the dynamics of price deviations across exchanges or their determinants. Nevertheless, in the research on price discovery and dynamics of the cryptocurrency, the Kimchi premium has been mentioned and discussed while some literature directly tackles the Kimchi premium. In one strand of literature, Kimchi premium is a consequence of market segmentation in combination with capital control limiting cross-border arbitrage trading. Choi et al. (2018) argue that capital controls amplify market frictions from the microstructure of the bitcoin network that limits the ability of arbitrageurs to exploit price differences. Lee and Oh (2022) attribute the premium to the upsurge of overseas remittances to China due to capital controls and financial regulations exploited by foreign arbitrageurs. Makarov and Schoar (2019, 2020) attribute the premium to poorly functioning financial institutions and capital controls in many countries including South Korea, especially in the presence of market segmentation. They also claim that the premium widens in times of appreciation of cryptocurrencies and hence it reflects in part bubbles.

Another strand of literature focuses more on a bubble aspect of the Kimchi premium arising from severe speculative investment motives. Lee et al. (2019) find that the Kimchi premium reflects bubbles mostly driven by Korean domestic factors and can be abused by arbitrageurs. They also claim that suitable government regulation could eliminate bubbles. Eom (2021) considers Kimchi premium as a proxy for bubbles in the Korean cryptocurrency market. Oh (2018) directly focuses on Kimchi premium and its determinants. However, he considers the case where cryptocurrency is used as another type of currency. He claims that the factors influencing the premium include the cryptocurrency's rate of return, as well as exchange rates and interest rates associated with standard currencies.

However, it is worth mentioning that the extant literature relies on either descriptive or linear regression modelling, which has a clear limitation when analysing the nonlinear dynamics of the underlying processes. For instance, Eom (2021) and Choi et al. (2018) use linear regression to analyse the dynamics of the premium. Makarov and Schoar (2019) follow Hasbrouck (1995) on the vector error correction model via cointegration and Makarov and Schoar (2020) combine factor analysis with the price decomposition in Hasbrouck (1995). While linear modelling often suffices to analyse the dynamics, we note that nonlinear modelling is necessary for instance, when Augmented Dickey–Fuller and KPSS tests yield opposite results.

In our empirical study, we find evidence of nonlinearity in the dynamics of the Kimchi premium. In the short run, the premium fluctuates and displays a movement of random walk while in the long run, it tends to a long-run steady-state level of 1.24% for Bitcoin. We can conclude that there exists a long-run positive Kimchi premium, which supports some degree of market segmentation. This implies that this arbitrage opportunity may be exploited by investors who are able to bypass capital controls and market segmentation. In addition, we show that the change in volatilities of cryptocurrency prices, turnover, exchange rate, market capitalisation, crypto market indices, leverage effects and trends are major determinants of the premium.

Consistent with the claims of the literature that the premium depends on important market frictions such as trading fees, we find that the threshold defining the positive side of the random walk regime increases with a higher miner fee rate, and the lower bound of the regime also becomes slightly more negative as the miner fee rate increases. While the increased miner fee adds difficulty to arbitrage trading, the arbitrageurs react slowly when the miner fee is high and when the Kimchi premium is positive.

In what follows, Section 2 provides data description along with their notable features and the summary statistics. Section 3 provides the analysis of the dynamics of Kimchi premium and its determinants. Section 4 concludes.

2. Data: Bitcoin and Ethereum

2.1. Data description

Cryptocurrency prices and Kimchi premium. We collect the daily Korean-won denoted data primarily from Dunamu Inc, a fintech company located in the Republic of Korea, which owns and operates a digital asset exchange called "UPbit". We also use daily Korean-won denoted US market data from coinmarketcap.com and daily foreign exchange rate data from the Bank of Korea. The data ranges over the period from 26 Apr 2018 to 31 Jul 2022. For our analysis, we consider the two most representative and biggest (in market capitalisation) cryptocurrencies: Bitcoin (BTC) and Ethereum (ETH). The cryptocurrency-related variables we consider are daily high and low prices, closing prices, volume (the US, Korea), volatility (the US, Korea), market capitalisation, a type of transaction fee called the miner fee, an array of indices, say UBMI, Digital Asset Fear and Greed Index, Momentum Top 5, Low Volatility Top 5, which will be explained shortly.[1]

From the above raw data, for our analysis, we calculate various variables for Bitcoin and Ethereum as follows. The Kimchi premium, denoted as $Kimp_t$, is calculated as

$$Kimp_t = \frac{P_t^{KR} - P_t^{US}}{P_t^{US}} \times 100, \tag{2.1}$$

where P_t^j is the daily Korean-won denoted closing price of cryptocurrency (Bitcoin or Ethereum) traded in the Korean market ($j = KR$) and in the US market ($j = US$).

As seen from Figs. 1 and 2, Bitcoin and Ethereum display similar patterns. In most cases, it appears that the Kimchi premium tends to get larger when the price and volatility increase. However, the premium gets larger even when the price decreases. Unlike the claims of the literature that the premium widens in times of the appreciation of cryptocurrencies, i.e. the premium as bubbles, we find that the premium widens even in times of abrupt depreciation of cryptocurrencies.

Cryptocurrency-related variables as regressors. For the analysis of the dynamics of Kimchi premium and its determinants, we examine a variety of variables that are also considered as factors in the analysis of the Kimchi premium in other cryptocurrency literature, e.g. Oh (2018) and Eom (2021). Several cryptocurrency indices are also considered following the finance literature on the analysis of return premium. Most of the variables are obtained in both US and Korean exchanges unless it is stated otherwise. To begin with, we use the daily simple returns R_t and volatility for both Bitcoin and Ethereum. We obtain the variable $Illiqudity_t$ calculated based on Amihud (2002) such that $Illiquidity_t = 365|R_t|/vol_t \times 100$ where vol_t is the daily trading volume at

[1] The cryptocurrency exchanges operate 24/7 and therefore, the closing prices of both the Korean-won denoted cryptocurrencies are based on coinmarketcap.com provided latest data in range (UTC time).

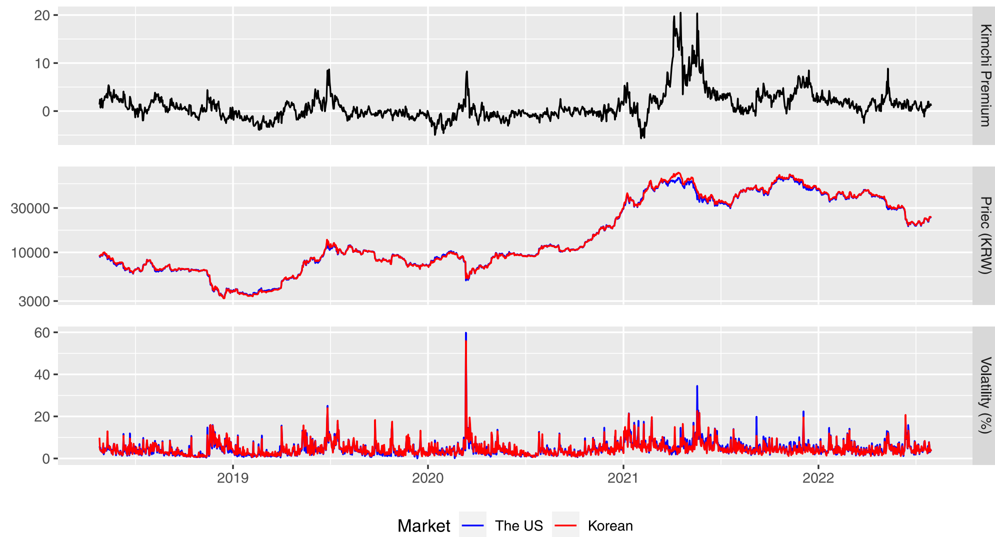

Fig. 1. Kimchi premium, prices and volatility of BTC.

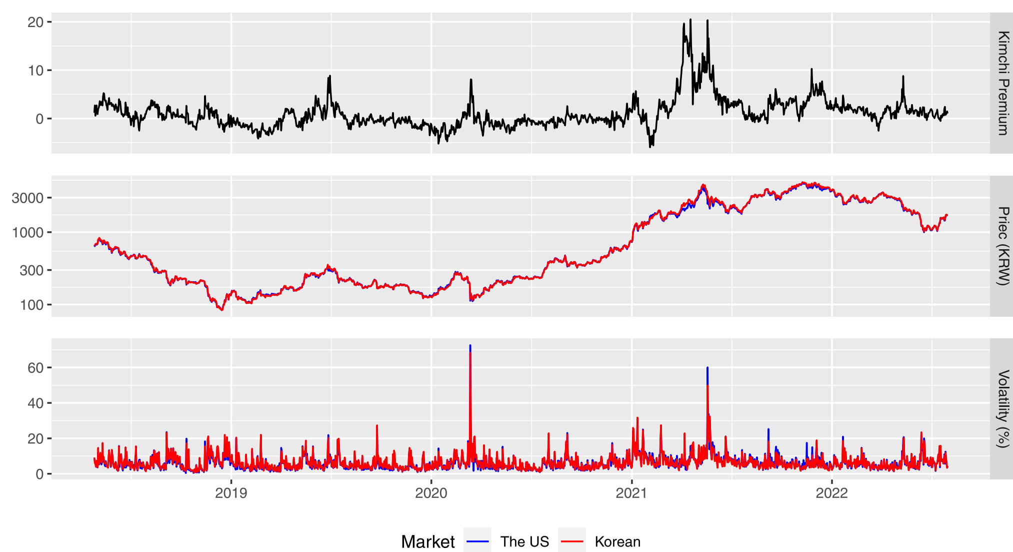

Fig. 2. Kimchi premium, prices and volatility of ETH.

time t converted from Korean-won to USD using the exchange rate from the Bank of Korea. Other variables we use are the market capitalisation at time t, cap_t and the turnover, trn_t such that $trn_t = vol_t/cap_t \times 100$. We also consider a trend variable t and the sign of the return of cryptocurrencies in the previous period. Figs. 3 and 4 display the time series of some of the covariates we consider.

We also take into account various indices prepared by the Korean exchange "Upbit" as explanatory variables for the Kimchi premium. Financial ratios derived from these cryptocurrency-related indices are regarded as significant factors in the analysis of returns and premiums. These are as follows. UBMI (Upbit Market Index) is the weighted average index of all the digital currencies traded in the UPbit exchange. It is calculated based on the composite market index methodology. Upbit Momentum Top 5 Index is a strategy index that holds 5 types of digital assets (equally weighted) with excellent 30-day performance among the Upbit Top 30 Index. Upbit Low Volatility Top 5 Index is a strategy index that holds 5 types of digital assets (equally weighted) with the least volatility among the Upbit Top 30 Index. Digital Asset Fear and Greed Index is an index calculated based on five different levels of sentiment, from extreme fear to extreme greed along with the volatility-volume points and momentum points of digital assets. These indices are depicted in Fig. 5.

2.2. Summary statistics

Tables 1 and 2 show the summary statistics of the variables for our analysis of Bitcoin and Ethereum respectively.

There are notable observations worth mentioning here. Cryptocurrencies display highly volatile price movements. Over the period we consider, the price of Bitcoin (Ethereum) ranges from $US\$3,241$ ($US\$84$) to $US\$68,581$ ($US\$4,884$) in the US market, and from $US\$3,200$ ($US\$84$) to $US\$72,005$ ($US\$4,907$) in the Korean market. Its daily returns and volatility of Bitcoin (Ethereum) peak at 17.13% (23.21%) and 59.84% (72.64%) respectively in the US market and 15.75% (25.98%) and 55.90% (68.29%) in the Korean market. During the period, the US market provides much bigger liquidity compared to the Korean market. Take Bitcoin as an example, the Kimchi premium ranges from −5.69% to 20.5% and is mostly positive and persistent as

Fig. 3. Model covariates BTC.

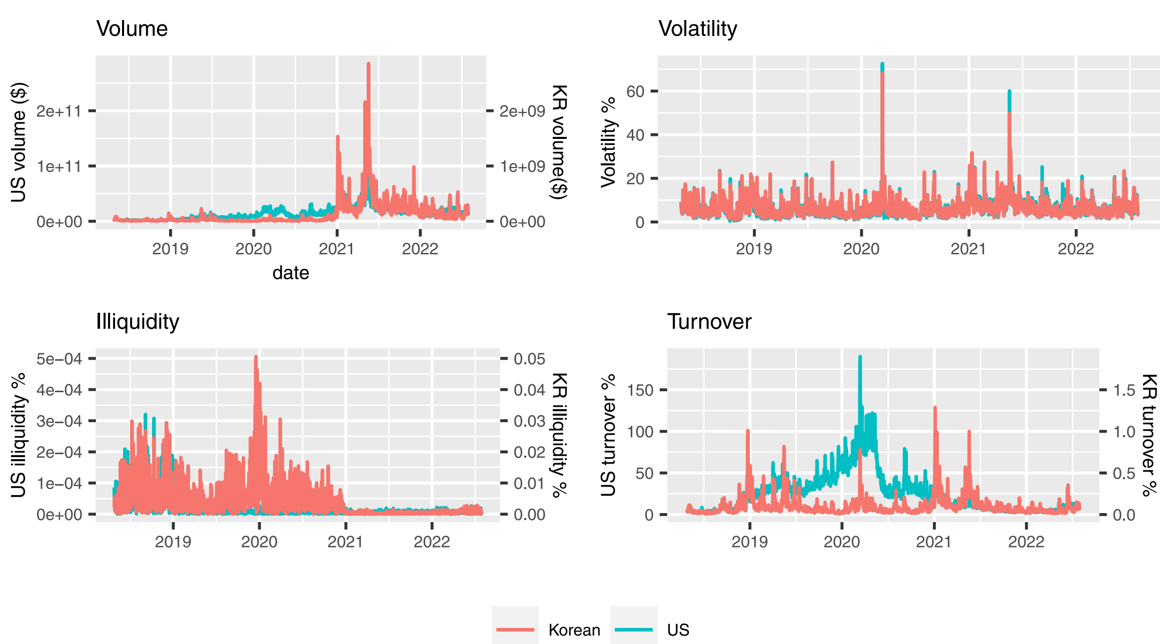

Fig. 4. Model covariates ETH.

seen in Fig. 1. Unlike most of the literature predicting high Kimchi premium in times of rapid growth in the prices of cryptocurrencies, the premium widens sometimes when the price drops significantly as well, for instance, in early 2020. This might indicate that capital controls could aggravate the price deviations incurring significant arbitrage opportunities, and not always representing bubbles in cryptocurrencies.

2.3. Tests: Unit root and regime-switching

We conduct unit root tests to check whether the Kimchi premium exhibits a unit root. We employ Kwiatkowski–Phillips–Schmidt–Shin (KPSS, Kwiatkowski et al., 1992) and Augmented Dickey–Fuller (ADF, Said and Dickey, 1984) tests. The number of lags we choose is 7, following Kwiatkowski et al. (1992) and other previous simulation exercises, but the conclusion does not differ with different choices of lag. The KPSS and ADF tests yield the opposite results. The calculated KPSS test statistic value is 0.7557 for Bitcoin and 0.7885 for Ethereum with both p-values smaller than 0.01, which indicates the rejection of the null of trend stationarity. The calculated ADF test statistic value ranges over −4.48 (lag 7) to −5.87 (lag 1) for Bitcoin and −4.46 (lag 7) to −5.99 (lag 1) for Ethereum with all of the p-values smaller than 0.01, which indicates the rejection of the presence of a unit root, in favour of trend stationarity. These conflicting test results could indicate more complicated nonlinear dynamics of Kimchi premium,

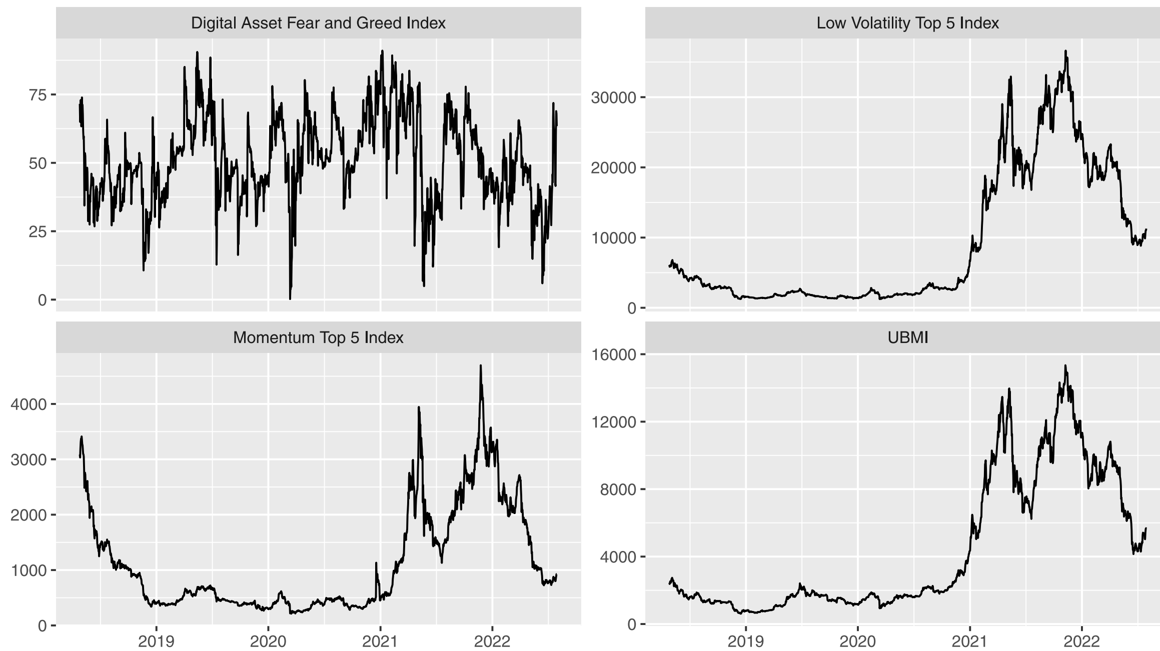

Fig. 5. Four market indices.

Table 1
BTC.

	Mean	SD	Min	Median	Max
Kimchi premium(%)	1.09	3.01	−5.69	0.47	20.50
Trading volume (US$m) - US	27,482	20,263	2938	25,076	351,396
Trading volume (US$m) - KR	185	241	7	70	1780
Volatility(%) - US	4.66	3.67	0.01	3.80	59.84
Volatility(%) - KR	4.66	3.47	0.56	3.77	55.90
Return(%) - US	0.075	3.768	−44.844	0.088	17.133
Return(%) - KR	0.074	3.391	−40.184	0.068	15.746
Illiquidity(%) - US	5.39e−6	8.61e−6	1.37e−8	2.71e−6	8.83e−5
Illiquidity(%) - KR	1.12e−3	1.46e−3	0.0e+0	5.62e−4	1.06e−2
Turnover(%)- US	9.95	7.63	1.81	8.10	72.96
Turnover(%)- KR	0.0412	0.0325	0.0044	0.0324	0.3419
Price(USD) - US	21,209	18,138	3241	10,327	68,581
Price(USD) - KR	21,742	19,018	3200	10,386	72,005
Market capitalisation (US$bn) - US	395	344	56	188	1294
Index: UBMI	4425	4073	614	1939	15,347
Index: Momentum Top 5	1137	936	212	680	4699
Index: Low Volatility Top 5	9130	9820	1239	2998	36,630
Index: Digital Asset Fear and Greed	51.0	15.6	0.2	50.0	91.0

Table 2
ETH.

	Mean	SD	Min	Median	Max
Kimchi premium(%)	1.06	3.05	−5.96	0.45	20.53
Trading volume (US$m) - US	13,963	11,080	1082	12,007	84,541
Trading volume (US$m) - KR	122	220	1	29	2855
Volatility(%) - US	6.27	4.66	0.37	5.13	72.64
Volatility(%) - KR	6.48	4.57	0.72	5.38	68.29
Return(%) - US	0.077	4.985	−53.444	0.093	23.208
Return(%) - KR	0.076	4.596	−48.948	0.131	25.984
Illiquidity(%) - US	1.84e−5	3.32e−5	4.20e−8	7.84e−6	3.20e−4
Illiquidity(%) - KR	4.17e−3	5.89e−3	0.0e+0	1.78e−3	5.06e−2
Turnover(%)- US	24.89	23.89	2.12	19.21	190.03
Turnover(%)- KR	0.0957	0.1078	0.0048	0.0643	1.2881
Price(USD) - US	1147	1290	84	387	4884
Price(USD) - KR	1179	1339	84	387	4907
Market capitalisation (US$bn) - US	134	154	9	43	578
Index: UBMI	4425	4073	614	1939	15,347
Index: Momentum Top 5	1137	936	212	680	4699
Index: Low Volatility Top 5	9130	9820	1239	2998	36,630
Index: Digital Asset Fear and Greed	51.0	15.6	0.2	50.0	91.0

Table 3
Test results for existence of regime-switching (Hansen, 1996).

Crypto	Type[a]	Stats	P-values
BTC	$SupLM^{h}$[b]	21.89	0.000
	$ExpLM^{h}$	7.84	0.000
	$AveLM^{h}$	11.98	0.000
ETH	$SupLM^{h}$	9.69	0.078
	$ExpLM^{h}$	2.74	0.075
	$AveLM^{h}$	4.43	0.078

[a] Details on types can be found in Hansen (1996).
[b] h: Heteroskedasticity robust.

which necessitates accommodating nonlinear dynamics of the premium process.

There are various tests for the presence of threshold effect like Hansen (1996) and Lee et al. (2011). Since it is known that the coefficients associated with regime changes are unidentified when there is no regime, these tests are necessary in order to justify regression analysis for the presence of nonlinearity via threshold regression. The various test statistics and p-values for the presence of regime-switching can be found in Table 3 (Detailed explanations regarding an array of testing types can be found in Hansen (1996)). All of the tests for Bitcoin reject the null of linear stationarity at the 5% level, in favour of the existence of thresholds. The tests for Ethereum reject the null of linear stationarity at the 10% level, with p-values around 0.075, presenting a valid, albeit weaker suggestion towards regime-switching behaviour.

We use KPSS tests to check the stationarity of the control variables. For variables that rejected the stationarity null, we take the first difference of these variables and test again to check if stationarity is met and if another differencing is required. We find the trading volume (US, KR), volatility (US), illiquidity (US, KR), turnover (US), exchange rate, the market capitalisation and all of the four indices require one first difference. The tests on the turnover and volatility of the Korean market do not reject the stationarity null, but we take one first difference of them for interpretation and comparison purposes. The analysis of the following sections, including the labels in tables, refers to the differenced series.

3. Dynamics of Kimchi premium and its determinants

As seen from the previous section, unit root tests, tests for the presence of regime-switching and visual inspection of the time series of the Kimchi premium necessitate the consideration of the nonlinear dynamics inherent in the Kimchi premium. To this end, we employ the threshold regression model in order to capture nonlinear dynamics, see e.g. Tong and Lim (1980) and Hansen (2000). Our threshold regression model is particularly useful when there exists nonlinear dynamics that may involve unit root dynamics temporarily. Furthermore, the threshold effect reflects the possibility of heterogeneous dynamics of Kimchi premium due to cryptocurrency market frictions such as transaction fees and regulatory hurdles. Based on the model estimation, we also predict a long-run steady state for the Kimchi premium. Later we extend the standard threshold model to the threshold index model proposed by Seo and Linton (2007) and Lee et al. (2021) to accommodate the dependence of the premium threshold on other characteristics like miner fees.

3.1. Modelling of Kimchi premium dynamics: Threshold regression

To investigate whether there exists nonlinearity in the dynamics of Kimchi premium, we consider the following threshold regression aligned with the rationale discussed in Section 1:

$$y_t = y_{t-1} + \delta_1 y_{t-1}\mathbf{1}(y_{t-1} \geq \gamma_1) + \delta_2 y_{t-1}\mathbf{1}(y_{t-1} < \gamma_2) + \rho' control_t + e_t, \quad (3.1)$$

where $\mathbf{1}(\cdot)$ is the indicator function such that $\mathbf{1}(A) = 1$ when A is true and 0 otherwise, γ_1 and γ_2 are the upper and lower thresholds respectively with $\gamma_1 > 0 > \gamma_2$. y_t denotes $Kimp_t$, the Kimchi premium defined in Eq. (2.1), and $control_t$ is a vector of a variety of covariates described in the previous section and transformed appropriately to be stationary.

Model (3.1) allows for nonlinearity and the regime-switching where the premium from the previous period falls between the lower and upper thresholds constitutes the baseline regime, with its coefficient fixed at 1, reflecting a unit root process. If the benefit from arbitrage trading does not offset various trading hurdles which would be reflected in the thresholds, there is no reason for investors to exploit the Kimchi premium, so it follows a random walk process with no arbitrage trading across different markets. Model (3.1) also accommodates asymmetry in mean-reverting speed in the realms of positive and negative premiums by allowing for potential differences between the coefficients δ_1 and δ_2 associated with the upper and the lower regimes respectively. We expect them to be both negative to reflect the mean-reverting nature, such that the coefficient in the upper regime is $1+\delta_1 < 1$ and the coefficient in the lower region is $1+\delta_2 < 1$. The model also accommodates the fact that the movement of Kimchi premium may display a long-run behaviour, possibly moving around a long-run equilibrium. Such movement may not be present in every time period. With the existence of transaction hurdles such as transaction fees and regulatory restrictions, the investors only operate when the benefits of investing overtake the transaction costs or exceed different levels of costs. Thus, we specify different regimes of threshold to account for different patterns of arbitrage.

3.2. Estimation results and analysis

We estimate the model by the nonlinear least squares following Hansen (2000). We consider the following two scenarios and report the results for Bitcoin and Ethereum: (1) without any control variable in the specification and (2) including control variables in the model specifications. The regression results can be found in Table 4. All estimations are implemented in the programming language R (R Core Team, 2023) and the table is created with the `stargazer` package (Hlavac, 2022). In what follows, we primarily focus on the scenario including a set of control variables (Columns (2) for Bitcoin and (4) for Ethereum) and briefly mention the results of the scenario without any control variable since the model specifications including control variables are more appropriate given the much better fit and quite similar interpretations for the others.

Since the limit distribution is not normal, we employ a bootstrap method to estimate the lower and upper quantiles of the bootstrap distribution of threshold estimates of γ_1 and γ_2 as follows. Model (3.1) is estimated to obtain estimates of all the parameters first and obtain the residual $\hat{e}_t$. The bootstrap residual $\hat{e}_t^*$ is constructed from resampling $\hat{e}_t$ with replacement. Bootstrap data y_t^* is constructed using $\hat{e}_t^*$, which is then plugged into the left-hand side to (re)estimate $\hat{\delta}_1^*$, $\hat{\delta}_2^*$, $\hat{\rho}^*$, $\hat{\gamma}_1^*$ and $\hat{\gamma}_2^*$. We repeat the process 1000 times and plot the bootstrap confidence region in Fig. 6. The 95% confidence region of the thresholds of Bitcoin does not overlap with zero in any direction, suggesting the thresholds for Bitcoin are significantly different from zero. On the other hand, while the positive threshold γ_1 for Ethereum is significantly different from zero, the confidence region of the negative threshold γ_2 for Ethereum overlaps with zero, suggesting the lower threshold of Ethereum is not significantly different from zero and possibly it lies around zero, meaning that the Kimchi Premium for Ethereum is mean reverting as soon as it drops below 0.

For Bitcoin and Ethereum, regardless of which control variables are used, we find that the coefficient estimates for different regimes are statistically significant. Furthermore, the coefficient estimates for the upper and lower regimes differ considerably showing that the mean-reverting speed is faster in the region of negative premiums. This is compatible with persistent positive Kimchi premiums compared to the negative ones. When the magnitude of the Kimchi premium for Bitcoin

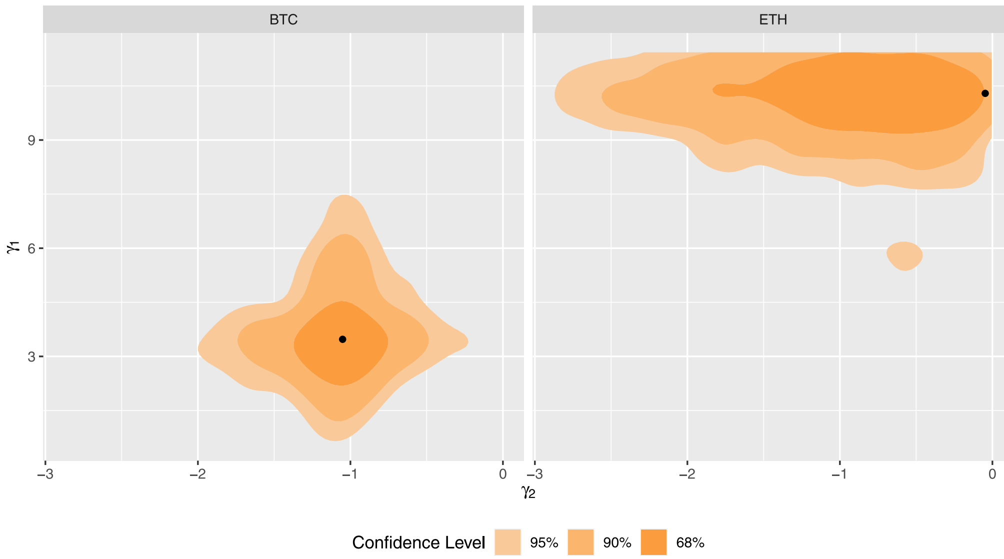

Fig. 6. Bootstrap confidence interval of the thresholds.

in the previous period falls in the region between −1.05 and 3.47 percentage, the process follows a unit root process, otherwise, it is an AR(1) with an AR coefficient of 0.96 and 0.87 in the regions of the upper (positive) and the lower (negative) premiums. In the short run, the process of Kimchi premium follows a random walk while in the long run, it tends to a long-run steady state level, 1.24%.

The estimation sheds light on the determinants for Kimchi premium. We find that the volatilities of cryptocurrency prices and turnover, market capitalisation, crypto market indices, leverage effects and the time trend have an influence on the formulation of the premium. Volatility in both the US and Korean markets make a significant impact on the premium in the opposite direction. More specifically, one percentage point change in volatility change yields a higher Kimchi premium by 0.095 percentage points in the US market while the same amount decreases the premium by 0.103 percentage points in the Korean market. If we view the Kimchi premium as a bubble, for the period of higher change in volatility in the US market, the price in the US market will quickly and better reflect the true value of the cryptocurrency, while the price in the Korean market adjusts slower, resulting in a widened gap. When the volatility in the Korean market increases eventually, the gap becomes smaller as it adjusts to the true level with a smaller bubble.

In addition, exchange rates are denoted as Korean won against the US dollar and hence the depreciation of the Korean won (increase in the exchange rates) incurs the selling of Bitcoin in the Korean market for purchase of Bitcoin in the US market leading to the decrease in Kimchi premium, which is compatible with its significant negative coefficient for Bitcoin. Turnover in the Korean market has a significant positive impact compatible with the Kimchi premium as a bubble in Eom (2021). The negative return in the previous period ($Return_{t-1} < 0$) incurs a significant negative impact on the premium which can be related to either the leverage effect or the premium as a bubble. The negative impact of market capitalisation is compatible with the positive impact of turnover since turnover is calculated as trading volume divided by market capitalisation.

We find similar results for the dynamics of Kimchi premiums associated with Ethereum (Column (4)). The process follows a unit root process when the Kimchi premium for Ethereum falls in the region between −0.05 and 10.29 percentage, otherwise, it is an AR(1) with an AR coefficient of 0.94 and 0.88 in the regions of the upper (positive) and the lower (negative) premiums. Again, the mean-reverting traction is stronger in the region of negative premiums less than the lower threshold compared to the positive premiums. While almost all the determining factors that are significant in the model of Bitcoin are still significant in the model of Ethereum, illiquidity in either the US market or the Korean market does not seem to have any impact on the Kimchi premium of Bitcoin but it (in both markets) does have an effect on the premium for Ethereum, which is due in part to less liquid Ethereum market compared to much liquid Bitcoin market as can be seen from illiquidity variables for Bitcoin and Ethereum in Tables 1 and 2.

For the scenario without any control variable, the aforementioned results for regime related coefficients apply but the fit is much weaker than the scenario where relevant control variables are included and the difference between the upper and the lower thresholds, i.e. the middle regime is much wider with the confidence region for the threshold variables being much wider as well.

3.3. Long-run steady state for Kimchi premium

As seen from Table 4, we report the long run steady states of Kimchi premiums of Bitcoin and Ethereum for the model specifications under consideration. Our results show the presence of persistent positive Kimchi premiums, which could be understood as the violation of the law of one price. The long run steady state implied by an estimated nonlinear dynamic models such as our threshold regression is nontrivial to derive analytically. Following Taylor et al. (2001) and Bec et al. (2010) based on Monte Carlo simulations, we construct the long-run steady-state level for Kimchi premium as follows. Based on the model (3.1) and estimates in Table 4, we simulate the process for the long horizon T, which is long enough for the Kimchi premium to stabilise for multiple repetitions M. Hence, we can obtain M number of long-run levels of Kimchi premium over which we average to obtain an estimate of the long-run steady state. The value of other variables are taken to be their sample means. We set $T = 2000$ and $M = 500$ and the predicted long-run steady-state levels for Kimchi premium of Bitcoin and Ethereum with all the control variables included are 1.24% and 2.56% respectively.

3.4. Half-lives

We also compute the half-lives resulting from the four models estimated above. Note that the half-lives and the long-run steady state are akin properties of the same process. As is the case with the aforementioned long run steady state, the procedure resembles that of Bec et al. (2010). Similar to the calculation of long-run steady state,

Table 4
Estimation result.

	BTC (1)	BTC (2)	ETH (3)	ETH (4)
$y_{t-1}\mathbf{1}(y_{t-1} \geq \hat{\gamma}_1)$	−0.090***	−0.038***	−0.096***	−0.055***
	(0.013)	(0.008)	(0.014)	(0.012)
$y_{t-1}\mathbf{1}(y_{t-1} < \hat{\gamma}_2)$	−0.135***	−0.132***	−0.140***	−0.121***
	(0.032)	(0.023)	(0.031)	(0.027)
ΔTrading volume US		−0.167		−0.169
		(0.168)		(0.198)
ΔTrading volume KR		−0.074		−0.046
		(0.087)		(0.102)
ΔVolatility US		0.095***		0.055***
		(0.015)		(0.015)
ΔVolatility KR		−0.103***		−0.041**
		(0.018)		(0.017)
ΔIlliquidity US		1860		1858**
		(3285)		(912)
ΔIlliquidity KR		−22.581		−7.857
		(17.821)		(4.866)
ΔTurnover US		0.005		0.003
		(0.011)		(0.005)
ΔTurnover KR		6.771***		1.576***
		(1.595)		(0.478)
ΔExchange rate		−0.033***		−0.024***
		(0.004)		(0.005)
ΔMarket capitalisation US$bn		−0.075***		−0.096***
		(0.003)		(0.005)
ΔDigital Asset Fear and Greed		−0.041***		−0.065***
		(0.005)		(0.006)
ΔUBMI		0.009***		0.004***
		(0.0004)		(0.0003)
ΔMomentum Top 5		−0.004***		−0.0003
		(0.0003)		(0.0004)
ΔLow Volatility Top 5		−0.001***		0.0000
		(0.0001)		(0.0001)
$Return_{t-1} < 0$		−0.132***		−0.142***
		(0.039)		(0.047)
Time		0.0001**		0.0001*
		(0.00004)		(0.0001)
Constant	−0.039	−0.031	−0.046	−0.046
	(0.031)	(0.046)	(0.033)	(0.058)
$\hat{\gamma}_1$	10.71	3.47	10.99	10.29
$\hat{\gamma}_2$	−0.03	−1.05	−0.04	−0.05
Long run equilibrium	2.87	1.24	2.7	2.55
R^2	0.042	0.515	0.043	0.379
Adjusted R^2	0.041	0.510	0.042	0.371

* p<0.1.
** p<0.05.
*** p<0.01.
Notes: a. Columns (1) and (2) for Bitcoin and Columns (3) and (4) for Ethereum; b. Columns (1) and (3) for a scenario without any control variable and Columns (2) and (4) for a scenario with a set of potential control variables included; c. Δ denotes the differenced variables.

Table 5
Half-lives (days).

Shock size (%)	BTC (1)	BTC (2)	ETH (3)	ETH (4)
1	23.77	19.6	21.19	26.62
5	23.46	18.8	18.4	23.41
20	22.29	18.46	20.49	26.31
40	22.77	18.11	21.36	27.31

we simulate from the model (3.1) based on estimates in Table 4. For each model specification, we generate two series of length $T = 100$ with identical errors except that the first series has an extra additive shock at the beginning. The value of the shock is chosen to be 40%, 20%, 5%, 1% of the initial value of Kimchi premium. The initial value is the first value of the sample period. Note the initial values of both Bitcoin and Ethereum are positive, so the shocks are also positive. Then, we calculate the difference between the two series and repeat this procedure 1000 times. The average of the difference between the two series at each time point forms our estimate of the impulse response function. The half-life is chosen to be the minimum number of periods for the impact of the shock to reach half of the initial impact. Since we can only observe data in the discrete time unit of days but the impact can be reduced by half at any time, interpolation between the first day that the impact drops below half and the previous day is used to find the half-life in between. The result can be found in Table 5.

The half-lives for Bitcoin seem to be less variant to different shock sizes. The exception is when the shock size is only 1% of the initial value of the series. In this scenario, the half-life tends to be longer. This is because the initial shock is so weak that the premium falls in the random walk region specified in the model. When the shock is so small that the Kimchi Premium is not moving outside of the random walk region, there is no mean-reverting force to calm the shock down to zero, so the shock can last longer. The same observation can be made for Ethereum. In addition, apart from the smallest shock, the larger the shock, the longer the half-life for Ethereum. We can draw a connection between half-lives and the thresholds. Comparing the process BTC(1) and BTC(2), the half-lives are longer with a wider random walk region when no control variables are added. This is because when the threshold is larger, it is easier for the Kimchi premium to move freely inside the random walk region, without being pulled back to zero, resulting in a longer period for the impact of the shock to be halved.

3.5. Fees and Kimchi premium

Kimchi premium can be considered as an outcome of the presence of market friction. Trading fees are considered as one form of market friction and therefore, we examine whether trading fees could determine the level of Kimchi premium threshold value that determines the type of regimes. To this end, we employ the threshold regression with a threshold index proposed in Seo and Linton (2007) to investigate the potential relationship between trading fees and regimes. It allows for the unknown threshold to depend on other observable variables and has been extended by Lee et al. (2021) and Lee and Wang (2023) to more flexible functional forms.

Unlike traditional financial assets, two types of fees are of major concern in cryptocurrency trading: miner fee and service fee. The miner fee is charged by miners, who are providers of specialised computer hardware that confirm and finalise transactions of cryptocurrency on the corresponding network. It is not a fixed rate but fluctuates depending on the number of transactions waiting to be processed at a specific time. Higher miner fees imply an increased "demand" (crypto-transactions waiting to be processed) and a relatively reduced "supply" (miners available to process these transactions). Consequently, transactions initiated by investors will experience slower settlement speeds due to a shortage of "settlers" (miners). Essentially, the increased miner fees, regardless of the location of the exchange, can be seen as heightened market friction, which impedes arbitrage trading. More specifically, for investors to seize arbitrage opportunities, they need to trade in both markets. However, higher miner fees, as a result of the interplay between the aforementioned supply and demand, reduce the incentive of arbitrageurs to trade, which then causes an enlarged random walk region. The service fee, on the other hand, is charged by third-party service providers (e.g. the exchange) on top of the miner fee. It is often a fixed amount levied on the transaction value for a given exchange. Although this amount is different across different exchanges, the service fees of different exchanges change in the same direction over time, thus making the relative difference close to being constant. This means the effect of differences in the service fee contributes to the same impact towards Kimchi premiums over time and can be captured by the constant intercept term in Model (3.1). In addition, as investors recognise that the service fee only depends on the transaction amount, their willingness to arbitrage does not change over time because of the service fee. We focus our analysis on the miner fee for this reason.

The miner fee rate is calculated as the total daily fee (collected from blockchain.com) divided by the total volume of both markets and

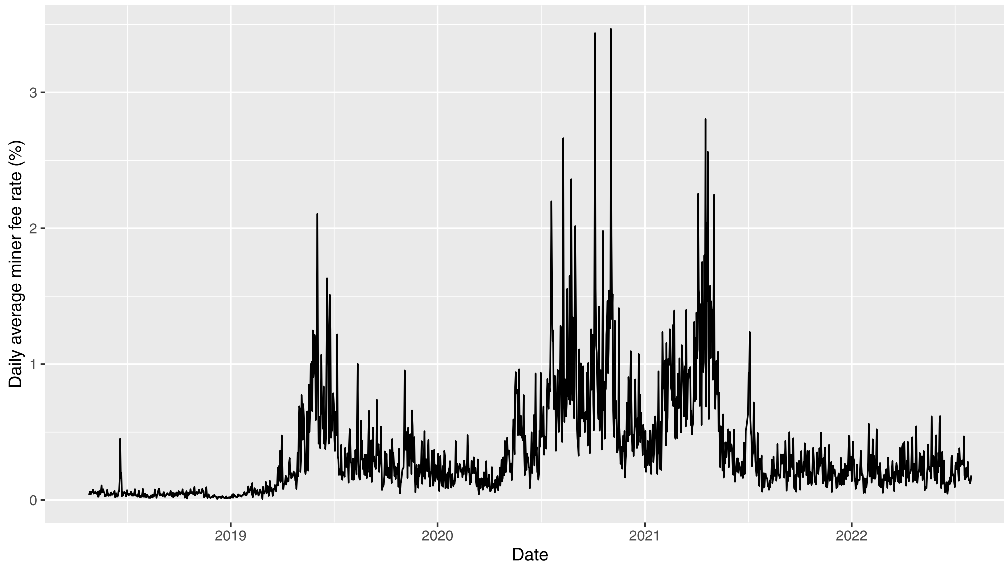

Fig. 7. Daily average miner fee rate.

Table 6
Model estimation when thresholds depend on the miner fee rate.

Variable	Estimate	Standard error	P-value	Confi. int. lower	Confi. int. higher
Constant	−0.016	0.046	0.720	−0.106	0.073
$y_{t-1}\mathbf{1}(\lvert y_{t-1}\rvert \geq \hat{\gamma}_{11} + \hat{\gamma}_{12} z_t)$	−0.042	0.008	0.000	−0.057	−0.027
$y_{t-1}\mathbf{1}(\lvert y_{t-1}\rvert < \hat{\gamma}_{21} + \hat{\gamma}_{22} z_t)$	−0.121	0.023	0.000	−0.166	−0.075
ΔTrading volume US	−0.158	0.168	0.346	−0.487	0.171
ΔTrading volume KR	−0.081	0.087	0.351	−0.252	0.090
ΔVolatility US	0.093	0.015	0.000	0.064	0.123
ΔVolatility KR	−0.101	0.018	0.000	−0.136	−0.066
ΔIlliquidity US	1924	3282	0.558	−4514	8363
ΔIlliquidity KR	−22.978	17.807	0.197	−57.907	11.950
ΔTurnover US	0.005	0.011	0.609	−0.015	0.026
ΔTurnover KR	6.778	1.593	0.000	3.653	9.904
ΔExchange rate	−0.033	0.004	0.000	−0.041	−0.025
ΔMarket capitalisation US$bn	−0.075	0.003	0.000	−0.080	−0.070
ΔDigital Asset Fear and Greed	−0.041	0.005	0.000	−0.050	−0.031
ΔUBMI	0.009	0.000	0.000	0.008	0.010
ΔMomentum Top5	−0.004	0.000	0.000	−0.004	−0.003
ΔLow Volatility Top5	−0.001	0.000	0.000	−0.001	−0.001
$Return_{t-1} < 0$	−0.127	0.039	0.001	−0.204	−0.050
Time	0.000	0.000	0.009	0.000	0.000

plotted in Fig. 7. It is an approximation of the real miner fee rate since part of trading is made outside of exchanges, for example in the over-the-counter (OTC) market. The actual miner fee rate would be lower but the approximation is a good indicator of the general tendency.

Consider the threshold regression model following Seo and Linton (2007):

$$\begin{aligned} Kimp_t =& \beta_0 + Kimp_{t-1} + \delta_1 Kimp_{t-1} 1\left\{Kimp_{t-1} > \gamma_{11} + \gamma_{12} z_t\right\} \\ &+ \delta_2 Kimp_{t-1} 1\left\{Kimp_{t-1} < \gamma_{21} + \gamma_{22} z_t\right\} + \rho' control_t + e_t \end{aligned} \tag{3.2}$$

where z_t is the daily average miner fee rate. We estimate the above model for Bitcoin using the grid search following Seo and Linton (2007) and its results are given in Fig. 8, Tables 6 and 7. The grid search is constrained such that the threshold with parameters γ_{11} and γ_{12} is above zero when evaluated at every sample point of miner fee rate and the threshold with parameters γ_{21} and γ_{22} is below zero. This is according to the assumption that the Kimchi premium follows a unit root process in a region around zero, even though this region can change over different values of the miner fee rate.

In the left panel of Fig. 8 and Table 7, we can see that the positive threshold has a positive slope ($\hat{\gamma}_{12} = 325.5$) on the miner fee rate, and from Table 6 we can see that the Kimchi premium is indeed mean reverting with a coefficient of 0.96. This means the higher the miner fee rate, the larger the unit root region, and the more likely a higher Kimchi premium persists. This can be explained by the incentives of arbitrageurs. For the Kimchi premium to present a mean reverting behaviour, investors will need to recognise and take advantage of the arbitrage opportunity, and trade in both markets. When the miner fee goes up, they would be less willing to do so, thus resulting in a wider unit root region where the Kimchi premium moves freely.

The negative threshold, on the other hand, has a much smaller slope in absolute term and seems almost flat. This means the unit root region is not symmetric around zero. When the Kimchi Premium drops below the negative threshold, it will move towards 0 with a coefficient of 0.879, faster than when the Kimchi Premium is above the positive threshold. This is consistent with the observation that the Kimchi premium tends to be positive in the long run.

4. Conclusion

On January 10, 2024, the U.S. Securities and Exchange Commission (SEC) approved the listing and trading of several spot bitcoin exchange-traded product (ETP) shares. This signifies a significant step towards cryptocurrencies becoming an accessible investment asset class for ordinary investors. However, unlike other asset classes, research on

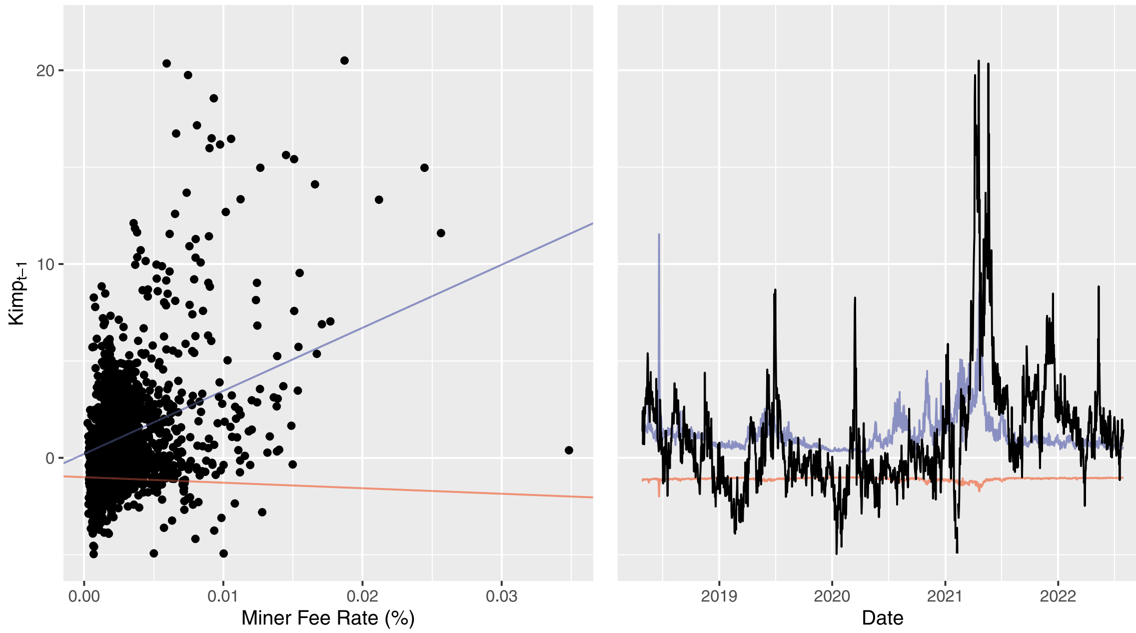

Fig. 8. Daily average miner fee rate, Kimchi premium and the thresholds. In the left panel, the Kimchi Premium at lag 1 is plotted against the Daily average miner fee rate. The positive threshold (blue) and negative threshold (red) are functions of the miner fee rate. In the right panel, the Kimchi premium of Bitcoin, the positive threshold (blue) and the negative threshold (red) are plotted over time. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 7
Estimated parameters and statistics.

Name	Value
$\hat{\gamma}_{11}$	0.2
$\hat{\gamma}_{12}$	325.5
$\hat{\gamma}_{21}$	−1
$\hat{\gamma}_{22}$	−28.5
Observations	1556
R^2	0.516
Adjusted R^2	0.510

the features of cryptocurrencies as investment assets is still in its early stages.

Against this backdrop, we study "Kimchi premium" and its dynamics and investigate its determinants. Our empirical study identifies the change in volatilities of cryptocurrency prices, turnover, exchange rate, market capitalisation, crypto market indices, leverage effects and trends driving Kimchi premium. Contrary to existing literature based on linear modelling, the presence of nonlinearity in the dynamics of Kimchi premium is apparent. Specifically, the Kimchi premium behaves like a random walk process when its value falls within an interval around zero, yet exhibits a mean reverting pattern when its value exceeds a positive threshold or falls below a negative threshold, where the two thresholds are asymmetric around zero. The negative threshold is closer to zero and the region below the negative threshold shows a stronger mean reverting force compared to the positive threshold. Consequently, this nonlinear dynamic exhibits a persistent positive long-run steady state, which indicates the violation of the law of one price.

As a form of market friction, we identify that the miner fee, as a type of trading fee specific to cryptocurrency transaction, influences arbitrage trading in cryptocurrency markets. As the miner fee goes up, the random walk region becomes wider, particularly because the positive threshold becomes larger, instead of the negative threshold. This observation is consistent with a positive long run level, and confirms the role of market friction in the arbitrage trading.

While our focus is on univariate time series modelling, exploring multivariate time series modelling and its dynamics could offer insights into features of cryptocurrency markets, such as spillovers and network relationships among various cryptocurrencies. We leave this as a topic for future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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